EQUATION OF MOTION WITH IMPLICIT FUNCTIONS OF PHASE COORDINATES VELOCITIES AND ITS APPLICATIONS FOR ENGINEERING CONSTRUCTIONS RUPTURE LIFE EVALUATION

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ABSTRACT

The generalized mathematical problem of image point motion is formulated for phase coordinate velocities as implicit functions which are defined by two coupled initial-boundary-value problems. Applications of this problem are presented for rupture life evaluation of engineering constructions damaged due to high temperature creep.

INTRODUCTION

The well-known in nonlinear dynamics state (phase) space conception has outstanding resources for generalized mathematical representation of different physical problems. Feature of problems generalized mathematical representation has especially meaning for development of software utilities necessary for this problem computer solution [1].

Engineering construction rupture life evaluation is an urgent scientific problem [2]. This problem consists in determination of the time from the beginning of operating to the limiting state occur. It is means that system has some state at the beginning time, that system state can be changed with the course of time, and that system has some limited state which can be lead over some time period. This considerations are naturally connected with state (phase) space conception in nonlinear dynamics. It is allows facilities to assume that state space conceptions could be applicable for engineering constructions rupture life evaluation. This paper objective consists in mathematical formulating of engineering constructions rupture life evaluation as problems about imaging point moving in suitable phase space.

1. ABSTRACT MATHEMATICAL FORMULATION OF THE PROBLEM

Structural element (body) is presented as number of points i.e. as geometric images of them material particles constituent. Set of points is consisted as geometrical area Υ with boundary surface υ (fig. 1). Position vector of the point of body is determined using curvilinear coordinates x^1, x^2, x^3:

$$\vec{r} = \vec{r}\left(x^1, x^2, x^3\right). \tag{1}$$

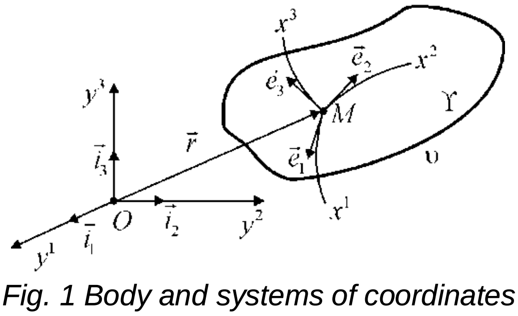

Fig. 1 Body and systems of coordinates

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Let vector $\mathbf{w} \in R^N$ (here and further R denotes real number field) is introduced as quantitative estimation of distance between state of structural element and its limited state. In analogy of damage parameter all-known in creep problems [4] components of the vector $\mathbf{w} \in R^N$ are

$$0 \le w^i \le 1, i = 1, 2, \ldots, N. \tag{2}$$

where $w^i = 0, i = 1, 2, \ldots, N$ values are corresponded to initial state of structural element at time $t = 0$ and $w^i = 1$ are corresponded to structural element limited state at the time $t = t_*$.

Values (2) are defined for all points of body and could be changed during time

$$w^i = w^i\left(t, \vec{r}\right), i = 1, 2, \ldots, N. \tag{3}$$

Rupture life exhausting in every point of body is clearly presented by relations (3) as values (2) time depending and as geometric interpretation on phase plane in the case of $N = 2$ (fig. 2).

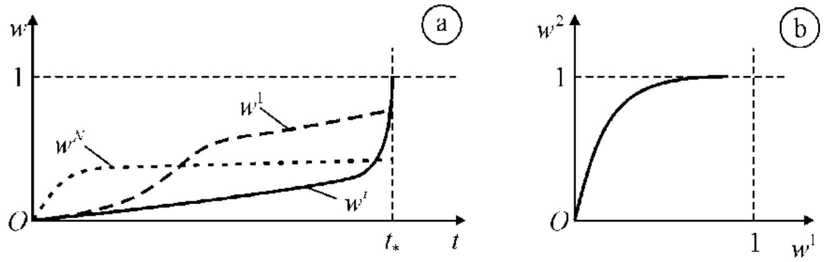

Fig. 2 Graphic presentation of structure elements rupture life exhausting as time depending (a) and as phase path (b)

