Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

NONLINEAR NORMAL MODES OF FORCED VIBRATIONS IN ROTOR SYSTEMS

N.V. Perepelkin, Yu.V.Mikhlin[1]
National Technical University "Kharkov Polytechnic Institute", Kharkov, Ukraine

ABSTRACT

Forced vibrations of the disk elastic rotor on nonlinear flexural base for a case of internal resonance are considered. The gyroscopic moments are taken into account. The Shaw-Pierre conception of nonlinear normal vibration modes and the modified Rausher method are used to construct resonance forced vibrations.

INTRODUCTION

Rotor systems are important elements of machines and mechanisms. Different nonlinear effect must be taken into account in analysis of dynamical behavior of such systems. Moreover, internal resonances in the rotor systems dynamics must be taken into account too. One selects some principal publications on the rotor nonlinear dynamics. V.A. Grobov [1] suggested apply asymptotic methods to analyze the rotating shafts dynamics. A.P. Filippov [2] analyzed non-stationary vibrations of one disc rotor with nonlinear flexible base assuming that one support is nonlinear. V.V. Bolotin [3] took into account nonlinear inertia in the model of one disk rotor. Different models of rotor vibrations and analysis of motions stability are treated in the book [4]. Non-stationary vibrations of rotor interacting with limited power supply are considered in [5]. In [6,7] it is investigated the periodic and chaotic vibrations in the model of the Laval-Jeffcott rotor with two degree-of-freedoms with the internal resonance phenomena, using asymptotic method. Note that in many publications mostly the simplest model of the Laval-Jeffcott rotor is considered, when for the centrally mounted disk, the system is symmetric and the first two fundamental translational and rotational motions are decoupled and can be considered separately.

The Shaw-Pierre nonlinear modes of rotors accounting gyroscopic terms are considered in the paper [8]. In the present paper nonlinear forced vibrations of rotor taking into account gyroscopic effects and nonlinear flexible base are considered. An asymmetrical disposition of the disk in the shaft is considered. The Shaw-Pierre nonlinear normal modes (NNMs) together with the modified Rausher method are used to construct resonance vibrations. In contrast to results presented in [8], here it is constructed NNMs is a system having the internal resonance. This situation is always realized in the rotor system with the isotropic-elastic shaft and the isotropic-elastic supports.

1. THE SHAW-PIERRE NONLINEAR NORMAL VIBRATION MODES

Nonlinear normal vibrations modes (NNMs) are a generalization of the normal vibrations of linear systems. In the normal mode, a finite-dimensional system behaves like a conservative one having a single degree of freedom [9,10]. A generalization of the NNMs conception to non-autonomous systems is possible too. In [11,12] the authors reformulated the concept of NNMs for a general class of nonlinear discrete oscillators. The analysis is based on the computation of invariant manifolds of motion on which the NNMs take place.

To use this approach the original ODE system must be presented of the next standard form,

$$\frac{dx}{dt} = y, \frac{dy}{dt} = f(x, y) \tag{1}$$

[1] Corresponding author. Email muv@kpi.kharkov.ua

where $\mathrm{x} = \{\mathrm{x}_1 \quad \ldots \quad \mathrm{x}_N\}^{\mathrm{T}}$ is a vector of the generalized coordinates, $\mathrm{y} = \{\mathrm{y}_1 \quad \ldots \quad \mathrm{y}_N\}^{\mathrm{T}}$ is a vector of the generalized velocities, and $\mathrm{f} = \{\mathrm{f}_1 \quad \ldots \quad \mathrm{f}_N\}^{\mathrm{T}}$ is a vector of the forces. One chooses a couple of new independent phase variables (*u, v*), so-called ***master coordinates***, where *u* is some ***dominant generalized coordinate,*** and *v* ***is the corresponding generalized velocity***. By the Shaw-Pierre approach, the nonlinear normal mode is such regime when all generalized coordinates and velocities are univalent functions of the selected couple of variables, named master coordinates. Denoting these master coordinates as the coordinate and velocity with the index 1, one writes the nonlinear normal mode of the form:

