Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

NONLINEAR PHYSICS AND ENERGY LOCALIZATION IN PERIODIC SYSTEMS

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ABSTRACT

The recent progress in the study of the energy localization and solitons in a variety of nonlinear systems where the effects of *discreteness* and *periodicity* become important, is overviewed.

The recent progress in the study of the energy localization and solitons in a variety of nonlinear systems where the effects of *discreteness* and *periodicity* become important, is overviewed. This panoramic presentation will cover (i) generation and control of optical gap solitons in waveguide arrays and photonic lattices, including the most recent observation of polychromatic gap solitons and dynamics localization of light generated by a supercontinuum source, (ii) localized matter waves of Bose-Einstein condensates in two- and three-dimensional optical lattices, (iii) discrete localized modes in composite metamaterials and nanophotonic structures, and (iv) energy localization in carbon nanotubes and graphene nanoribbons.

First of all, the most important recent advances in nonlinear photonics where many of novel theoretical findings have been verified in experiment, is emphasized. This includes the observation of surface solitons in one- and two-dimensional photonic lattices, the observation of polychromatic "rainbow" gap solitons in photonic lattices generated by a supercontinuum source [1], the generation of topologically stable spatially localized multivortex solitons, etc.

One of the recent concepts in the theory of nonlinear waves is associated with a novel type of broad nonlinear states which appear in the gaps of the bandgap spectra of periodic systems such as light waves in periodic photonic lattices and Bose-Einstein condensates in optical lattices. These localized states cannot be treated by familiar multi-scale asymptotic expansion techniques, and they can be better understood as truncated nonlinear Bloch waves [2]. I demonstrate that these self-trapped localized nonlinear modes can be found in one-, two-, and three-dimensional periodic potentials, and they have been readily observed in experiments on nonlinear self-trapping of matter waves in one-dimensional optical lattices.

Finally, the energy localization in graphene structures and demonstrate the existence of spatially localized nonlinear modes in the form of discrete breathers in carbon nanotubes and nanoribbons [3], is discussed. In nanotubes with the chirality index $\boldsymbol{(m,0)}$ there exist three types of discrete breathers associated with longitudinal, radial, and torsion anharmonic vibrations, however only *twisting breathers* survive in a curved geometry remaining long-lived modes even in the presence of thermal fluctuations.

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IRREVERSIBLE TRANSFER OF VIBRATION ENERGY IN LINEAR AND NONLINEAR COUPLED PARAMETRIC SYSTEMS

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ABSTRACT

We present a novel principle of vibration energy trapping based on vibration analogue of nonadiabatic Landau-Zener tunneling. We demonstrate analytically and numerically that in a system of two weakly coupled pendulums or oscillators, linear or nonlinear, an efficient irreversible transfer of vibration energy from one subsystem to another occurs when the coupled subsystems pass through the internal resonance. The internal resonance takes place due to parametric drive when the length, mass or spring stiffness of at least one of the pendulums or oscillators varies in the course of vibrations. Nonlinear effects result in a separatrix mode of vibration energy transfer, in the vicinity of which the irreversible character of the energy transfer is substantially enhanced.

INTRODUCTION

Tunneling is one of the most striking manifestations of quantum behavior and has been the subject of extensive research both in fundamental and applied physics. A well-known generic example of tunneling phenomenon is Landau-Zener tunneling (LZT), in which a quantum system subject to an external force tunnels across an energy gap between anti-crossing energy levels [1,2]. Quantum LZT was observed in semiconductor superlattices for electrons, as well as in optical lattices for ultracold atoms and Bose-Einstein condensates. In the case of electrons in semiconductor superlattice, the external force responsible for nonadiabatic energy-level crossing and LZT is exerted by an external electric field. LZT of optical waves was observed in optical lattices [3] and optical waveguide arrays [4]. Recently, LZT of bulk and surface acoustic waves in ultrasonic superlattices was predicted and observed [5,6]. Effective external forces in optical or acoustic LZT are produced by the perturbation of the corresponding optical or ultrasonic lattice.

