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# DYNAMIC INSTABILITIES OF A SEISMICALLY EXCITED SHELL WITH SHAKER-SHELL INTERACTION MODELING

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ABSTRACT

The present paper is focused on the experimental and theoretical analysis of circular cylindrical shells under seismic excitation. The shell axis is vertical, it is clamped at the base and connected to a rigid body on the top; the base provides a vertical seismic excitation. The goal is to investigate the shell response when a resonant forcing is applied: the first axisymmetric mode is excited around the resonance at relatively low frequency and low amplitude of excitation. A violent resonant phenomenon is experimentally observed as well as an interesting saturation phenomenon close to the previously mentioned resonance. A theoretical model is developed to reproduce the experimental evidence and provide an explanation of the complex dynamics observed experimentally; the model takes into account geometric shell nonlinearities, electrodynamic shaker equations and the shell shaker interaction.

## INTRODUCTION

Circular cylindrical shells are important elements in many Engineering fields e.g. Aerospace, Nuclear, Civil; examples of applications are: building vaults, heat exchangers, aircraft fuselages, missile and space vehicle structures, structural and non-structural car elements, tanks, pipelines. In many fields the need of more and more efficient structures in terms of strength and weight led to a strong reduction of safe factors; one of the direct consequences of weight reduction is the increasing of vibration problems.

In order to give to the reader a complete view of the research carried out in the previous decades about topics strictly related with the present work, a deep description of the literature is given in the following.

Readers interested to deepen the literature are suggested to read Refs. [1-6]: as noted by Babcock [2], the literature regarding shell modeling is perhaps too wide as thousands papers can be found on the subject. On the other hand some topics of extreme importance need further investigations: dynamic stability, post-critical behavior, sensitivity to imperfections, nonlinear vibrations and fluid structure interaction.

Kubenko and Koval'chuk [7] published an interesting review on nonlinear problems of shells, where several results were reported about parametric vibrations; in such review the limitations of reduced order models were pointed out. Babich and Khoroshun [8] presented results obtained at the S. P. Timoshenko Institute of Mechanics of the National Academy of Sciences of Ukraine over 20 years of research; the authors focused the attention on the variational–difference methods; more than 100 papers were cited. Kubenko and Koval'chuck [9] analyzed the stability and nonlinear dynamics of shells, following the historical advancements in this field, about 190 papers were deeply commented; they suggested, among the others, the effect of imperfections as an important issue to be further investigated.

Pellicano [10] presented experimental results about violent vibration phenomena appearing in a shell with seismic excitation and carrying a rigid mass on the top. In correspondence of the resonance

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of the first axisymmetric mode, which involves mainly the translation of the top mass, a huge out of plane vibration (more than 2000g) is detected, with a relatively low excitation (about 10g).

Pellicano [11] developed a new method, based on the Nonlinear sanders Koiter theory, suitable for handling complex boundary conditions of circular cylindrical shells and large amplitude of vibrations. The method is based on a mixed expansions considering orthogonal polynomials and harmonic functions. Among the others, the method showed good accuracy also in the case of a shell connected with a rigid body; this method is the starting point of the model developed for the present research.

In the present paper, experiments are carried out on a circular cylindrical shell, made of a polymeric material (P.E.T.) and clamped at the base by gluing its bottom to a rigid support. The axis of the cylinder is vertical and a rigid disk is connected to the shell top end. In Ref. [11] this problem was fully analyzed from a linear point of view.

Here nonlinear phenomena are investigated by exciting the shell using a shaking table and a sine excitation. Shaking the shell from the bottom induces a vertical motion of the top disk that causes axial loads due to inertia forces. Such axial loads generally give rise to axial symmetric deformations; however, in some conditions it is observed experimentally that a violent resonant phenomenon takes place, with a strong energy transfer from low to high frequencies and huge amplitude of vibration. Moreover, an interesting saturation phenomenon is observed: the response of the top disk was completely flat as the excitation frequency was changed around the first axisymmetric mode resonance.

