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CONSTRUCTION OF TRANSIENT IN MECHANICAL SYSTEMS

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ABSTRACT

The transient in a system containing a linear oscillator, linearly coupled to an essentially nonlinear attachment with a comparatively small mass, is considered. A damping is taken into account. A transfer of energy from the initially perturbed linear subsystem to the nonlinear absorber is observed. The modified multiple scales method is used to construct a process of transient in the system under consideration. Numerical simulation confirms an efficiency of the analytical construction. A similar construction is made to describe the transient in a system which contains a linear oscillator and a vibro-impact attachement with a comparatively small mass. A transient in such system under the external periodical excitation is considered too.

INTRODUCTION

An investigation of transient is important in engineering, in particular, in problem of absorption. Over the past years different new devices have been used for the vibration absorption and for the reduction of the transient response of structures [1-5 et al.]. It seems useful to study nonlinear passive absorbers for this reduction.

In presented paper the transient in a system containing a linear oscillator, linearly coupled to an essentially nonlinear attachment with a comparatively small mass, is considered. A damping is taken into account. It is assumed that some initial excitation implies vibrations of the linear oscillator. The multiple scales method [6] is used to construct a process of transient in some nonlinear systems. A transfer of energy from the initially perturbed linear subsystem to the nonlinear attachment is observed. A similar construction is made to describe the transient in a system which contains the main linear subsystem and a vibro-impact absorber with a comparatively small mass. Both an exact integration with regards to conditions of impact, and the multiple scales method are used for this construction. The transient in such system under the external periodical excitation is considered too. Numerical simulation confirms an efficiency of the analytical construction in all considered systems.

1. TRANSIENT IN A SYSTEM CONTAINING AN ESSENTIALLY NONLINEAR ATTACHMENT

Let us consider a system with two connected oscillators, namely, one linear and one nonlinear with a comparatively small mass, which can be considered as absorber of the linear oscillator vibrations (Fig.1). Here M is a mass of the main linear subsystem, m is a mass of the nonlinear attachment, ω^2, γ and c characterize elastic springs, δ characterizes the linear dissipation force. To emphasis a smallness of some inertial and elastic characteristics of the attachment, as well a smallness of the dissipation force, the next transformations will be used: $m \to \varepsilon m$, $c \to \varepsilon c$, $\gamma \to \varepsilon\gamma$, $\delta \to \varepsilon^2\delta$, where ε is the small parameter ($\varepsilon << 1$). So, the system under consideration is described by the following differential equations:

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$$\begin{cases} \varepsilon m\ddot{x} + \varepsilon c x^3 + \varepsilon^2 \delta \dot{x} + \varepsilon\gamma(x-y) = 0, \\ M\ddot{y} + \omega^2 y + \varepsilon^2 \delta \dot{y} + \varepsilon\gamma(y-x) = 0, \end{cases} \quad (1)$$

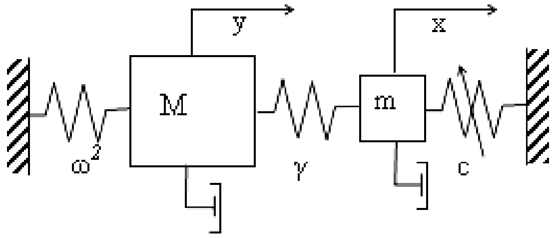

Fig. 1. The syste m with two connected oscillators

The solution of the system (1) will be found by the multiple-scale method. One has

$$x = x_0(t_0,t_1,t_2,\ldots) + \varepsilon x_1(t_0,t_1,t_2,\ldots) + \varepsilon^2 x_2(t_0,t_1,t_2,\ldots) + \ldots$$

$$y = y_0(t_0,t_1,t_2,\ldots) + \varepsilon y_1(t_0,t_1,t_2,\ldots) + \varepsilon^2 y_2(t_0,t_1,t_2,\ldots) + \ldots, \quad (2)$$

where

$$t_0 = t, t_1 = \varepsilon t, t_2 = \varepsilon^2 t, \ldots, t_n = \varepsilon^n t, \ldots, \quad \frac{d}{dt} = \frac{\partial}{\partial t_0}\frac{dt_0}{dt} + \frac{\partial}{\partial t_1}\frac{dt_1}{dt} + \frac{\partial}{\partial t_2}\frac{dt_2}{dt} + \ldots =$$

$$= \frac{\partial}{\partial t_0} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2} + \varepsilon^3\frac{\partial}{\partial t_3} + \ldots = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \varepsilon^3 D_3 + \ldots$$