The common feature of the aforementioned examples of nonadiabatic LZT is the irreversible (and almost unidirectional) exchange of energy between two states caused by external forces or perturbations. The possibility of this type of exchange would also be desirable in vibrating mechanical systems, e.g., in towers or in an airplane's wings. Here the impact excitation threatening the structural integrity of the system must be irreversibly transferred to a sacrificial subsystem. It turns out that a system governed by equations similar to that of a quantum system can in fact be designed. We noticed earlier a profound analogy between adiabatic quantum tunneling and energy exchange between weakly coupled oscillators, both linear and nonlinear [7]. In this work we present a vibration analogue of nonadiabatic quantum Landau-Zener tunneling that reveals a new type of energy trapping. We demonstrate analytically and numerically that a Landau-Zener-like transition can take place in a system of two weakly coupled oscillators. This can occur when the length, mass or spring stiffness of at least one of the oscillators varies during vibration. In result, an efficient irreversible transfer of vibration energy from one oscillator to another takes place when the coupled subsystems pass through the internal resonance. Such mechanical oscillatory systems represent new types of energy traps. These can be easily generalized for the dynamic protection of more complex systems from vibro-impact actions, with numerous potential applications in nano-, micro-, and macromechanics. Nonlinear effects can enhance the irreversible character of the vibration energy transfer.

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1. IRREVERSIBLE TRANSFER OF VIBRATION ENERGY IN LINEAR COUPLED PARAMETRIC SYSTEMS

We consider a system of two plane pendulums with lengths l_1 and l_2, and masses m_1 and m_2, weakly coupled by a spring (with a comparable with l_1 equilibrium length). The Lagrange function of the system is written as follows:

$$L = \frac{1}{2}\left(m_1 l_1^2 \left(\frac{d\varphi_1}{dt}\right)^2 + m_2 l_2^2 \left(\frac{d\varphi_2}{dt}\right)^2 \right) - g\left[m_1 l_1 (1-\cos\varphi_1) + m_2 l_2 (1-\cos\varphi_2)\right] - \frac{1}{2} k_{12} \left(l_1 \sin\varphi_1 - l_2 \sin\varphi_2\right)^2 \quad (1)$$

where φ_1 and φ_2 are the deflection angles, and k_{12} is the spring constant. Let l_1 be a constant and l_2 be a function of time. Then the corresponding equations of motion are:

$$\begin{aligned} & m_1 l_1^2 \frac{d^2\varphi_1}{dt^2} + m_1 g l_1 \sin\varphi_1 + k_{12} l_1 \cos\varphi_1 \left(l_1 \sin\varphi_1 - l_2 \sin\varphi_2\right) = 0 \\ & m_2 l_2^2 \frac{d^2\varphi_2}{dt^2} + 2 m_2 l_2 \frac{dl_2}{dt}\left(\frac{d\varphi_2}{dt}\right) + m_2 g l_2 \sin\varphi_2 + k_{12} l_2 \cos\varphi_2 \left(l_2 \sin\varphi_2 - l_1 \sin\varphi_1\right) = 0 \end{aligned} \quad (2)$$

We assume that

$$l_2(t) = l_1(1 + \Delta_2(t)) \quad (3)$$

where $\Delta_2(t)$ describes a (relatively small) change in time of l_2. In order to avoid a superfluous decrease in l_2, in the following we assume that

$$\Delta_2(t) = \delta_2 - f_2 T_2 \tanh(t/T_2) \quad (4)$$

where δ_2 and f_2/ω_1 are independent small parameters of the same sign, $\omega_1 = \sqrt{g/l_1}$.

Since the LZT is basically linear phenomenon, we start with the analysis of linearized Eqs. (2) for the case of $\varphi_1 << 1$ and $\varphi_2 << 1$. There are several ways to proceed from two real equations of the second order (2) to four complex equations of the first order. Following the approach used in [8], we introduce two *complex envelopes* a_1 and a_2 of the real deflection angles φ_1 and φ_2:

$$\varphi_{1,2} = \frac{1}{2}\left[a_{1,2} e^{-i\omega_1 t} + a_{1,2}^* e^{i\omega_1 t}\right] \quad (5)$$

where we assume that $d a_{1,2}/d t << \omega_1 a_{1,2}$. As follows from Eq. (5), the real part of the variable a_i determines the envelope of φ_i, while its imaginary part determines the envelope of the dimensionless time derivative $d\varphi_i/d(t\omega_1)$, $i = 1,2$. These properties of complex envelopes allow us to easily relate the envelope modulus $|a_i|$ with the vibration energy of the linearized i-th pendulum: $E_i = 0.5 g l_1 m_i |a_i|^2$.

