Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI 2010
September 21-24, 2010, Kharkov, Ukraine

NONLINEAR PARAMETRIC VIBRATIONS OF CYLINDRICAL SHELLS

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ABSTRACT

Donnell's equations are used to predict nonlinear vibrations of cylindrical shells, which are excited by parametric dynamical load. The finite degree-of-freedom dynamical system of cylindrical shells is derived. The nonlinear modes of the shell with dissipation and without one are analyzed by harmonic balance method. These nonlinear modes correspond to the standing waves in the shell. Traveling waves are analyzed in detail.

INTRODUCTION

Thin-walled structures are widely used in aerospace, nuclear, civil and mechanical engineering. Longitudinal periodic loads usually act on the shells and leads to complex dynamical behavior of the systems. Many efforts were made to study this behavior. Parametric oscillations of simply supported cylindrical shells are modeled by two interacting modes (asymmetric and axisymmetric ones) in [1]. Donnell's shallow shell equations were used to study parametric oscillations of cylindrical shells [2] and the fundamental role of axisymmetric modes in evaluating the parametric instability bounds is treated. The effect of initial imperfections on parametric oscillations of simply supported cylindrical shells was studied by Koval'chuk and Krasnopol'skaya [3]. Kubenko et al. [4] obtained theoretically and experimentally the frequency response and the region of the main parametric resonance of simply supported cylindrical shells. Pellicano et al. [5] analyzed nonlinear oscillations and dynamic instability of simply supported cylindrical shells under the action of longitudinal dynamic forces. The dynamic stability of cylindrical shells under the action of both static and periodic axial loads is treated in [6]. Analysis of nonlinear modes of cylindrical shells, which are described by three mode model, is considered in the paper [7]. Detailed reviewer of cylindrical shell dynamics is presented in [8].

Nonlinear dynamics of cylindrical shells in the case of the main parametric resonance is treated in the present paper. Cylindrical shells have dense frequency spectrum. Therefore, the case, when the three eigenfrequencies of conjugate modes are close, is considered. This case occurs frequently in shell dynamics. These three conjugate modes are taken into account in analysis of the main parametric resonance.

1. PROBLEM FORMULATION AND MAIN EQUATIONS

The simply supported cylindrical shell without imperfections is considered. The following time periodic distributed parametric load acts on the shell (Fig.1):

$$N_x(t) = N_1 \cos 2\nu t\,,\ N_1 = const > 0 \tag{1}$$

where ν is an excitation frequency. The vibrations of shell have moderate amplitudes. Then the strains are small and displacements are moderate and the strains- displacement relations are nonlinear. The strains and stresses satisfy the Hooke's law. In this case the following Donnell's equations describe the shell vibrations adequately [1, 5]:

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$$\frac{D}{h}\nabla^4 w+\rho\frac{\partial^2 w}{\partial t^2}=\frac{\partial^2 F}{R\partial x^2}+\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 F}{\partial y^2}-2\frac{\partial^2 w}{\partial x\ \partial y}\frac{\partial^2 F}{\partial x\partial y}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 F}{\partial x^2}\right)$$

$$\frac{1}{E}\nabla^4 F=-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}+\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{2}$$

where w is displacements of the middle surface points in the radial directions; x, y are longitudinal and circumferential coordinates; R, ρ are mean shell radius and material density; E,μ are Young's modulus and Poisson's ratio; F is an in-plane stress function; $D=Eh^3/\left(12\left(1-\mu^2\right)\right)$ is a flexural rigidity.

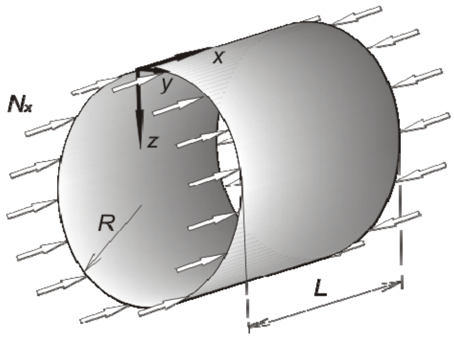


Fig. 1 Cylindrical shell

The conjugation vibrations modes $\cos sy \sin rx$ and $\sin sy \sin rx$ have the same frequencies of cylindrical shells vibrations. If a shell performs nonlinear vibrations, these modes can be excited jointly. As follows from the shell analysis [9], wide class of cylindrical shells has three close eigenfrequencies of conjugate modes. In future analysis the main parametric resonance is considered $\nu\approx\omega_i; i=1,2,...,6$, where $\omega_{2i-1}=\omega_{2i}; i=\overline{1,3}$ are equal eigenfrequencies of conjugate modes. Three conjugate modes are taken into account in the expansion of the displacements in the radial directions. Then the dynamic flexure w can be presented as:

$$w=\sum_{i=1}^{3}\left(f_{2i-1}\cos s_i y+f_{2i}\sin s_i y\right)\sin rx+f_7\ \sin^2 r\ x+f_8 \tag{3}$$

where $s_i=n_i/R;\ r\ =m\ \pi/L; i=1,2,3;$ n_i is numbers of waves in circumference directions; m is a number of half-waves in x direction. The summand $f_7\sin^2 r\ x$ describes asymmetry of dynamic flexure with respect to a middle surface. The term f_8 describes displacements in radial directions of shell face sections points. This term does not depend on circumference coordinate y. Therefore, the face sections can "breathe" [4].

The in-plane stress function F is determined from the second equation of the system (2), satisfying the periodicity conditions of the circumference displacements. The stress function is substituted into the first equation of the system (2) and the Galerkin method is applied to the resulted equation. Assuming that $\ddot{f}_7=0$, $\ddot{f}_8=0$ [11], the finite-degree-of-freedom shell model with respect to the dimensionless variables and parameters has the following form:

$$\ddot{f}_i\ +\omega_i^2 f_i\ +f_i\ R_i\left(f_1\ ,...,f_6\right)+G_i\left(f_1\ ,...,f_6\right)+\chi_i N_x f_i\ =0,\quad i=1,2,...,6 \tag{4}$$

2. NONLINEAR MODES AND HARMONIC BALANCE ANALYSIS

The nonlinear dynamics of the system (4) is analyzed in this section. The equations

$$f_{2i-1}=\pm f_{2i},\ i=1,2,3 \tag{5}$$

are exact solutions of the system (4). If the solutions (5) are substituted into (4), the following dynamical system is derived:

$$\ddot{f}_i + \omega_i^2 f_i + f_i \tilde{R}_i \left(f_1 , f_3 , f_5 \right) + \tilde{G}_i \left(f_1 , f_3 , f_5 \right) + \chi_i N_x f_i = 0, \quad i = 1,3,5 \tag{6}$$

The solutions (5) are called nonlinear modes. These nonlinear modes are straight lines in configuration space. The dynamical system (6) describes the motions on nonlinear modes.

The harmonic balance method is used to study the motions on the nonlinear modes (6). As the nonlinear modes for the main parametric resonance are considered, the motions are presented as:

$$f_i = A_i \cos(\nu t) + B_i \sin(\nu t), i = 1,3,5 \tag{7}$$

Now (7) is substituted into (6) and the amplitudes of harmonics $\cos(\nu t)$ and $\sin(\nu t)$ are equated. As a result the following system of nonlinear algebraic equations is derived (values η_{ij}, χ_i depend on the shell parameters):

$$A_i \left(\omega_i^2 - \nu^2 + \eta_{ii} A_i^2 + \frac{1}{2} \sum_{j=1,3,5} \eta_{ij} \left(3A_j^2 + B_j^2\right) + \frac{1}{2} \chi_i N_1 \right) + G_i^{(A)} \left(A_1 , A_3 , A_5 , B_1 B_3 B_5 \right) = 0$$

$$B_i \left(\omega_i^2 - \nu^2 + \eta_{ii} B_i^2 + \frac{1}{2} \sum_{j=1,3,5} \eta_{ij} \left(3B_j^2 + A_j^2\right) - \frac{1}{2} \chi_i N_1 \right) + G_i^{(B)} \left(A_1 , A_3 , A_5 , B_1 B_3 B_5 \right) = 0, i = 1,3,5 \tag{8}$$

The following groups of solutions exist in the system (8):