$$x_1 = u, y_1 = v, x_i = X_i(u,v), y_i = Y_i(u,v) \ (i \neq 1) \tag{2}$$

Computing derivatives of all variables in the system (1), and taking into account that *u = u(t)* and *v = v(t)*, then substituting the obtained expressions to the system (2), one has the following system of partial derivation equations:

$$\begin{aligned} &\frac{\partial X_i}{\partial u} v + \frac{\partial X_i}{\partial v} f_1(x,y) = Y_i(u,v), \\ &\frac{\partial Y_i}{\partial u} v + \frac{\partial Y_i}{\partial v} f_1(x,y) = f_i(x,y), \\ &i = \overline{1..N}. \end{aligned} \tag{3}$$

One presents the system solution in the form of the power series by new independent variables *u* and *v*:

$$\begin{aligned} x_i &= X_i(u,v) = a_{1i}u + a_{2i}v + a_{3i}u^2 + a_{4i}uv + a_{5i}v^2 + \ldots, \\ y_i &= Y_i(u,v) = b_{1i}u + b_{2i}v + b_{3i}u^2 + b_{4i}uv + b_{5i}v^2 + \ldots. \end{aligned} \tag{4}$$

The series (4) are introduced to equations (3), then coefficients in terms of the same degree by independent variables, are equated. So, a system of recurrent algebraic equations can be written. Coefficients of the series (4) can be determined from these equations, and, as a result, the corresponding nonlinear normal mode is obtained.

In a case of internal resonance it can observe an interaction of two NNMs. So, four phase coordinates are active, and they must be chosen as master coordinates. In this important case all other phase coordinates are presented as univalent functions of the selected four coordinates. Namely this situation occurs in the problem of the rotor dynamics which will be considered later.

2. USE OF THE MODIFIED RAUSHER METHOD TO CONSTRUCT FORCED VIBRATION MODES

One considers the nonlinear dynamical system under an external periodical excitation, which is written in principal (normal) coordinates of the following standard form:

$$\begin{cases} \dot{q}_1 = s_1 \\ \dot{s}_1 = -\nu_1^2 q_1 - f_1(\bar{q},\bar{s}) + F_1\cos(\Omega t) \\ \dot{q}_2 = s_2 \\ \dot{s}_2 = -\nu_2^2 q_2 - f_2(\bar{q},\bar{s}) + F_2\cos(\Omega t) \\ \ldots \\ \dot{q}_N = s_N \\ \dot{s}_N = -\nu_N^2 q_N - f_N(\bar{q},\bar{s}) + F_N\cos(\Omega t) \end{cases} \tag{5}$$

where $\bar{q} = \{q_1, q_2, ..., q_N\}^T$, $\bar{s} = \{s_1, s_2, ...s_N\}^T$. It is assumed that the frequencies ν_1 and ν_2 are close, and they are close to the external frequency, Ω. In this case two active coordinates, $q_{1,2}$, and two corresponding velocities, $s_{1,2}$, may be taken as independent master coordinates to construct expansions which are analogous to the series (4).

One assumes that there is a representation of the master coordinates in the form of the following Fourier series:

$$
\begin{aligned}
q_1 &= A_1\cos(\Omega t)+B_1\sin(\Omega t)+A_2\cos(2\Omega t)+\\
&\quad +B_2\sin(2\Omega t)+A_3\cos(3\Omega t)+B_3\sin(3\Omega t)+\ldots\\
s_1 &= B_1\Omega\cos(\Omega t)-A_1\Omega\sin(\Omega t)+2B_2\Omega\cos(2\Omega t)-\\
&\quad -2A_2\Omega\sin(2\Omega t)+3B_3\Omega\cos(3\Omega t)-3A_3\Omega\sin(3\Omega t)+\ldots\\
q_2 &= \ldots\\
s_2 &= \ldots
\end{aligned}
\tag{6}
$$

