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GEOMETRICALLY NONLINEAR TRANSVERSAL VIBRATIONS OF PLIABLE TO SHEAR AND COMPRESSION PLATES

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ABSTRACT

A system of differential equations that describes nonlinear transversal vibrations and takes into account pliability to transversal shear and compression strains of composite plates is given. The parametrical analysis of dependence on fundamental frequency and amplitude of transversal vibrations of a strip-plate for hinge- fixed or hold rigidly plate is carried out.

INTRODUCTION

The laminated elements from composites are widely used in various designs and technical means under intensive cyclic loading. This loading can cause different bending proportional to the plate thickness what stipulates the geometrically non-linear character of the strain state. Therefore, to prevent the resonance phenomena the fundamental frequencies in such cases should be calculated using the geometrically nonlinear relations of the plate dynamics which take into account the pliability to transversal shear and compressive strains.

The majority of studies on nonlinear dynamics of thin-walled elements of design are based on the Karman quadratic theory being the generalization of the classical linear Kirchhoff-Lave theory for geometric nonlinearity [1]. In some works the relations of nonlinear technical theory were used, the basis of which forms the Timoshenko model [1–4]. However the theories grounding on the hypotheses of these authors do not take full account of the peculiarities of behavior of composites. Therefore this paper utilizes a mathematical model of dynamic deformation of plates, which considers the above peculiarities [5, 6]. The influence of boundary conditions on amplitude-frequency characteristics during nonlinear vibrations of composite plates has been analyzed on this basis.

1. STATEMENT OF THE PROBLEM

Consider a composite plate of thickness $2h$ with effective elastic characteristics and averaged material density ρ, related to the Cartesian coordinate system x_i $(i=1,2,3)$. Assume that one dimension of the plate exceeds considerably the other one. Then its dynamic geometrically nonlinear stress-strain state depends only on one spatial coordinate $x_1 = x$ in its median plane. The equations of plate motion in this case may be written as [5]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x^2} = \frac{1}{c_1^2}\frac{\partial^2 u}{\partial t^2}, \tag{1}$$

$$\frac{\partial^2 \gamma}{\partial x^2} - æ_1^2\left(\gamma + \frac{\partial w}{\partial x}\right) = \frac{1}{c_1^2}\frac{\partial^2 \gamma}{\partial t^2}, \tag{2}$$

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$$\frac{\partial^2 u}{\partial x^2}\frac{\partial w}{\partial x}+\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}+\frac{3}{2}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}+æ_2^2\frac{\partial}{\partial x}\left(\gamma+\frac{\partial w}{\partial x}\right)=\frac{1}{c_1^2}\frac{\partial^2 w}{\partial t^2}, \qquad (3)$$

where u,w are relatively tangential and transversal displacements of the plate median plane, γ is the angle of rotation of normal element to the median plane before deformation, $c_1=\sqrt{\overline{B}/2\rho h}$ is the velocity of longitudinal waves propagation in the plate, $æ_1^2=\Lambda/\overline{D}$, $æ_2^2=æ_1^2h^2/3$, $\overline{B}=\frac{2Eh}{(1-\nu^2)}(1+\alpha)$, $\overline{D}=\frac{h^2}{3}\overline{B}$, $\Lambda=2k'hG'$, $k'=14/15$, $\alpha=\frac{(1+\nu)(\nu')^2}{1-\nu-2\nu\nu'}\frac{E}{E'}$.

Here E,ν are Young's modulus and Poisson's ratio in the median and equidistant to it planes; E',ν' are the same values in the planes perpendicular to the median plane; G' – is the transversal shear modulus.

The origin of coordinate x in the middle of the plate sides perpendicular to it the sides is arranged at a distance $2a$ one from another. Then in the case of a hinge fixing of these sides the boundary conditions are:

$$u(\pm a,t)=0,\ w(\pm a,t)=0,\ M(\pm a,t)=0, \qquad (4)$$

and for hold rigidly sides the boundary conditions are defined as

$$u(\pm a,t)=0,\ w(\pm a,t)=0,\ \gamma(\pm a,t)=0. \qquad (5)$$

The system (1)–(3) together with boundary conditions (4) or (5) forms a mathematical model of free geometrically nonlinear transversal vibrations of hinge-fixed or hold rigidly of the composite strip-plates which undergo deformations of transversal shear and compression.

The model presented describes also the forced longitudinal and shear vibrations generated by free transversal vibrations. They are also agreed with the results of investigations of quadratically nonlinear waves in elastic bodies as in Ref [7].