A semi-analytical approach is proposed for reproducing experimental results and giving a deeper interpretation of the observed phenomena. The shell is modeled using the nonlinear Sanders Koiter shell; in modeling the system the effect of the top disk was accounted for applying suitable boundary conditions and considering its inertial contribution; moreover, the interaction between the shell-disk and the electro-dynamic shaking table was included in the modeling. The shell displacement fields are represented by means of a mixed series (harmonic functions and orthogonal polynomials), which are able to respect exactly geometric boundary conditions; an energy approach, based on the Lagrangian equations, is used to obtain a set of ODE that represent the original system with good accuracy.

Comparisons between experiments and numerical results show a good behavior of the model, numerical analyses furnish useful explanations about the instability observed experimentally.

## 1. THE PROBLEM: DESCRIPTION AND EXPERIMENTAL RESULTS

The system under investigation is described in Figures 1 and 2; a circular cylindrical shell, made of a polymeric material (P.E.T.), is clamped at the base by gluing its bottom to a rigid support ("fixture"); the connection is on the lateral surface of the shell, in order to increase the gluing surface, see Figure 1. A similar connection is carried out on the shell top; in this case the shell is connected to a disk made of aluminium alloy, such disk is not externally constrained; therefore, it induces a rigid body motion to the top shell end.

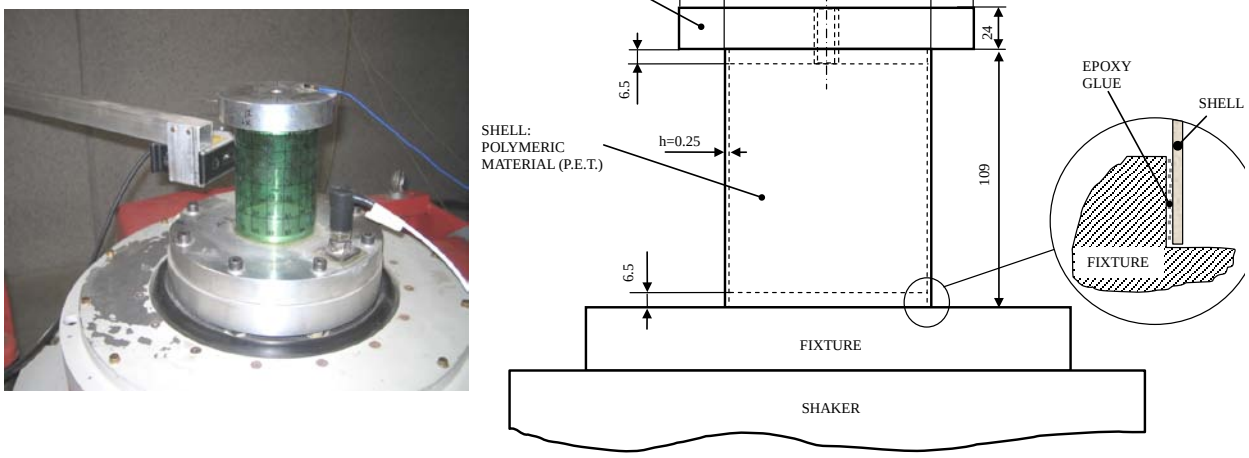


*Fig. 1 Experimental setup*

The fixture is bolted to a high power shaker (LDS V806, 13000N peak force, 100g maximum acceleration, 300kg payload, 1-3000Hz band frequency); such shaker is used to excite the shell from the base.

### 1.1 Experimental results

The behavior of the system is investigated when the base excitation is harmonic and close to the resonance of the first axisymmetric mode; indeed, in such conditions experiments, evidenced strong nonlinear responses.