It is necessary to take into account accumulation of the irreversible deformations, degradation of structures materials, micro- and macro-defects creation and progress with time for rupture life evaluation. Let these processes quantitative characteristics are denoted as v^1, v^2, …, v^S and are considered as components of vector $\mathbf{v} \in R^S$. It is necessary to take into account state structural element as deformable body for rupture life evaluation too. Let quantitative characteristics of structure element as deformable body are denoted as u^1, u^2, …, u^Q and are considered as components of vector $\mathbf{u} \in R^Q$. Components of vectors $\mathbf{v}$ and $\mathbf{u}$ are

$$v^i = v^i\left(t, \vec{r}\right), i = 1, 2, \ldots, S. \tag{4}$$

$$u^i = u^i\left(t, \vec{r}\right), i = 1, 2, \ldots, Q. \tag{5}$$

Finite set of characteristics (5) existing is based on the macroscopic definability axiom and local effect principle all-known in mechanics of deformable bodies. Finite set of characteristics (3) and (4) existing is postulated similar.

Introduced as quantitative estimations of distance between state of structural element and limited state values (2) are presented by relations (3). State in the point of body is defined by values (4) and (5). Invariance of the values (3) under time keeping, coordinate systems and choice of point reduces to relation

$$\mathbf{w} = \mathbf{w}\left(\tilde{\mathbf{v}}, \tilde{\mathbf{u}}\right). \tag{6}$$

where $\tilde{\mathbf{v}}$, $\tilde{\mathbf{u}}$ - invariants of vectors $\mathbf{v}$ and $\mathbf{u}$.

Mappings $\tilde{\mathbf{v}} = \tilde{\mathbf{v}}\left(\mathbf{v}\right)$, $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}\left(\mathbf{u}\right)$ define from infinite sets of vectors $\mathbf{v}$ and $\mathbf{u}$ invariants necessary for structural element limit state defining only and in common case are surjection's.

Relations (4)-(6) are equivalent of (3). Thus rupture life evaluation problem is reduced to vectors $\mathbf{v}$, $\mathbf{u}$ and $\mathbf{w}$ determination and phase dependence of life time exhausting analyses (fig. 2).

Differential equations for vector $\mathbf{v}$ determination are usually based on test data such as creep and long-term strength curves, corrosion-fatigue crack velocity curves, long-term stress corrosion cracking curves etc. Vector $\mathbf{v}$ velocity is not explicitly dependent on time, but depended on vector $\mathbf{v}$, its spatial coordinates partial derivatives for given vector $\mathbf{u}$:

$$\begin{cases} \frac{\partial \mathbf{v}}{\partial t} = \mathbf{k}(\mathbf{v}, \mathbf{D} \cdot \mathbf{v}; \mathbf{u}),\ \mathbf{v}(0, \vec{r}) = \mathbf{v}_0(\vec{r})\ \forall \vec{r} \in \Upsilon; \\ \mathbf{B_D} \cdot \mathbf{v}(t, \vec{r}) = \mathbf{v_B}(t, \vec{r}),\ \mathbf{v_B}(0, \vec{r}) = \mathbf{v}_0(\vec{r})\ \forall \vec{r} \in \upsilon, \end{cases} \tag{7}$$

where $\mathbf{k}(\mathbf{v}, \mathbf{D} \cdot \mathbf{v}; \mathbf{u})$ - given velocity of vector $\mathbf{v}$; $\mathbf{D}$ - given spatial coordinates partial derivatives included operator; $\mathbf{v}_0(\vec{r})$ - given at initial time $t = 0$ vector $\mathbf{v}$; $\mathbf{B_D}$, $\mathbf{v_B}$ - given operator and vector are corresponded to boundary conditions.