2. CONSTRUCTION OF SOLUTION

In Ref. [5] the fundamental frequency-to-amplitude of nonlinear vibrations ratio of the plate with hinge fixing of the edges $x=\pm a$:

$$\omega^2=\omega_0^2\left(1+\frac{3}{4}KA^2\right), \qquad (6)$$

has been analyzed , where ω is the fundamental frequency of nonlinear natural vibrations of the plate, A is the dimensionless amplitude, $\omega_0=c_2\lambda^2/\sqrt{æ_1^2+\lambda^2}$ is the fundamental frequency of linear natural vibrations of the plate, $c_2=\sqrt{\Lambda/2\rho h}$ is the velocity of shear wave propagation in the plate, $\lambda=\pi/2a$,

$$K=K_c(1+4\beta),\qquad \beta=\frac{\pi^2}{12}(h/a)^2\frac{1}{k'}(E/G')\frac{1+\alpha}{1-\nu^2}. \qquad (7)$$

The equality (7) has the same form as in Ref. [1] for the plate sufficiently long in one direction with hinge-fixed edges when the classical theory is applied. For motionless hinges the value of the coefficient $K_c=3$ was obtained in Ref. [1]. If in equality (7) passing to the limit is performed

$$\lim_{\mathrm{E/G'}\to 0} K(\mathrm{E/G'})=K_c=3 \qquad (8)$$

we can obtain the analogous result.

For the fundamental frequency of nonlinear transversal vibrations of the plate hold rigidly on the edges to be found, it is necessary to choose the unknown functions in (1)–(3) in such way that the boundary conditions (5) were satisfied

$$w = W(t)cos^2 \lambda x, \quad \gamma = Y(t)sin 2\lambda x, \quad u = U(t)sin 4\lambda x \ . \tag{9}$$

Neglecting in Eq. (2) the inertia of the element normal to the median plane [5], we obtain:

$$\mathrm{Y(t)} = \frac{æ_1^2 \lambda}{æ_1^2 + 4\lambda^2} W(t). \tag{10}$$

To define the function $U(t)$ from (1) we have an ordinary differential equation

$$\ddot{U}(t) + 4\, \omega_u^2\, U(t) = \frac{1}{4}\, \lambda\, \omega_u^2\, W^2(t),$$

the solution of which is written in the form

$$U(t) = C_1\, sin\, \omega_u t + C_2\, cos\, \omega_u t + \frac{1}{8}\, \lambda\, \omega_u \int_0^t W^2(\tau) sin\, \omega_u (t - \tau) d\tau\,, \tag{11}$$

where $\omega_u = 2\lambda c_1$ is the fundamental frequency of linear longitudinal vibrations of the plate.

From the initial condition at moment $t = 0$ the velocity of points of the median plane along the axis is equal to zero and the median plane itself takes the form of the surface

$$w(0,x) = W(0) cos^2 \lambda x\,,$$

we can define the integration constants

$$C_1 = 0\,, \qquad\qquad C_2 = \frac{1}{16}\, W^2(0). \tag{12}$$

If we introduce the dimensionless values into consideration

$$\xi(t) = W(t)/2h, \qquad\qquad \eta(t) = U(t)/2a \tag{13}$$

by substitution (10) and (11), with regard for (12) in (3), after application of the Bubnov-Galerkin procedure [1], we obtain the integro-differential equation for the function of dimensionless bending of the nonlinear transversal vibrations of the plate considered:

$$\ddot{\xi}(t) + (\omega_0^1)^2 \xi(t) + \frac{(\omega_0^1)^2}{2} K\, \xi(t) \Big\{ \xi^2(t) - \Big[\xi^2(0) cos\ \omega_u t + \\ + \omega_u \int_0^t \xi^2(\tau)\, sin\ \omega_u (t - \tau) d\tau \Big] \Big\} = 0\,, \tag{14}$$

where $\omega_0^1 = \dfrac{4\sqrt{3}}{3} c_2\, \lambda^2 / \sqrt{æ_1^2 + 4\lambda^2}$ is the fundamental frequency of free linear transversal vibrations of the plate hold rigidly on the edges;

$$K_1 = K_{c1}(1 + 4\beta). \tag{15}$$

The passing to the limit in (15) as the parameter pliability to transversal shear strains

E/G' approaches zero

$$\lim_{E/G' \to 0} K_1(E/G') = K_{c1} = 3/4 \tag{16}$$

yields the classical result from Ref. [1].