Substituting Eqs. (3) and (5) in linearized Eqs. (2), we get the following two evolution equations for the complex envelopes a_1 and a_2 in the main approximation with respect to small parameters δ_2, f_2/ω_1, and $k_{12}/\mu\omega_1^2$ ($\mu = 1/(1/m_1 + 1/m_2)$ is a reduced mass of m_1 and m_2):

$$\begin{aligned} & i\frac{da_1}{dt} = \frac{k_{12}}{2 m_1 \omega_1}(a_1 - a_2) \\ & i\frac{da_2}{dt} = \frac{k_{12}}{2 m_2 \omega_1}(a_2 - a_1) - \frac{1}{2}\omega_1 \Delta_2(t) a_2 \end{aligned} \quad (6)$$

We also obtain two corresponding equations for the complex-conjugated envelopes a_1^* and a_2^*. The total vibration energy of the coupled pendulums is given by $0.5gl_1(m_1|a_1|^2 + m_2|a_2|^2)$, which is the integral of motion. For $t < T_2$ and $\Delta_2(t)$ given by Eq. (4), Eqs. (6) coincide with the modified description of the quantum Landau-Zener-like transition [3]. The multiple scale expansion procedure, presented, e.g., in [9], leads to similar complex evolution equations.

The same Eqs. (6) describe the dynamics of the complex envelopes of the displacements u_1 and u_2 of two oscillators with masses m_1 and m_2 and springs with equal coefficient of stiffness κ_1 or two oscillators with equal masses m_1 and springs with coefficients of stiffness κ_1 and κ_2, weakly coupled by a spring with coefficient of stiffness $k_{12} << \kappa_1$, when either m_2 or $1/\kappa_2$ changes in time according to Eq. (3) ($l_{1,2}$ should be replaced by $m_{1,2}$ or $1/\kappa_{1,2}$). Introducing two complex envelopes a_1 and a_2 of the real displacements u_1 and u_2 according to Eq. (5), under the same assumption $da_{1,2}/dt << \omega_1 a_{1,2}$ we obtain LZT-like Eqs. (6) for the complex envelopes a_1 and a_2. Here now $\omega_1 = \sqrt{\kappa_1/m_1}$ and parameter m_2 equals m_1 in the evolution equation for a_2.

The asymptotic analytical solution of Eqs. (6) for large positive t with $\Delta_2(t)$ given by Eq. (4) and the initial conditions $|a_1(-\infty)|^2 = 1, a_2(-\infty) = 0$ can be written as:

$$|a_1(\infty)|^2 = \exp(-R), \quad R = \pi k_{12}^2/(m_1 m_2 |f_2| \omega_1^3) \tag{7}$$

This equation describes the part of the initial vibration energy that is retained asymptotically in pendulum 1.

To check the efficiency of the system of pendulums in the capacity of an energy trap, we calculated the time evolution of vibration energies of the coupled pendulums from the solution of linearized Eqs. (2) for the deflection angles φ_1 and φ_2 and compare it with the numerical solution of LZT-like Eqs. (6) for the complex envelopes a_1 and a_2. Since the damping of low-frequency vibrations of pendulums is very small, the effect of damping on the energy exchange between pendulums can be neglected in the main approximation.

In Fig. 1 we plot the vibration energies E_1 and E_2 of pendulums 1 and 2 with $m_1 = m_2$ and their total energy E_T versus time from the solution of linearized Eqs. (2) (lines 1, 2 and 3) alongside with a solution of LZT-like Eqs. (6) (lines 4 and 5) and with the LZT-like prediction, given by Eq. (7), for the part of initial vibration energy which is retained asymptotically in pendulum 1, line 6. The initial conditions correspond to the impact excitation of pendulum 1. The following realistic parameters and initial conditions were taken: $l_1 = 0.305$ m, $m_1 = 0.244$ kg, $k_{12} = 0.785$ N/m, $\delta_2 = 0.22$, $f_2 = 0.062\ s^{-1}$ and $T_2 = 15.6$ s, and

$$\begin{aligned} &\varphi_1(0) = 0, \varphi_2(0) = 0, \dot{\varphi}_2(0) = 0, \dot{\varphi}_1(0) = 0.61 \text{ rad/s} \\ &ia_1(0) = \dot{\varphi}_1(0)/\omega_1, a_2(0) = 0 \end{aligned} \tag{8}$$