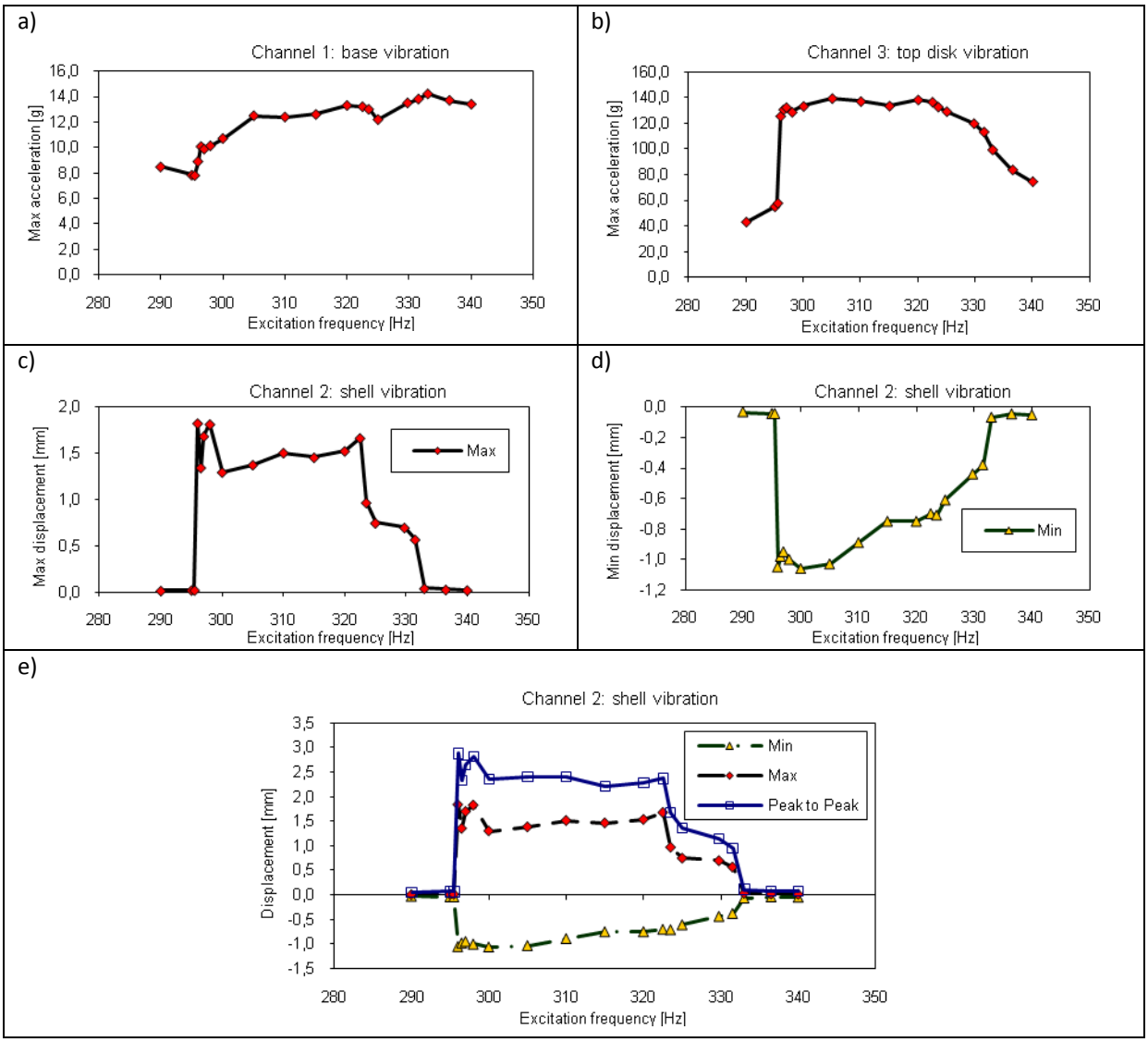


*Fig. 2 Experimental results, harmonic excitation, amplitude of vibrations: a) base excitation amplitude (acceleration [g]), b) top disk amplitude (acceleration [g]), c) response on the shell mid-span (displacement [mm], positive inward), d) minimum response of the shell mid-span (displacement [mm], negative outward), e) maximum, minimum and peak to peak of the shell response at the mid-span.*

The goal is a deep understanding of nonlinear phenomena appearing when the first axisymmetric mode is resonant: experiments evidenced that, when the shell is excited harmonically from the base, with an excitation frequency close to the first axisymmetric mode, complex dynamic scenarios appear and the energy pumped in the system at low frequency spreads over a wide range of the spectrum.

Tests are carried out using a seismic sine excitation, close to the resonance of the first axisymmetric mode (m=1, n=0).

The complexity and violence of vibrations due to nonlinear phenomena gave several problems to closed loop controllers of the shaking table; therefore, an open loop approach was chosen, the control is the input voltage of the shaker amplifier.