If we integrate the equations (14) by the full period of vibrations $T = 2\pi/\omega$ neglecting appropriate of infinitesimal values, as in Refs. [1, 5], we obtain the relation like expression (6)

$$\omega^2 = (\omega_0^1)^2 \left(1 + \frac{3}{4} K_1 A^2\right). \tag{17}$$

3. ANALYSIS OF THE RESULTS

Introduce the notations μ_1 and μ_2 for the value ω to corresponding fundamental frequencies of natural free vibrations of the plate ratio

$$\mu_1 = \frac{\omega}{\omega_0} = \sqrt{1 + \frac{3}{4} K A^2}, \quad \mu_2 = \frac{\omega}{\omega_0} = \sqrt{1 + \frac{3}{4} K_1 A^2} \tag{18}$$

and consider the value

$$\eta = \frac{\omega_0^1}{\omega_0} = \frac{4\sqrt{3}}{3} \sqrt{\frac{1 + 4æ(h/a)^2}{1 + æ(h/a)^2}}, \quad æ = \frac{\pi^2}{4} \frac{E}{G'} \frac{1}{k'} \frac{1+\alpha}{1-\nu^2}. \tag{19}$$

It is obvious that for $h/a << 1$ and limited value of pliability to transversal shear E/G'

$$\eta \approx 4\sqrt{3}/3 \approx 2{,}31. \tag{20}$$

In coordinates μ, A $(\mu = \mu_1, \mu_2)$, we construct the backbone curves [1], illustrating the dependences between the dimensionless frequencies μ_1, μ_2 and the dimensionless amplitude A. Moreover, for one coordinate we have the following dependence:

$$\mu_2 = \eta\, \mu_1. \tag{21}$$

The coefficient η we shall call the influence coefficient on the amplitude-frequency characteristics of hold rigidly edges which is compared with the hinge fixed. Figs. 1, 2 present the backbone curves for $h/a = 0{,}1$, $\nu = \nu' = 0{,}375$ for different values E/G': $E/G' = 0$ shear and compression strains are absent, $E/G' = 2(\nu + 1)$ for isotropic material, $E/G' = 10$ and 60.

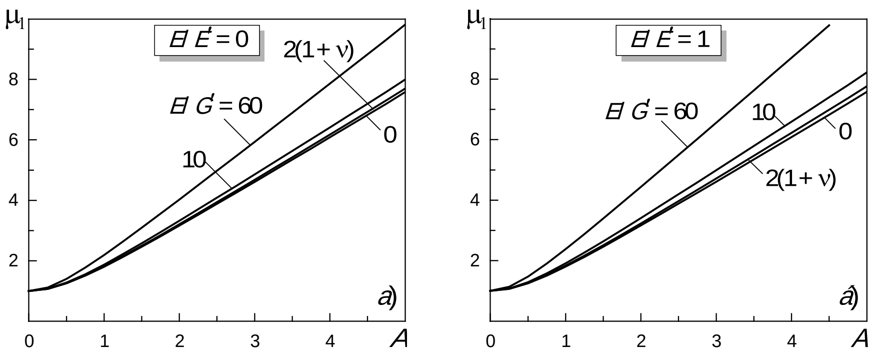

Fig.1. The dimensionless frequency μ_1 vs. the dimensionless amplitude A neglecting (a) and transversal compression for different values E/G' (б).

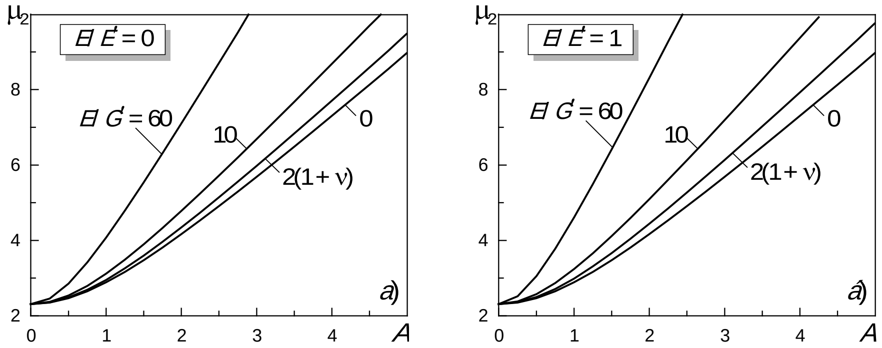

Fig.2. The dimensionless frequency μ_2 vs. the dimensionless amplitude A neglecting (a) and transversal compression for different values E/G' (б).

When parameters h/a and ν are given we have observed a considerable influence of the edges fixing type and pliability parameter E/G' on the value μ_1 and μ_2 for $1 \le A \le 5$ in comparison with classical results for $E/G' = 0$.

4. CONCLUSION

Taking into account the pliability to transversal shear and compression strains for nonlinear vibrations of composite plates, we can increase the rigidity of the dynamic system considered. In defining the frequency of nonlinear vibrations of composite plates with amplitude close to five thicknesses, it is necessary to utilize the refined mathematical model. Provided that the edges are hold rigid the fundamental fundamental frequency increases by 2.31 times in comparison with hinge fixed edges of the plate.

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