When slave coordinates are essentially smaller than the master coordinates, we can obtain such trigonometric expansions from the next ODE system:

$$
\begin{cases}
\dot{q}_1 = s_1\\
\dot{s}_1 = -\nu_1^2 q_1 - f_1(\overline{q},\overline{s}) + F_1\cos(\Omega t)\\
\dot{q}_2 = s_2\\
\dot{s}_2 = -\nu_2^2 q_2 - f_2(\overline{q},\overline{s}) + F_2\cos(\Omega t)\\
\left.\begin{aligned} q_i &= 0\\ s_i &= 0\end{aligned}\right\},\ i=\overline{3,N}
\end{cases}
\tag{7}
$$

One has from here, using some trigonometric transformations that

$$
\cos(\Omega t) = \alpha_1 q_1 + \alpha_2 s_1 + \alpha_2 q_2 + \alpha_3 s_2 + \alpha_5 q_1^2 + \alpha_6 s_1^2 + \ldots \tag{8}
$$

This relation is substituted to right parts of the equations (5); it corresponds to the principal idea of the Rausher method. As a result, the autonomous system is obtained:

$$
\begin{cases}
\dot{q}_1 = s_1\\
\dot{s}_1 = -\nu_1^2 q_1 - f_1(\overline{q},\overline{s}) + F_1\left(\alpha_1 q_1 + \alpha_2 s_1 + \alpha_2 q_2 + \alpha_3 s_2 + \alpha_5 q_1^2 + \ldots\right)\\
\ldots\\
\dot{q}_N = s_N\\
\dot{s}_N = -\nu_N^2 q_N - f_N(\overline{q},\overline{s}) + F_N\left(\alpha_1 q_1 + \alpha_2 s_1 + \alpha_2 q_2 + \alpha_3 s_2 + \alpha_5 q_1^2 + \ldots\right)
\end{cases}
\tag{9}
$$

In the system (9) the NNMs by Shaw-Pierre can be constructed from the equations similar to (3). But in a case of the internal resonances the four independent coordinates are used, and the corresponding equation in partial derivatives must be used. These equations are not presented here. Solution of these equations is obtained in form of the Taylor series:

$$
\begin{cases}
q_n = a^{(n)}_{1,0,0,0}q_1 + a^{(n)}_{0,1,0,0}s_1 + a^{(n)}_{0,0,1,0}q_2 + a^{(n)}_{0,0,0,1}s_2 + a^{(n)}_{2,0,0,0}q_1^2 + \ldots\\
s_n = b^{(n)}_{1,0,0,0}q_1 + b^{(n)}_{0,1,0,0}s_1 + b^{(n)}_{0,0,1,0}q_2 + b^{(n)}_{0,0,0,1}s_2 + b^{(n)}_{2,0,0,0}q_1^2 + \ldots
\end{cases},\ n=\overline{3,N}
\tag{10}
$$

Then the expansions (10) are substituted to the system (5). It permits to reduce the n-DOF system to the two-DOF one. Two master coordinates are obtained from this system. So, the solution (6) is made more precise.

The pointed out series of operations can be repeated some times to reach a necessary exactness.

As some simple example, a system of three oscillators, connected by elastic springs, one of them is nonlinear, is considered. Equations of motion are the following:

$$
\begin{cases}
m_1\ddot{x}_1 + \beta\dot{x}_1 + c_1 x_1 + c_2(x_1 - x_2) + \gamma x_1^3 = 0\\
m_2\ddot{x}_2 + \beta\dot{x}_2 + c_2(x_2 - x_1) + c_3(x_2 - x_3) + c_5 x_2 = 0\\
m_3\ddot{x}_3 + \beta\dot{x}_3 + c_3(x_3 - x_2) + c_4 x_3 = f\cos(\Omega t)
\end{cases}
\tag{11}
$$

It is assumed that two vibration modes of the lineariazed system (11) have close frequencies. Use of the proposed approach permits to construct NNMs of the non-autonomous system (11). A transformation to principal coordinates q_1, q_2, q_3 is made, where two first coordinates correspond to

modes with close frequencies. The frequency response of the system is obtained. In the Fig.1 the frequency response for the first harmonic of the principal coordinate q_3 is shown. The entire line is obtained by the NNMs approach, and the dashed line is obtained by the harmonic balance method. Numerical simulation confirms a good exactness of the proposed approach too.