Two accelerometers and a Laser displacement sensor are used to measure the accelerations of the base, the top, and the displacement of the shell lateral surface: channel 1 (accelerometer Wilcoxon Research S 100 C) records the base acceleration (the excitation) due to the shaking table; channel 2 records the displacement of the shell in radial direction (Micro Epsilon optoNCDT 2200 Laser displacement sensor); channel 3 records the top disk acceleration (PCB M352C65 micro accelerometer); see Figure 1.

Figures 2 a-e represent the amplitudes of vibration in terms of acceleration (base and top disk vibration) or displacement (measured on the lateral surface of the shell, the vertical position is on the middle): during experiments the input voltage was sinusoidal ($v(t)=v0\sin(2\pi f t)$, $v0=0.07V$) and the frequency was moved step by step (stepped sine approach) starting from high frequency, 340Hz, and reducing up to 290Hz.

Channel 1 (Figure 2a) shows that the maximum excitation (base motion) is between 8 and 14 g; from such data one can guess that there is a strong interaction between the shaker and the shell-disk, i.e. inertia forces generated by the top disk and the shell vibration influence the shaker response; it is worthwhile to remember that the shaker control is open loop.

The top disk vibration (channel 3, figure 2b) increases as the first axisymmetric mode resonance is approached, from 340 to 333Hz the top disk response follows the usual behaviour expected by a linear resonance. However, from 333 Hz to 320 Hz the slope of the curve changes, when the excitation frequency is less than 322 Hz the top disk vibration amplitude remains flat, this happens up to 295 Hz; below such frequency the top disk response amplitude drops down suddenly and then follows a regular (linear like) behaviour.

The behaviours of base and top disk are strictly related to the dynamic phenomena appearing on the shell. Let us now follow results presented in Figure 2c, where the maximum amplitude of vibration (positive for inward shell deflection) are shown. For excitation frequencies higher than 333 Hz the shell vibration is small, about 0.04 mm (about 16% with respect to the shell thickness, 0.25mm), see Figure 2c; such small amplitude indicates that the shell deflection remains in the linear field, as nonlinearities generally arise when the deformation is of the order of the shell thickness. Reducing the excitation frequency below 333Hz, the shell vibration amplitude suddenly grow up, at 331.5 Hz the amplitude is 0.57 mm, the increment is 1325% passing from 333 Hz to 331.4 Hz (about 0.5% frequency variation); such data show that a new dynamic phenomenon appears suddenly. Another jump in the shell response is observed from 325 Hz (0.75 mm amplitude) to 320 Hz (1.53 mm), i.e. 104% increment in terms of amplitude in 5 Hz. Reducing the excitation frequency to 300 Hz does not cause a big changing in the response, which remains almost flat; from 300 Hz to 296 Hz the amplitude oscillates around 1.5 mm; then at 295 Hz the phenomenon suddenly disappears (0.022mm amplitude).

Figure 2d shows the behaviour of the minimum shell vibration (negative means outward deflection), the behaviour of the minimum vibration is similar to the maximum, but the magnitude of the minimum is smaller than the maximum. This is not surprising, it is well known that, when the amplitude of vibration is equal or larger than the shell thickness, the shell behaves nonlinearly; moreover, the shell is stiffer in outward than in inward direction.

It is to note that the dynamic phenomenon is extremely violent, it is accompanied by a strong noise (hear protections are needed), the acceleration generated on the shell are surprisingly huge. For example if the amplitude is 3 mm, and we suppose the vibration is purely harmonic at 300 Hz, an approximate estimation of the acceleration is about 1100 g! Such estimate does not consider that the shell response is no more sinusoidal, conversely it is non stationary and broad band, this means that the response spectrum contains high frequency components that can lead to a further increment of the acceleration. Some initial experiments carried out using accelerometers for the lateral shell vibration measurement, evidenced accelerations up to 2000g! See also Ref. [10]. This explains the need of a Laser Displacement sensor, such huge levels of acceleration exceed the maximum range of common accelerometers and make quite difficult the connection of the accelerometer to the shell; only micro accelerometers can be used here due to the small mass of the shell, such sensors can be connected using wax or glue, both types of connections cannot resist to huge accelerations and generally the accelerometer detaches from the shell after few seconds.