1.1). $A_1 \neq 0;\ A_3 = A_5 = 0;\ B_i = 0;$ 1.2). $B_1 \neq 0;\ B_3 = B_5 = 0;\ A_i = 0;$
2.1). $A_3 \neq 0;\ A_1 = A_5 = 0;\ B_i = 0;$ 2.2). $B_3 \neq 0;\ B_1 = B_5 = 0;\ A_i = 0;$
3.1). $A_5 \neq 0;\ A_1 = A_3 = 0;\ B_i = 0;$ 3.2). $B_5 \neq 0;\ B_1 = B_3 = 0;\ A_i = 0;$
4.1). $A_1 \neq 0;\ A_5 \neq 0;\ A_3 = 0;\ B_i = 0;$ 4.2). $B_1 \neq 0;\ B_5 \neq 0;\ B_3 = 0;\ A_i = 0;$
5.1). $A_1 \neq 0;\ A_3 \neq 0;\ A_5 \neq 0;\ B_i = 0;$ 5.2). $B_1 \neq 0;\ B_3 \neq 0;\ B_5 \neq 0;\ A_i = 0$, $i = 1,3,5$ (9)

Now every group of solutions is considered separately. Fixing the value ν with a certain step size, the solutions are determined from the system of nonlinear algebraic equations (8). The solutions (1.1 – 4.2) can be determined analytically. The solutions (5.1, 5.2) are analyzed numerically by the Newton method with respect to $A_1 , B_1 , A_3 , B_3 , A_5 , B_5$.

Now the nonlinear vibrations of cylindrical shells are considered accounting energy dissipation. Then the linear damping is added into the system (4). The resulted system has the following form:

$$\ddot{f}_i + \xi_i \dot{f}_i + \omega_i^2 f_i + f_i R_i \left(f_1 ,..., f_6 \right) + G_i \left(f_1 ,..., f_6 \right) + \chi_i N_x f_i = 0, \quad i = 1,2,...,6. \tag{10}$$

Note, that the equations $f_{2i-1} = \pm f_{2i};\ i = 1,2,3$ are exact solution of the system (10). These solutions correspond to nonlinear modes. Moreover, these nonlinear modes coincide with the nonlinear modes of the system without dissipation (6). The harmonic balance method is used to study these nonlinear modes and the system motions are presented in the form (7). Then the system of nonlinear algebraic equations with respect to amplitudes of harmonics (7) is derived as:

$$A_i \left(\omega_i^2 - \nu^2 + \eta_{ii} A_i^2 + \frac{1}{2} \sum_{j=1,3,5} \eta_{ij} \left(3A_j^2 + B_j^2\right) + \frac{1}{2} \chi_i N_1 \right) + B_i \xi_i \nu + G_i^{(A)} \left(A_1 , A_3 , A_5 , B_1 B_3 B_5 \right) = 0$$

$$B_i \left(\omega_i^2 - \nu^2 + \eta_{ii} B_i^2 + \frac{1}{2} \sum_{j=1,3,5} \eta_{ij} \left(3B_j^2 + A_j^2\right) - \frac{1}{2} \chi_i N_1 \right) - A_i \xi_i \nu + G_i^{(B)} \left(A_1 , A_3 , A_5 , B_1 B_3 B_5 \right) = 0, i = 1,3,5 \tag{11}$$

The following groups of solutions exist in the system (11):

$$1).\ A_1 \neq 0;\ B_1 \neq 0;\ A_3 = A_5 = 0;\ B_3 = B_5 = 0;\ 2).\ A_3 \neq 0;\ B_3 \neq 0; A_1 = A_5 = 0;\ B_1 = B_5 = 0;$$
$$3).\ A_5 \neq 0;\ B_5 \neq 0; A_1 = A_3 = 0; B_1 = B_3 = 0;\ 4).\ A_1 \neq 0;\ A_5 \neq 0;\ B_1 \neq 0;\ B_5 \neq 0;\ A_3 = 0; B_3 = 0;$$
$$5).\ A_1 \neq 0;\ A_3 \neq 0;\ A_5 \neq 0;\ B_1 \neq 0;\ B_3 \neq 0;\ B_5 \neq 0 \qquad (12)$$

The solutions (12) of the system (11) are analyzed numerically. Setting the parameter ν with a certain step, the system (11) are solved by the Newton method.