Differential equations for vector $\mathbf{u}$ determination are presented as equations of deformable body mechanics which are took into account influence of accumulation of the irreversible deformations, degradation of structures materials, micro- and macro-defects creation and progress with time on materials properties and on stress-strain state:

$$\begin{cases} \mathbf{J}(\mathbf{v}) \cdot \frac{\partial \mathbf{u}}{\partial t} + \mathbf{A}(\mathbf{v}) \cdot \mathbf{u} + \mathbf{C} \cdot \mathbf{v} = \mathbf{f}(t, \vec{r}),\ \mathbf{u}(0, \vec{r}) = \mathbf{u}_0(\vec{r})\ \forall \vec{r} \in \Upsilon; \\ \mathbf{B_A} \cdot \mathbf{u}(t, \vec{r}) = \mathbf{u_B}(t, \vec{r}),\ \mathbf{u_B}(0, \vec{r}) = \mathbf{u}_0(\vec{r})\ \forall \vec{r} \in \upsilon, \end{cases} \tag{8}$$

where $\mathbf{J}(\mathbf{v})$, $\mathbf{A}(\mathbf{v})$, $\mathbf{C}$ - operators and $\mathbf{f}$ - vector are corresponded to differential equations of deformable body mechanics which are took into account influence of accumulation of the irreversible deformations, degradation of structures materials, micro- and macro-defects creation and progress with time on materials properties and on stress-strain state; $\mathbf{u}_0(\vec{r})$ - given at initial time $t = 0$ vector $\mathbf{u}$; $\mathbf{B_A}$, $\mathbf{u_B}$ - given operator and vector are corresponded to boundary conditions.

Accountable micro- and macro-defects creation and progress with time factor require to take into account corresponded changing in body's area Υ and its boundary surface υ

$$\Upsilon = \Upsilon(\mathbf{v}); \upsilon = \upsilon(\mathbf{v}). \tag{9}$$

If vectors $\mathbf{w} \in R^N$ are introduced as state (phase) space then problem (6)-(9) could be considered as problem of imaging point moving with velocity:

$$\frac{\partial \mathbf{w}}{\partial t} = \frac{\partial \mathbf{w}}{\partial \tilde{\mathbf{v}}} \frac{\partial \tilde{\mathbf{v}}}{\partial \mathbf{v}} \frac{\partial \mathbf{v}}{\partial t} + \frac{\partial \mathbf{w}}{\partial \tilde{\mathbf{u}}} \frac{\partial \tilde{\mathbf{u}}}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial t}. \tag{10}$$

Right member of (10) could not been obtained explicitly and its determination consist in two coupled initial-boundary-value problems (7), (8) solving. In some elementary cases explicitly determination of velocity (10) however is possible.

2. SIMPLE PROBLEM ABOUT BINDERS RUPTURE LIFE EVALUATION

It is necessary to allow stress relaxation due to creep for binders (bolts, screws, studs) which are operated under higher temperatures conditions. Let equations of binder's bodies stress relaxation due to creep in simple case of homogeneous stress-strain state are presented for example as [4]:

$$\begin{cases} \frac{dc}{dt} = B\sigma^n, c(0) = 0; \\ \sigma + Ec = \sigma_0, \end{cases} \tag{11}$$

where $c = c(t)$ - creep deformation; $\sigma = \sigma(t)$ - stress; E - material modulus of elasticity; B, n - materials characteristics of creep; σ_0 - gripping stress.

It is obviously that equations (11) are particular case of equations (7), (8) in sense of $\mathbf{v} = c$, $S = 1$ and $\mathbf{u} = \sigma$, $Q = 1$. Rupture life of the high pressure vessels binder under high temperatures operated is limited by creep deformation and minimal griping secure tightness stresses:

$$N = 2 : w^1 = \frac{c}{c_*},\ w^2 = \frac{\sigma - \sigma_0}{\sigma_* - \sigma_0}, \tag{12}$$

where c_* - maximal creep which is allowed for given service conditions; σ_* - minimal griping stresses which secure tightness.