One obtains in zero approximation by the small parameter the next equation:

$$\varepsilon^0 : MD_0^2 y_0 + \omega^2 y_0 = 0.$$

The solution of this equation is the following:

$$y_0 = A_1(t_1,t_2,\ldots)\cos\psi_0, \quad \text{where } \psi_0 = \Omega t_0 + \varphi_0(t_1,t_2,\ldots), \quad \Omega^2 = \frac{\omega^2}{M}.$$

One has in the next approximation by the small parameter the following ODE system:

$$\varepsilon^1 : \begin{cases} mD_0^2 x_0 + c x_0^3 + \gamma(x_0 - y_0) = 0, \\ MD_0^2 y_1 + 2MD_0 D_1 y_0 + \omega^2 y_1 + \gamma(y_0 - x_0) = 0. \end{cases} \quad (3)$$

The presentation of the x_0 in the essentially nonlinear system of the zero approximation is chosen of the form

$$x_0 = B_1(t_1,t_2,\ldots)\cos\psi_0 + B_2(t_1,t_2,\ldots)\cos\psi_1$$

where $\psi_1 = \bar{\Omega}(t_1,t_2,\ldots)t_0 + \varphi_1(t_1,t_2,\ldots)$.

Equating cosine coefficients in the first equation and eliminating secular terms in the second one, we get nonlinear functional equations of the form:

$$\begin{cases} -mB_1\Omega^2 + c\left(\tfrac{3}{4}B_1^3 + \tfrac{3}{2}B_1B_2^2\right) + \gamma B_1 = \gamma A_1 \\ \gamma - m\bar{\Omega}^2 + \tfrac{3}{4}cB_2^2 + \tfrac{3}{2}cB_1^2 = 0 \end{cases}, \quad \begin{cases} 2MA_1\Omega\dfrac{\partial\varphi_0}{\partial t_1} + \gamma B_1 - \gamma A_1 = 0 \\ \dfrac{\partial A_1}{\partial t_1} = 0 \end{cases} \quad (4)$$

Thus $A_1 = A_1(t_2, t_3, \ldots)$, $\dfrac{\partial \varphi_0}{\partial t_1} = \dfrac{\gamma(A_1 - B_1)}{2MA_1\Omega}$.

Escaping calculations of the next approximations in the multiple scale method we give expressions for the amplitudes, frequencies and phases of zero-approximation x_0, y_0 of (2), namely:

$$B_2 = \overline{c}(t_2, t_3, \ldots)e^{-\frac{\delta}{2m}t_1}, \quad B_1 = c_0(t_2, t_3, \ldots) + c_2(t_2, t_3, \ldots)e^{-\frac{\delta}{m}t_1},$$

$$A_1 = \frac{\gamma - m\Omega^2}{\gamma}c_0 + \frac{3}{4\gamma}cc_0^3, \quad \overline{\Omega}^2 = (\gamma + (3/4)cB_2^2 + (3/2)cB_1^2)/m.$$

After time-averaging one has the following: $\overline{\Omega}^2 \cong (\gamma + (3/2)cc_0^2)/m$,

$$\varphi_0 = \frac{\gamma}{2M\Omega}t_1 - \frac{\gamma}{2M\Omega A_1}\left(c_0 t_1 - c_2\frac{m}{\delta}e^{-\frac{\delta}{m}t_1}\right) + c_2^*, \qquad \text{where } c_2 = \frac{\frac{3}{2}c\overline{c}^2 c_0}{m\Omega^2 - \gamma - \frac{9}{4}cc_0^2}.$$