The traveling waves for the main parametric resonance, which are described by the system (10), are considered taking into account dissipation. The harmonic balance method is used to study these motions and the system vibrations are presented as:

$$f_i = A_i \cos(\nu t) + B_i \sin(\nu t),\ f_{i+1} = A_i \sin(\nu t) + B_i \cos(\nu t),\ i = 1,3,5 \qquad (13)$$

Then the amplitudes of harmonics (13) are determined from the following system of nonlinear algebraic equations:

$$A_i \left(\omega_i^2 - \nu^2 + \eta_{ii} B_i^2 + \sum_{j=1,3,5} \eta_{ij} \left(A_j^2 + B_j^2 \right) \pm \frac{1}{2} \chi_i N_1 \right) \pm B_i \xi_i \nu + \tilde{G}_i^{(A)} \left(A_1, A_3, A_5, B_1 B_3 B_5 \right) = 0$$
$$B_i \left(\omega_i^2 - \nu^2 + \eta_{ii} A_i^2 + \sum_{j=1,3,5} \eta_{ij} \left(A_j^2 + B_j^2 \right) \pm \frac{1}{2} \chi_i N_1 \right) \pm A_i \xi_i \nu + \tilde{G}_i^{(B)} \left(A_1, A_3, A_5, B_1 B_3 B_5 \right) = 0, i = 1,3,5 \qquad (14)$$

The following groups of solutions exist in the system (14):

$$1).\ A_1 = B_1 \neq 0;\ A_3 = A_5 = B_3 = B_5 = 0;\ 2).\ A_1 = B_1 \neq 0; A_5 = B_5 \neq 0;\ A_3 = B_3 = 0;$$
$$3).\ A_1 = B_1 \neq 0; A_5 = B_5 \neq 0;\ A_3 = B_3 \neq 0 \qquad (15)$$

Altering the frequency of the parametric load ν, the system (14) is solved by the Newton method.

In order to analyze stability of periodic vibrations, the system of variational equations is derived and fundamental matrix is calculated numerically. Then the multipliers are obtained from the fundamental matrix [10].

3. NUMERICAL ANALYSIS OF VIBRATIONS

The shell with the parameters (16) [4] is considered. The frequencies of shell linear vibrations are also presented ($\mathrm{rad/s}$) (16).

$$h = 0.002\mathrm{m}, L = 0.4\mathrm{m}, R = 0.2\mathrm{m}, E = 2.1 \times 10^{11}\,\mathrm{N/m^2}, \mu = 0.3, \rho = 7850\mathrm{kg/m^3}, N_1 = 1.5 \times 10^6\,\mathrm{N/m}$$
$$\omega_{3,1} = 5636.3; \omega_{4,1} = 3745.3; \omega_{5,1} = \omega_0 = 3165.0; \omega_{6,1} = 3437.2; \omega_{7,1} = 4214.3; \omega_{8,1} = 5289.5 \qquad (16)$$

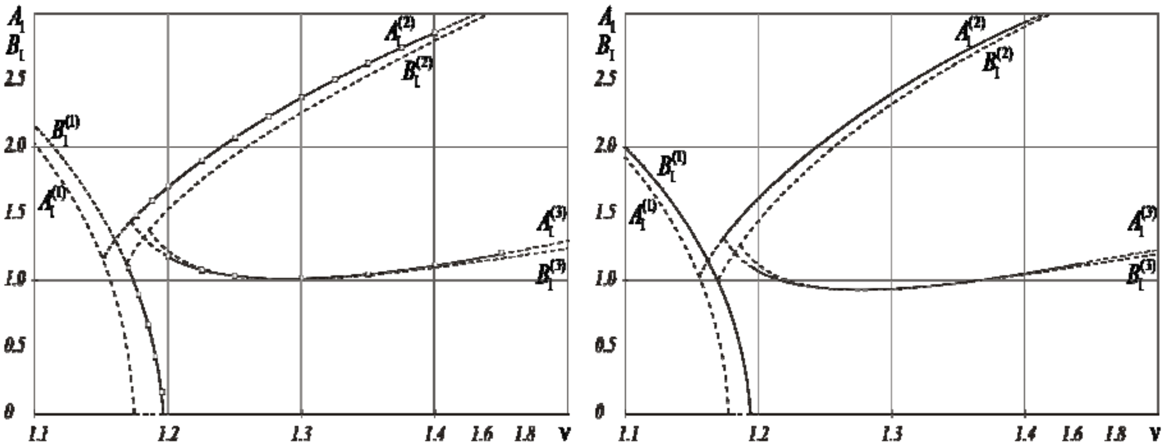
where the first subscript indicates the wave numbers in circumference direction and the second subscript shows the number of half-waves in x directions (Fig.1). In future nonlinear analysis the modes with the following parameters are taken: $n_1 = 4$; $n_2 = 5$; $n_3 = 6$; $m = 1$.