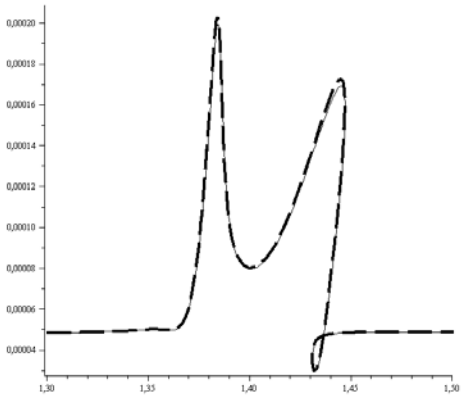

Fig. 1. Frequency response for the first harmonic of the generalized coordinate q_3 of the system (11).

3. PRINCIPAL MODEL OF THE ROTOR NONLINEAR DYNAMICS.

A model of the rotor dynamics with an asymmetrical disposition of the disk in the shaft is considered. Gyroscopic effects and nonlinear flexible base are taken into account. The fixed and moving coordinate systems and positional angles are shown in the Fig. 2.

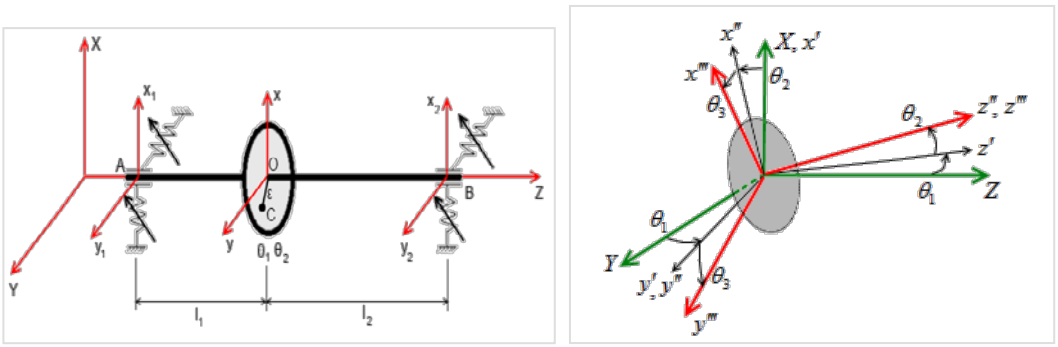

Fig.2. Principal model of the rotor dynamics. Fixed and moving coordinate systems.

Equations of the rotor motion can be written of the following form:

$$
\begin{aligned}
&m\ddot{x}+\rho_1\dot{x}+c_{11}\left(x-h_1x_2-h_2x_1\right)+c_{12}\left(\theta_2-\frac{x_2-x_1}{l}\right)=\varepsilon\Omega^2\cos\Omega t\\
&m\ddot{y}+\rho_1\dot{y}+c_{11}\left(y-h_2y_1-h_1y_2\right)-c_{12}\left(\theta_1+\frac{y_2-y_1}{l}\right)=\varepsilon\Omega^2 m\sin\Omega t\\
&I_e\ddot{\theta}_1+\rho_2\dot{\theta}_1+I_p\Omega\dot{\theta}_2+c_{22}\left(\theta_1+\frac{y_2-y_1}{l}\right)-c_{12}\left(y-h_2y_1-h_1y_2\right)=0\\
&I_e\ddot{\theta}_2+\rho_2\dot{\theta}_2-I_p\Omega\dot{\theta}_1+c_{22}\left(\theta_2-\frac{x_2-x_1}{l}\right)+c_{12}\left(x-h_1x_2-h_2x_1\right)=0\\
&\beta\dot{x}_1+\left(\frac{c_{12}}{l}-h_2c_{11}\right)\left(x-h_1x_2-h_2x_1\right)+\left(\frac{c_{22}}{l}-h_2c_{12}\right)\left(\theta_2-\frac{x_2-x_1}{l}\right)+c_x^{(1)}x_1+c_x^{(2)}x_1^3=0\\
&\beta\dot{y}_1+\left(\frac{c_{12}}{l}-h_2c_{11}\right)\left(y-h_2y_1-h_1y_2\right)+\left(h_2c_{12}-\frac{c_{22}}{l}\right)\left(\theta_1+\frac{y_2-y_1}{l}\right)+c_y^{(1)}y_1+c_y^{(2)}y_1^3=0
\end{aligned}
\qquad (12)
$$