## 2. THEORETICAL RESULTS

The theoretical model of the shaker response is developed as well as the theoretical shell modeling based on the Sanders-Koiter theory. A suitable interaction between shaker and shell is considered as well as a method for solving the governing equations, which consists of a system of nonlinear partial differential equations for the shell and linear ordinary differential equations for the

shaker. Here details are not reported for the sake of brevity; see Ref. [11] for details about the shell modeling.

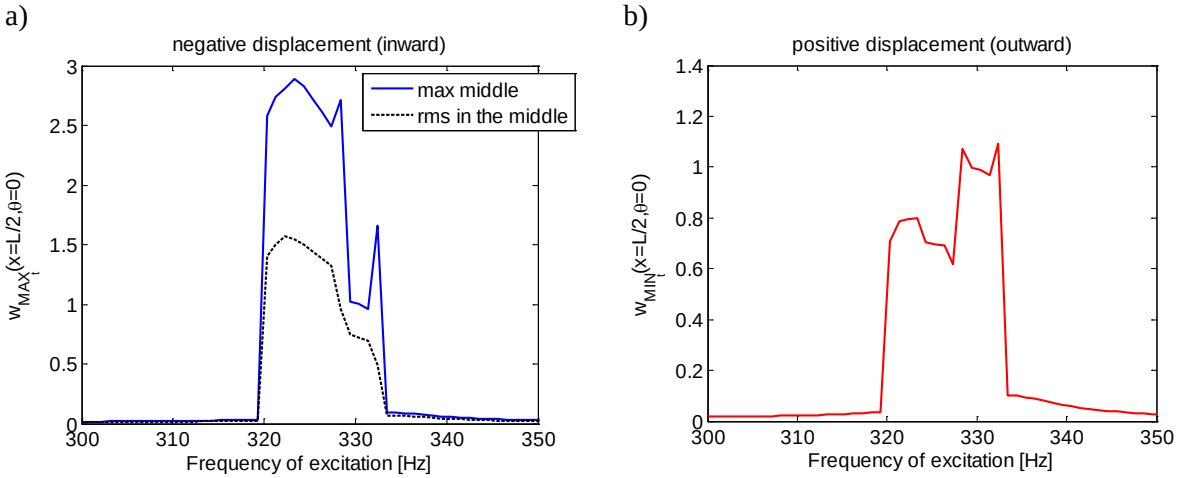


*Fig. 3 Amplitude frequency diagrams, numerical simulations, backward frequency sweep, shell vibration (mm). a) inward (negative, see Fig. 15) displacement and RMS(w); b) outward (positive) displacement. Position of the simulated point measurement:* $x = \frac{L}{2}, \theta = 0$*. Excitation source: 0.09V.*

Figure 3 shows results of simulations carried out considering an input voltage equal to 0.09V, this value is larger than the excitation used during the experiments (0.07V); however, below such value the numerical model did not detect any dynamic instability. Simulations are carried out by decreasing the excitation frequency. Figure 3a shows the amplitude of vibration of the shell in terms of max inward displacement and RMS, a measurement in the middle of the shell is simulated. Figure 3b shows the response in terms of max outward displacement. The simulation frequency interval is 300-350Hz; by decreasing the frequency the onset of instability is found at 333.4Hz, below such frequency the vibration amplitude is magnified, at 329,4Hz a second increment of the vibration amplitude is detected leading the maximum inward deflection to 2.7mm, a further reduction of the frequency does not cause a big amplitude variation up to 319.3Hz, where the vibration level drops down to small amplitudes. The behavior is coherent with experimental results (see Figure 2c), the numerical model overestimates the amplitude of vibration (experiments give 1.8mm max inward vibration) and underestimates the frequency range for which the instability appears (experimental instability range 295-333 Hz); this can be explained by the absence of companion modes and imperfections.