In such a way we have got the zero-approximation of sought solution containing four constants with respect to the variable t_0, namely,

$$c_1^* = c_1^*(t_3, t_4, \ldots), c_2^* = c_2^*(t_2, t_3, \ldots), c_3^* = c_3^*(t_2, t_3, \ldots), \overline{c} = \overline{c}(t_2, t_3, \ldots)$$

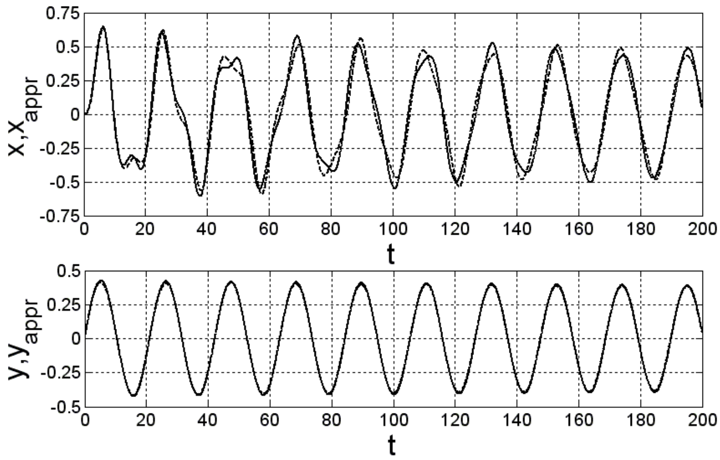

Fig. 2a.

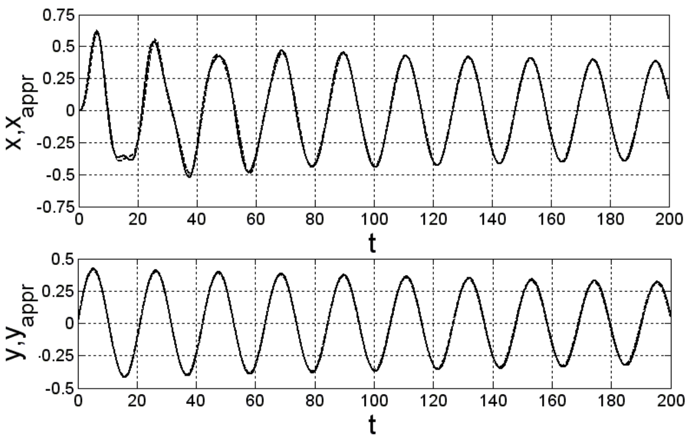

Fig. 2.b

Fig. 2. Comparison of results of analytical approximation (solid line) and ones obtained by Runge-Kutta procedure (dash line)

They were found numerically by Newton method from initial conditions:

$$x(0)=\dot{x}(0)=0\,,\ y(0)=0, \dot{y}(0)=V\ ,$$

which describe the impact initial excitation of vibrations in the linear subsystem.

In the Figure 2 results of comparing of the analytical solution with the numerical simulation obtained by using the Runge-Kutta procedure for different initial values are shown.

2. TRANSIENT IN THE VIBRO-IMPACT SYSTEM

One considers the 2-DOF vibro-impact system with the one-sided catch (Fig.3). This system contains the linear oscillator and the attachment with a comparatively small mass. It is presupposed to obtain analytical description of transient, both for free and forced oscillations, by using the multiple-scale method.

Equations of motion for the system under consideration in a case of the free vibrations are the following:

$$\begin{cases} \varepsilon m\ddot{x}+\varepsilon\gamma(x-y)+\varepsilon^2\delta\dot{x}=0; \\ M\ddot{y}+c^2y+\varepsilon\gamma(y-x)+\varepsilon^2\delta\dot{y}=0, \end{cases} \tag{5}$$

where all notations and transformations of parameters are the same as in the Section 2. The small parameter ($\varepsilon<<1$) is introduced to select a smallness of the attachment mass, the connection between oscillators and the dissipation force.