$$\beta\dot{x}_2+\left(-\frac{c_{12}}{l}-c_{11}h_1\right)\left(x-h_1x_2-h_2x_1\right)+\left(-\frac{c_{22}}{l}-h_1c_{12}\right)\left(\theta_2-\frac{x_2-x_1}{l}\right)+k_x^{(1)}x_2+k_x^{(2)}x_2^{\,3}=0$$

$$\beta\dot{y}_2+\left(-\frac{c_{12}}{l}-h_2c_{11}\right)\left(y-h_2y_1-h_1y_2\right)+\left(\frac{c_{22}}{l}+h_1c_{12}\right)\left(\theta_1+\frac{y_2-y_1}{l}\right)+k_y^{(1)}y_2+k_y^{(2)}y_2^{\,3}=0$$

where c_{11}, c_{12}, c_{22} are static coefficients of shaft stiffness; l is the shaft length; l_1, l_2 are distances of the disk up to left and right supports, correspondently; $h_1 = l_1 / l; h_2 = l / l_2$; $c_x^{(1)}, c_y^{(1)}$ are coefficients which characterize linear terms in the left support restoring force; $k_x^{(1)}, k_y^{(1)}$ are similar coefficients for the right support; $c_x^{(2)}, c_y^{(2)}$ are coefficients which characterize cubic terms in the left support restoring force; $k_x^{(1)}, k_y^{(1)}$ are similar coefficients for the right support; β is a coefficient of damping in supports; ρ_1, ρ_2 are coefficients of damping during the disk motion; m is the mass of the disk; ε is an eccentricity of the disk mass center.

4. FORCED VIBRATIONS IN ROTOR DYNAMICS.

The procedure, which was described in the Section 2, is used. As a result, nonlinear normal modes of the non-autonomous rotor system are obtained.

Numerical simulation of the rotor forced dynamics is made for the following values of the system parameters : m=15.3 kg, I_e=0.22 kg·m^2, I_p=0.441 kg·m^2, l=1 m, h_1=1/3, h_2=2/3, $c_x^{(1)}=c_y^{(1)}=k_x^{(1)}=k_y^{(1)}=9.8\cdot10^5$ N/m, $c_x^{(2)}=c_y^{(2)}=k_x^{(2)}=k_y^{(2)}=1.96\cdot10^{12}$ N/m^3, $\varepsilon=10^{-4}$ m, β=3000 N·s/m, ρ_1=1.5 N·s/m, ρ_2=1.5 N·s. Elastic shaft is described by following parameters: the Young's modulus $E=2.1\cdot10^{11}$ Pa, the cross-section radius r=0.015 m, the shaft is considered to be massless.

The phase trajectory of the obtained NNM for $\omega = 0.9922$, where ω is a ratio of the frequency of external excitation and the first frequency of the linearized system, is presented in the Fig. 3. Here the analytical solution is shown by points, and the numerical simulation is shown by the entire line.

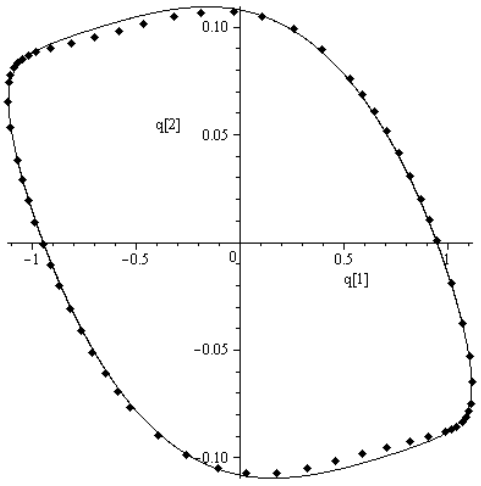

Fig. 3.The NNM phase trajectory. Analytical solution (points) and numerical simulation (entire line).