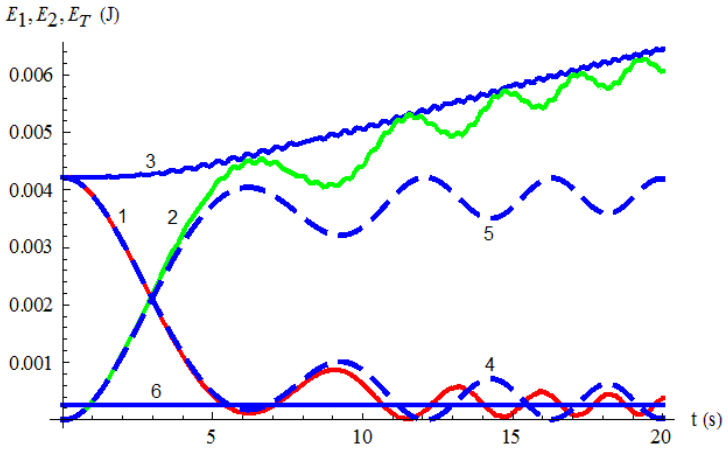

Fig. 1. Solid lines 1, 2 and 3: Vibration energies E_1 and E_2 of pendulums 1 and 2 and their total energy E_T versus time as solutions of linearized Eqs. (2). Dashed lines 4 and 5: Vibration energies of pendulums 1 and 2 as solutions of LZT-like envelope Eqs. (6). Solid line 6: Part of initial vibration energy which is retained asymptotically in pendulum 1, given by Eq. (7). Parameters used in the calculations are given by Eqs. (3), (4) and (8) in the case of equal pendulum masses.

As one can see, the irreversible and intensive energy flow from the pendulum 1 to the pendulum 2 occurs. One can also conclude from Fig. 1 that the LZT-like envelope equations (6) correctly reflect the regularities of the process during its initial stage, when the most intensive resonance energy transfer occurs. The LZT-like prediction for the part of initial vibration energy, which is retained asymptotically in pendulum 1, is also impressively confirmed in our simulations, although the factor R in Eq. (7) is not small ($R = 2.85$). According to our simulations, this value of *R* gives an approximate upper limit of the applicability of Eq. (7) for the considered classical systems. Large enough saturation time T_2 influences only the transient dynamics without affecting the asymptotic energy of pendulum 1.

From the physical point of view, the irreversible energy exchange revealed above can be considered as the targeted energy transfer (TET) [7,10]. The exact internal resonance is fulfilled when $l_2 = l_1$ (or $m_2 = m_1$, $\kappa_2 = \kappa_1$) and the eigenfrequencies of the coupled oscillators become equal (which occurs at $t = \delta_2 / f_2$). As the system moves out of resonance (for $t > \delta_2 / f_2$), there is no considerable reverse energy flow from pendulum 2 to pendulum 1. This phenomenon makes the second oscillator a vibration energy trap.

Our calculations also show that the use of the lower or larger mass of pendulum 2 does not essentially suppress the irreversible TET. By corresponding change of the parameters δ_2 and k_{12} together with the ratio between m_2 and m_1, we can obtain a good agreement with the LZT-like prediction given by Eq. (7) both for $m_2 < m_1$ (e.g., for $m_2 = 0.5 m_1$) and $m_2 > m_1$ (e.g., for $m_2 = 2 m_1$). Importantly in all the considered cases, the most interesting for possible applications time evolution and average asymptotic value of vibration energy of pendulum 1 are correctly described by conservative LZT-like equations (6), although the original classical system is a non-conservative one.

2. IRREVERSIBLE TRANSFER OF VIBRATION ENERGY IN NONLINEAR COUPLED PARAMETRIC SYSTEMS

Now we describe briefly the effect of nonlinear properties (anharmonicity) of the coupled pendulums or oscillators on the irreversible vibration energy exchange between them. The effect of nonlinearities on the energy transfer in the considered coupled parametric system, described by Eqs. (2), increases with the increase of initial pulse given to pendulum 1, which is proportional to $\dot{\varphi}_1(0)$. In Fig. 2(a) we present numerical solution of nonlinear Eqs. (2) for the time dependence of energies of the coupled pendulums in the case of relatively high initial pulse given to pendulum 1, $\dot{\varphi}_1(0) = 7.93$ *rad/s* for $m_1 = m_2$, when the rest of parameters is the same as in Fig. 1. Due to energy transfer from pendulum 1 and parametric drive, at $t \approx 15\,s$ pendulum 2 finds itself in the whirling mode, in which