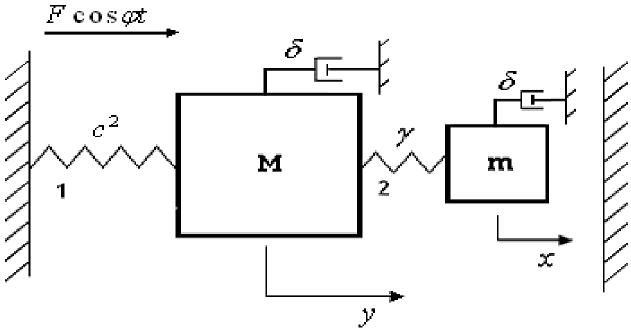

Fig. 3. The vibro-impact system under consideration

It is presupposed that an impact here is instantaneous. The restoration coefficient ($0\le e\le 1$) characterizes a lost of velocity in the instant of impact. One has the following conditions of the impact: $x(t_k^{\ +})=x(t_k^{\ -})=x_{\max}$, $\dot{x}(t_k^{\ +})=-e\dot{x}(t_k^{\ -})$, $y(t_k^{\ +})=y(t_k^{\ -})$, $\dot{y}(t_k^{\ +})=\dot{y}(t_k^{\ -})$.

Here: t_k is the impact instant, where k is a number of the impact; t_k^- is an instant before impact, t_k^+ is one after impact, $x_{\max}$ is a distance between the equilibrium state and the catch.

2.1 Free oscillations in vibro-impact systems

To construct an analytical solution by using the multiple scale method, the expansions (2) are used. In zero approximation by small parameter the next solution can be obtained:

$$y_0=A_0(t_1,t_2,t_3,...)\cos\Omega t_0+B_0(t_1,t_2,t_3,...)\sin\Omega t_0\ ,$$

where $\Omega_0^2=c^2/M$; $\begin{aligned} x_0&=\beta(A_0(t_1,...)\cos\Omega t_0+B_0(t_1,...)\sin\Omega t_0)+ \\ &+A_1(t_1,...)\cos\sqrt{\gamma/m}t_0+B_1(t_1,...)\sin\sqrt{\gamma/m}t_0, \end{aligned}$ $\beta=\dfrac{\gamma}{m(\gamma/m-\Omega_0^2)}$.

Conditions of secular terms elimination in the next approximation by the small parameter give us the following expressions for amplitudes of the zero approximation:

$$A_0=-C_1\sin\Omega_1t_1+C_2\cos\Omega_1t_1\,,\ B_0=C_1\cos\Omega_1t_1+C_2\sin\Omega_1t_1\,,$$

where $\Omega_1=\dfrac{\gamma(\beta-1)}{2M\Omega_0}$.

Taking onto account the next approximation, one has the approximate solution of the form:

$$x=\beta(\cos\Omega_2 t\cdot(-R_1C_1+R_2C_2)+\sin\Omega_2 t\cdot(R_2C_1+R_1C_2))+e^{\alpha\varepsilon t}\left\{C_3\sin\beta_3 t+C_4\cos\beta_3 t\right\},$$
$$y=C_1\sin\Omega_2 t+C_2\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{C_3\sin\beta_3 t+C_4\cos\beta_3 t\right\}, \tag{6}$$
$$R_1=\frac{\varepsilon\delta\Omega}{m\left(\gamma/m-\Omega^2\right)},\ R_2=1-\frac{2\varepsilon\Omega\Omega_1}{\gamma/m-\Omega^2},\ \beta_3=\sqrt{\gamma/m}-\beta_2\varepsilon,\ \Omega_2=\Omega-\varepsilon\Omega_1.$$

Impact conditions (4) give the next relations connecting coefficients C_i before (C_i^k) and after impact (C_i^{k+1}):