A comparison of the analytical and numerical forced NNM in time for the same ratio of the external frequency and the first linear frequency is presented in Fig. 4, where points correspond to the analytical results, and the entire line corresponds to the numerical simulation.

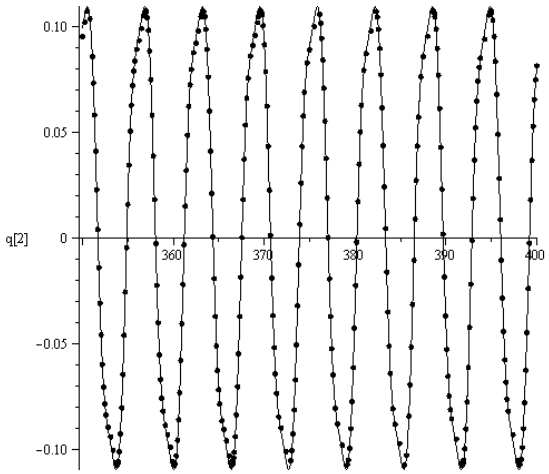

Fig. 4. Presentation of the NNM in time for the non-autonomous rotor system. Points correspond to the analytical solution; entire line corresponds to numerical simulation.

CONCLUSIONS

The forced vibrations of the non-autonomous rotor system for a case of the internal resonance is obtained by use of the nonlinear normal modes conception and the generalized Rausher method. Numerical simulation confirms an efficiency of the proposed analytical procedure.

ACKNOWLEDGMENTS

This work was supported in part by a grant М9403 ДР 0109U002426 from the Ministry of Science and Education of Ukraine and by a grant Ф28/257 from the National Academy of Science of Ukraine and Russian Foundation for Basic Research.

REFERENCES

[1] Grobov V.A. *Asymptotic Methods for Calculations of Bending Vibrations of Turbo- Machines Shafts*. Published by the USSR Academy of Sci., Moscow, 1961 (in Russian).
[2] Filippov A.P. *Vibrations of Elastic Systems*, Mashinostroenie, Moscow, 1970 (in Russian).
[3] Bolotin V.V. *Nonconservative Problems of the Theory of Elastic Stability*. Pergamon Press, New York, 1963.
[4] Tondl A. *Some Problems of Rotor Dynamics*, Chapman and Hall, London, 1965.
[5] Iwatsubo J., Kanki H., Kawai R. Vibration of asymmetric rotor through critical speed with limited power supply. *J. of Mechanical Engineering Science,* 14, pp.184-194, 1972.
[6] Ishida Y., Inoue T. Internal resonance phenomena of the Jeffcott rotor with nonlinear spring characteristics. *ASME J. of Vibration and Acoustics*, 126, pp. 476-484, 2004.
[7] Inoue T., Ishida Y. Chaotic vibrations and internal resonance phenomena in rotor systems. *ASME J. of Vibration and Acoustics*, 128, pp.156-169, 2006.
[8] Serra Villa C.V., Sinou J.-J., Thouverez F. The invariant manifold approach applied to nonlinear dynamics of a rotor-bearing system. *European J. of Mechanics A/ Solids*, 24, pp.676-689, 2005.
[9] Mikhlin Yu. Normal vibrations of a general class of conservative oscillators, *Nonlinear Dynamics*, 11, pp.1–16, 1996
[10] Vakakis A., Manevitch L., Mikhlin Yu., Pilipchuk V. and Zevin A. *Normal Modes and Localization in Nonlinear Systems*. Wiley, New-York, 1996.
[11] S. Shaw and C. Pierre. Nonlinear normal modes and invariant manifolds. *J. of Sound and Vibration*, 150, pp.170–173, 1991.
[12] S. Shaw and C. Pierre. Normal modes for nonlinear vibratory systems. *J. of Sound and Vibration*, 164, pp.85–124, 1993.