The dependence of the vibrations amplitudes A_1, B_1 on the frequency ν are presented on the frequency response (Fig.2a). The stable solutions are denoted by solid lines and the unstable solutions are shown by dashed lines. The branches of the frequency response (Fig.2a) are denoted by $A_1^{(1)}$, $B_1^{(1)}$ for the cases (1.1, 1.2) of the equations (9). In this case only one pair of the conjugate modes from the expansion (3) is active. The branches $A_1^{(2)}$, $B_1^{(2)}$ (Fig.2a) describe the motions with two pairs of conjugate vibrations modes. These solutions correspond to the cases (4.1) and (4.2) of the equations (9). The branches $A_1^{(3)}, B_1^{(3)}$ of the frequency response show the vibrations with three pairs of conjugate modes, which correspond to the cases (5.1) and (5.2) of the equations (9).

The direct numerical integrations of the system (6) at different values of frequency ν are carried out to confirm the analytical results. Using such approach, only stable solutions are derived. The data of the calculations are shown by small squares on Fig.2a. The results of the direct numerical integration are very close to the data, which are obtained by harmonic balance method.

Carrying out numerical integration on long time interval, the periodic solution is considered unstable, if the numerical trajectory escapes from the considered one to another trajectory. To study stability of the parametric vibrations the direct numerical integration of the differential equations (4) is carried out on the time interval $t \in \left[0; 2000\,\pi\nu^{-1}\right]$. The initial conditions are determined from the equations (7, 14).

The dynamics of the system with dissipation (10) on the nonlinear modes is presented on the frequency response (Fig.2b). The numerical analysis of the traveling waves is carried out. Fig.3 shows the frequency response of the traveling waves.



a) b)

Fig. 2 Frequency response of parametric vibrations on nonlinear mode of the system a) without dissipation, b) with dissipation

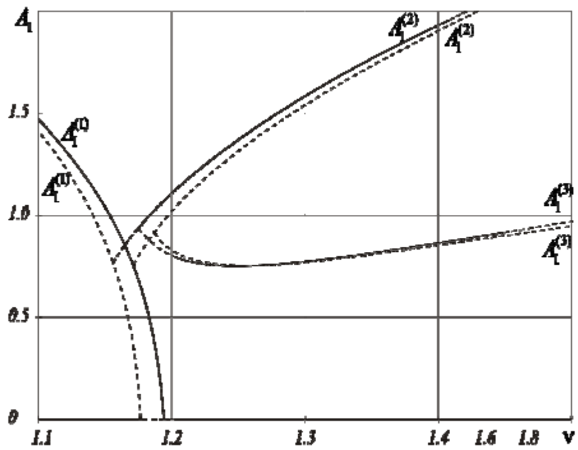


Fig. 3 Frequency response of the traveling waves of the system with dissipation

CONCLUSIONS

One and two conjugate modes approximations of shell vibrations are not enough to predict dynamics of wide class of cylindrical shells. This is explained by closeness of the eigenfrequencies of the different conjugate modes. In this case only many modes models of shells describe the parametric vibrations adequately. The following vibrations are analyzed in this paper: a) one pair of conjugate modes is active; b) two or three pairs of conjugate modes are active.

Nonlinear modes, which are straight lines in a configuration space, are observed for many modes shells dynamics. We stress, that the same nonlinear modes exist both in the system without damping and in the system with damping. The existence of such normal modes is explained by cyclic symmetry of cylindrical shells.

Nonlinear modes and traveling waves are some solutions of the dynamical system (4). The traveling waves are described by the equations (13). As follows from the results of the analysis, the normal modes and traveling waves exist in the frequency bands $\nu \in [1; 1.6]$ and $\nu \in [1.1; 1.8]$, respectively. Thus, the frequency band ν with two kinds of motions exists. Any one of these motions has a basin of attraction. Therefore, if the initial conditions belong to the basin of attraction of nonlinear mode or traveling waves, then nonlinear mode or traveling waves take place.

All frequency responses of nonlinear modes and traveling waves are qualitative similar. This is explained by similarity of the systems of nonlinear algebraic equations with respect to amplitudes.

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