Relations (12) are similar on relation (6) and allow to express vectors $\mathbf{v}$ and $\mathbf{u}$ (values c and σ) in term of vector $\mathbf{w}$:

$$c = c_* w^1;\ \sigma = \sigma_0 + \left(\sigma_* - \sigma_0\right) w^2 . \tag{13}$$

Relations (13) and equations (11) reduce to:

$$\begin{cases} \dfrac{dw^1}{dt} = c_* B\left(\sigma_0 + \left(\sigma_* - \sigma_0\right) w^2\right)^n, w^1(0) = 0; \\ \sigma_0 + \left(\sigma_* - \sigma_0\right) w^2 + E c_* w^1 = \sigma_0 . \end{cases} \tag{14}$$

High pressure and high temperature vessels binder is considered as example for [4, 5]:

$$E = 1.72 \cdot 10^5\,\text{MPa};\ n = 3.736;\ B = 3.798 \cdot 10^{-15}\,\text{MPa}^{-\text{n}}\,\text{/hours};\sigma_0 = 300\,\text{MPa};\ \sigma_* = 100\,\text{MPa} . \tag{15}$$

Results of equations (14) solving for input data (15) are presented in fig. 3. For taken as (15) input data rupture life of binders is near 1000 hours.

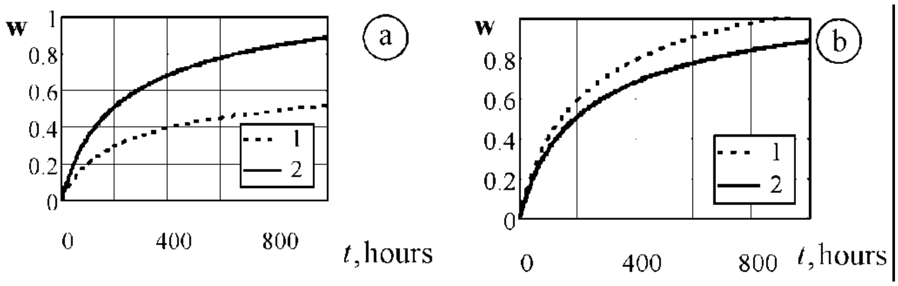

Fig. 3 Binder rupture life exhaustion w^1 (1), w^2 (2) for $c_* = 0.002$ (a) and $c_* = 0.001$ (b)

3. RUPTURE LIFE PROBLEMS ALGORITHMIZATION FOR STRUCTURE ELEMENTS DAMAGED DUE TO CREEP

Rupture life of structures elements in some case is assigned by creep deformations and damage parameter [3, 6]. In the case of infinitesimal deformations body's area Υ and its surface υ changing may be ignored. Equations for damaged due to creep structural elements could be presented in this case for Cartesian rectangular coordinates as

$$\begin{cases} \dfrac{\partial c_{ij}}{\partial t} = \dfrac{3}{2} \cdot \dfrac{B \cdot \left(\sigma_e^c\left(\sigma_{ij}\right)\right)^{n-1}}{(1-\omega^r)^m} \cdot \left(\sigma_{ij} - \tfrac{1}{3}\sigma_{kk}\right);\ c_{ij}\big|_{t=0} = 0; \\ \dot{\omega} = \dfrac{D\left(\sigma_e^\omega\left(\sigma_{ij}\right)\right)^k}{(1-\omega^r)^l};\ \omega\big|_{t=0} = 0; \end{cases} \tag{16}$$

$$\begin{cases} -d_{ijkl}\sigma_{kl} + \tfrac{1}{2}\left(\dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}\right) - c_{ij} = 0;\ \dfrac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0; \\ \left(\sigma_{ij} n_j\right)\big|_{\upsilon_p} = p_i;\ u_i\big|_{\upsilon_u} = u_i^*, \end{cases} \tag{17}$$