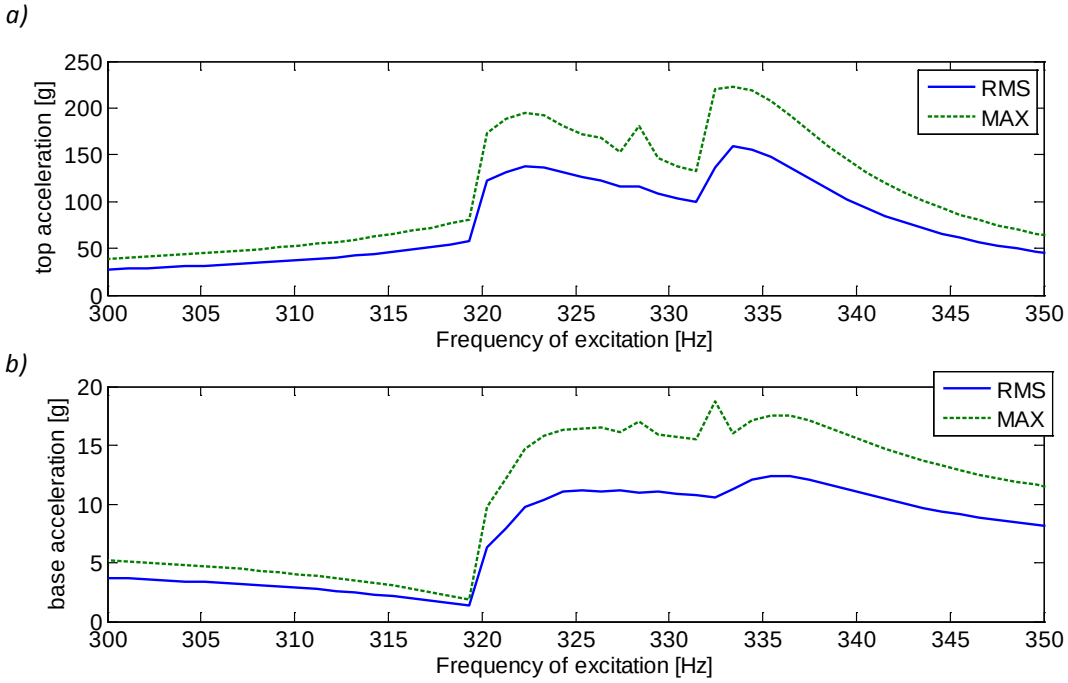


*Fig. 4 Amplitude frequency diagrams, numerical simulations, excitation source 0.09V. a) top disk acceleration, b) base acceleration.*

Figures 4a,b show the vibration amplitudes of top disk and base in terms of acceleration [g]; both maximum and RMS are considered. Similarly to experiments the first axisymmetric mode resonance is clearly visible outside the instability region; when the instability occurs the increment of vibration of the top disk, which is expected when the resonance condition is approached, is locked. Inside the instability region the disk acceleration remains almost flat. Figure 4b shows the base acceleration, it is never constant and is qualitatively similar to experiments. The model overestimates the top disk acceleration during the instability, which is about 90grms for experiments and fluctuates between 100 and 130grms for simulations. The simulation of the base is quantitatively quite close to experiments.

## CONCLUSIONS

In this paper an experimental investigation on the nonlinear dynamics of circular cylindrical shells excited by a seismic excitation is presented. A nonlinear model of the shell considering also the shell shaker interaction is developed.

Experiments clearly show a strong nonlinear phenomenon appearing when the first axisymmetric mode is excited: the phenomenon leads to large amplitude of vibrations in a wide range of frequencies, it appears extremely dangerous as it can lead to the collapse of the shell; moreover, it appears suddenly both increasing and decreasing the excitation frequency and is extremely violent. By observing a strong transfer of energy from low to high frequency a conjecture can be made about the nonlinear interaction among axisymmetric (directly excited) and asymmetric modes. A saturation phenomenon regarding the vibration of the top disk is observed, this is associated with the violent shell vibration; the shell behaves like an energy sink, absorbing part of the disk energy.

The theoretical model shows satisfactory agreement with experiments and clarifies the energy transfer mechanism from low frequency axisymmetric modes and high frequency asymmetric modes, confirming the conjecture arising by the experimental data analysis.

It is now clear that, in order to safely predict the response of a thin walled shell carrying a mass on the top, i.e. the typical aerospace problem for launchers, a nonlinear shell model is needed, but it is not enough: a further modeling regarding the shell mass interaction and the interaction between shell and excitation source is needed.

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