$$\begin{aligned}
&\beta(\cos\Omega_2 t\cdot(-R_1C_1^{k+1}+R_2C_2^{k+1})+\sin\Omega_2 t\cdot(R_2C_1^{k+1}+R_1C_2^{k+1}))+\\
&+e^{\alpha\varepsilon t}\left\{C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\right\}=\beta(\cos\Omega_2 t\cdot(-R_1C_1^k+R_2C_2^k)+\\
&+\sin\Omega_2 t\cdot(R_2C_1^k+R_1C_2^k))+e^{\alpha\varepsilon t}\left\{C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\right\}.\\
&\Omega_2\beta\left(-\sin\Omega_2 t\cdot(-R_1C_1^{k+1}+R_2C_2^{k+1})+\cos\Omega_2 t\cdot(R_2C_1^{k+1}+R_1C_2^{k+1})\right)+\\
&+e^{\alpha\varepsilon t}\left(\alpha\varepsilon\left\{C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\right\}+\beta_3\left\{C_3^{k+1}\cos\beta_3 t-C_4^{k+1}\sin\beta_3 t\right\}\right)=\\
&=-\mathrm{e}\Omega_2\beta\left(-\sin\Omega_2 t\cdot(-R_1C_1^k+R_2C_2^k)+\cos\Omega_2 t\cdot(R_2C_1^k+R_1C_2^k)\right)+\\
&+e^{\alpha\varepsilon t}\left(\alpha\varepsilon\left\{C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\right\}+\beta_3\left\{C_3^k\cos\beta_3 t-C_4^k\sin\beta_3 t\right\}\right)\\
&C_1^{k+1}\sin\Omega_2 t+C_2^{k+1}\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\right\}=\\
&=C_1^k\sin\Omega_2 t+C_2^k\cos\Omega_2 t+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\right\}\\
&\Omega_2\left(C_1^{k+1}\cos\Omega_2 t-C_2^{k+1}\sin\Omega_2 t\right)+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{\alpha\varepsilon\left(C_3^{k+1}\sin\beta_3 t+C_4^{k+1}\cos\beta_3 t\right)+\right.\\
&\left.+\beta_3\left(C_3^{k+1}\cos\beta_3 t-C_4^{k+1}\sin\beta_3 t\right)\right\}=\\
&=\Omega_2\left(C_1^k\cos\Omega_2 t-C_2^k\sin\Omega_2 t\right)+\varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{\alpha\varepsilon\left(C_3^k\sin\beta_3 t+C_4^k\cos\beta_3 t\right)+\right.\\
&\left.+\beta_3\left(C_3^k\cos\beta_3 t-C_4^k\sin\beta_3 t\right)\right\}.
\end{aligned} \tag{7}$$

The numeric simulation is realized for the next values of parameters: M=1, m=1, ε =0.01, δ=10, e=0.9, $x_{\max}$ =1.4, γ =1.5, c=1. Initial values simulate the instant impact to the linear subsystem: $x(0)=0,\ \dot{x}(0)=0,\ y(0)=0,\ \dot{y}(0)=\dot{V}_0=1$. Comparison of the analytical solution and numerical simulation shows a good exactness of the analytical approximation (Fig.4).

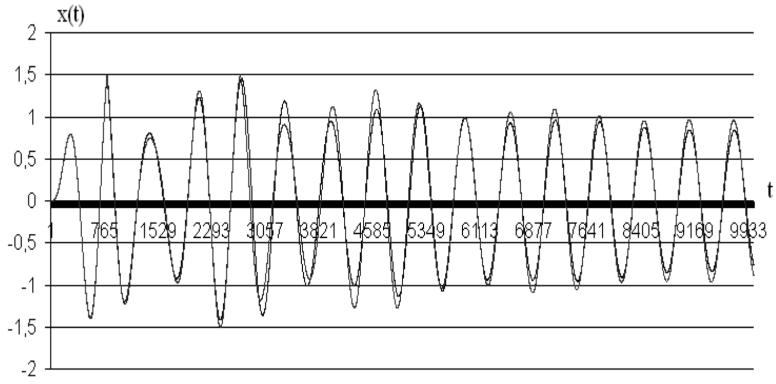

Fig. 4. Transient in a case of free oscillations in the vibro-impact system

2.2 Transient in a case of forced oscillations

One considers the same 2-DOF vibro-impact system in a case when an external periodic force acts to linear subsystem. The multiple scales method can be successfully used in this case too. In contrast with the solution, obtained in the sub-section 3.1, the part corresponding to the external excitation, has to be added. One writes the solution of the form:

$$x = \beta(\cos\Omega_2 t \cdot (-R_1 C_1 + R_2 C_2) + \sin\Omega_2 t \cdot (R_2 C_1 + R_1 C_2)) +$$
$$+ e^{\alpha\varepsilon t}\left\{C_3 \sin\beta_3 t + C_4 \cos\beta_3 t\right\} + \left(F_2 + \varepsilon F_5\right)\cos\varphi t + \varepsilon F_6 \sin\varphi t,$$
$$y = C_1 \sin\Omega_2 t + C_2 \cos\Omega_2 t + \varepsilon\beta_1 e^{\alpha\varepsilon t}\left\{C_3 \sin\beta_3 t + C_4 \cos\beta_3 t\right\} +$$
$$+ \left(F_1 + \varepsilon F_3\right)\cos\varphi t + \varepsilon F_4 \sin\varphi t, \tag{8}$$

where $F_1 = \dfrac{F}{\left(\Omega^2 - \varphi^2\right)}$, $F_2 = \dfrac{\gamma F_1}{m\left(\gamma/m - \varphi^2\right)}$, $F_3 = \dfrac{-\gamma(F_1 - F_2)}{M\left(\Omega^2 - \varphi^2\right)}$, $F_4 = \dfrac{2\varphi F_1}{\Omega^2 - \varphi^2}$,

$F_5 = \dfrac{\gamma F_3}{m\left(\gamma/m - \varphi^2\right)}$, $F_6 = \dfrac{\dfrac{\gamma}{m} F_4 + \left(2 + \dfrac{\delta}{m}\right) F_2 \varphi}{\gamma/m - \varphi^2}$.

Impact conditions (4) give some relations connecting coefficients C_i before (C_i^k) and after impact (C_i^{k+1}). These relations are not presented here.

Numerical simulation was made for the same parameters and initial values, as in the preceding sub-section. Comparison of the analytical solution and numerical simulation (Fig.5) shows a good exactness of the obtained analytical approximation. Vibrations of the linear subsystem with big mass are presented in the Fig. 5.

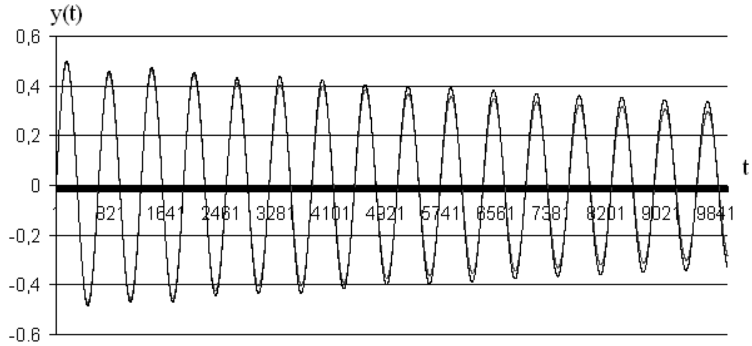

Fig. 5. Transient in a case of forced vibrations in the vibro-impact system.

CONCLUSIONS

It is shown an efficiency of the multiple-scales method to describe a transient in essentially nonlinear finite-DOF systems.

REFERENCES

[1] *Vibrations in engineering (K.V.Frolov, editor),* Mashinostroenie, Moscow, 1995 (in Russian).

[2] J. Shaw, S.Shaw, A.J.Haddow. On the response of the non-linear vibration absorber, *Int. Journal of Nonlinear Mechanics,* Vol. 24, pp. 281-293, 1989.

[3] M.P. Karyeaclis, T.K. Caughey. Stability of a Semi-active Impact Damper, *Trans. of the ASME. Journal of Applied Mechanics,* Vol. 56, pp. 926-940, 1989.

[4] K. Avarmov, Yu. Mikhlin. Snap-through truss as a vibration absorber, *Journal of Vibration and Control,* Vol. 10, N 2, pp. 291-308, 2004.

[5] Yu. Mikhlin, S. Reshetnikova. Dynamical interaction of an elastic system and essentially nonlinear absorber, *Journal of Sound and Vibration,* Vol. 283, pp. 91-120, 2005.

[6] A.H. Nayfeh, D. Mook. *Nonlinear Oscillations,* John Wiley, New York, 1984.