where $c_{ij} = c_{ij}(t, x_1, x_2, x_3)$ - creep deformations; $\omega = \omega(t, x_1, x_2, x_3)$ - damage parameter; σ_{ij} - stresses; $\sigma_e^c(\sigma_{ij})$, $\sigma_e^\omega(\sigma_{ij})$ - stresses' equivalents; B, D, n, r, m, k, l - material creep and damage characteristics; d_{ijkl} - material elasticity characteristics; u_i - displacements; f_i - external volume forces; p_i - given on $\upsilon_p \subset \upsilon$ surface-distributed force; u_i^* - given on $\upsilon_u \subset \upsilon$ displacements.

Comparison between equations (16), (17) and generalized equations (7), (8) reduce that vectors $\mathbf{v}$ and $\mathbf{u}$ can be assigned as

$$\begin{cases} \mathbf{v} = \begin{pmatrix} c_{11} & c_{22} & c_{33} & c_{12} & c_{13} & c_{23} & \omega \end{pmatrix}^T; \\ u = \begin{pmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{13} & \sigma_{23} & u_1 & u_2 & u_3 \end{pmatrix}^T. \end{cases} \tag{18}$$

Spatial coordinate's derivatives are not entered into the equations (16) and forces of inertia are not entered into the equations (17). Thus equations (16), (17) are the particular realization of the generalized equations (7), (8):

$$\frac{\partial \mathbf{v}}{\partial t} = \mathbf{k}(\mathbf{v}; \mathbf{u}), \mathbf{v}(0, \vec{r}) = \mathbf{0} \ \ \forall \vec{r} \in \Upsilon, \tag{19}$$

$$\begin{cases} \mathbf{A} \cdot \mathbf{u} + \mathbf{C} \cdot \mathbf{v} = \mathbf{f}(\vec{r}) \forall \vec{r} \in \Upsilon; \\ \mathbf{B_A} \cdot \mathbf{u}(\vec{r}) = \mathbf{u_B}(\vec{r}) \ \forall \vec{r} \in \upsilon. \end{cases} \tag{20}$$

Operator $\mathbf{k}(\mathbf{v}; \mathbf{u})$ from equations (16) is nonlinear, but operators $\mathbf{A}$, $\mathbf{C}$, $\mathbf{B_A}$ from equations (17) are linear.

Unknown solution of the problem (19), (20) is presented by approximations with given systems of spatial coordinate's trial functions and unknown coefficients as functions of time

$$\begin{cases} \mathbf{v}(t, \vec{r}) \approx \mathbf{V}_n(\vec{r}) \cdot \mathbf{v}_n(t); \mathbf{v}_n(0) = \mathbf{0}; \\ \mathbf{u}(t, \vec{r}) \approx \mathbf{u}_\upsilon(\vec{r}) + \mathbf{U}_n(\vec{r}) \cdot \mathbf{u}_n(t), \end{cases} \tag{21}$$

where n - count of trial functions; $\mathbf{V}_n(\vec{r})$, $\mathbf{U}_n(\vec{r})$ - matrices of given trial functions; $\mathbf{u}_\upsilon(\vec{r})$ - given vector which distribute boundaries values into area; $\mathbf{v}_n(t)$, $\mathbf{u}_n(t)$ - vectors are consisted of unknown approximations coefficients.

Trial functions are chose such that all boundary conditions (20) will be identically satisfied

$$\forall \mathbf{u}_n(t): \ \mathbf{B_A} \cdot (\mathbf{U}_n(\vec{r}) \cdot \mathbf{u}_n(t)) = \mathbf{u_B}(\vec{r}) - \mathbf{B_A} \cdot \mathbf{u}_\upsilon(\vec{r}). \tag{22}$$

Trial functions identically satisfied conditions (22) can be constructed by R-functions methods [7] for free-form body's area and arbitrarily given boundary conditions.

Let to apply orthogonal property between trial functions and obtained for approximations (21) equations (19), (20) misalignments. Linear properties of $\mathbf{A}$ and $\mathbf{C}$ operators reduce to

$$\begin{cases} \mathbf{K}_n \dfrac{d\mathbf{v}_n}{dt} = \mathbf{k}_n(\mathbf{v}_n; \mathbf{u}_n); \\ \mathbf{A}_n \mathbf{u}_n + \mathbf{C}_n \mathbf{v}_n = \mathbf{f}_n, \end{cases} \tag{23}$$

where $\mathbf{K}_n$, $\mathbf{A}_n$ - quadratic non-singular matrices and $\mathbf{C}_n$ - rectangular in common case matrix; $\mathbf{f}_n$ - vector; $\mathbf{k}_n(\mathbf{v}_n; \mathbf{u}_n)$ - nonlinear vector-function:

$$\mathbf{K}_n = \int_\Upsilon \mathbf{V}_n^T \cdot \mathbf{V}_n d\Upsilon; \ \mathbf{k}_n(\mathbf{v}_n; \mathbf{u}_n) = \int_\Upsilon \mathbf{V}_n^T \cdot \mathbf{k}(\mathbf{V}_n \cdot \mathbf{v}_n; \mathbf{u}_\upsilon + \mathbf{U}_n \cdot \mathbf{u}_n) d\Upsilon;$$

$$\mathbf{A}_n = \int_\Upsilon \mathbf{U}_n^T \cdot (\mathbf{A} \cdot \mathbf{U}_n) d\Upsilon; \ \mathbf{C}_n = \int_\Upsilon \mathbf{U}_n^T \cdot (\mathbf{C} \cdot \mathbf{V}_n) d\Upsilon = 0; \ \mathbf{f}_n = \int_\Upsilon \mathbf{U}_n^T \cdot (\mathbf{f} - \mathbf{A} \cdot \mathbf{u}_\upsilon) d\Upsilon.$$

Vectors $\frac{d\mathbf{v}_n}{dt}$ and $\mathbf{u}_n$ can be solved from equations (23):

$$\begin{cases} \frac{d\mathbf{v}_n}{dt} = \mathbf{K}_n^{-1} \cdot \mathbf{k}_n\left(\mathbf{v}_n;\mathbf{u}_n\right); \\ \mathbf{u}_n = \mathbf{A}_n^{-1} \cdot \left(\mathbf{f}_n - \mathbf{C}_n \mathbf{v}_n\right). \end{cases} \tag{24}$$

Relations (24) reduce to Cauchy problem in standard form suitable for computing:

$$\begin{cases} \frac{d\mathbf{v}_n}{dt} = \mathbf{K}_n^{-1} \cdot \mathbf{k}_n\left(\mathbf{v}_n\right); \\ \mathbf{v}_n\left(0\right) = \mathbf{0}, \end{cases} \tag{25}$$

where $\mathbf{k}_n\left(\mathbf{v}_n\right) = \mathbf{k}_n\left(\mathbf{v}_n;\mathbf{u}_n = \mathbf{A}_n^{-1} \cdot \left(\mathbf{f}_n - \mathbf{C}_n \mathbf{v}_n\right)\right)$.

Thus creep-damage problem (16), (17) is reduced to standard Cauchy problem (25) and it solution can obtained using any all-known computational methods, Runge-Kutta for example.

CONCLUSIONS

Engineering constructions rupture life evaluation problems are formulated as imaging point moving problems in phase space with vectors which are estimation of distance between current state of structural element and its limited state. Velocities of phase coordinates in imaging point equations of moving are implicit functions defined by two coupled initial-boundary-value problems. Applications for engineering constructions structural elements damaged due to high temperatures creep are discussed.

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