the reverse energy flow to pendulum 1 is suppressed. Such transition to the whirling mode of pendulum 2 is clearly seen in Fig. 2(b), which shows time dependence of pendulums deflection angles. Thus figure 2(a) demonstrates that nonlinear effects, together with optimized initial conditions and other parameters of the system, can make the energy transfer very effective: almost 100% energy of pendulum 1 is irreversibly transferred to pendulum 2 in 10 seconds.

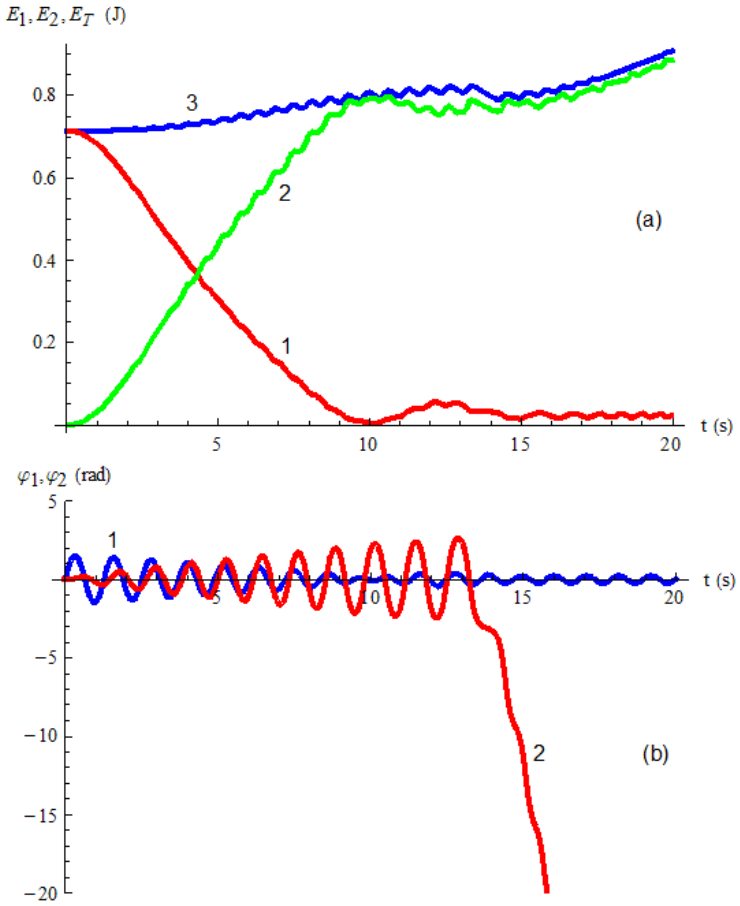

Fig. 2 (a) Vibration energies E_1 and E_2 of pendulums 1 and 2 and their total energy E_T versus time as solutions of nonlinear Eqs. (2), lines 1, 2 and 3, respectively. (b) Deflection angles φ_1 and φ_2 versus time as solutions of nonlinear Eqs. (2), lines 1 and 2, respectively. Parameters used in the calculations are given by Eqs. (3), (4) and (8) in the case of $\dot{\varphi}_1(0) = 7.93$ rad/s and equal pendulum masses.

For the high enough initial pulse given to pendulum 1, it will immediately be excited to the whirling mode, in which further energy transfer to pendulum 2 is strongly suppressed. This means that the considered parametric system is characterized by an effective *TET separatrix*, which detaches two modes with almost complete and strongly suppressed incomplete energy exchange. TET separatrix is known for the energy transfer in *passive* nonlinear systems, in which the nonlinearity substantially changes the rate and completeness of the TET through the self-trapping of energy in one of the coupled subsystems [7,10]. Our results demonstrate that nonlinearities of the coupled elements can substantially affect TET in the *active* (parametric) systems also.

CONCLUSIONS

We present a novel principle of trapping of the vibration energy. This principle is based on the profound analogy that we have found between the irreversible transfer of the vibration energy in a classical parametric system and quantum nonadiabatic Landau-Zener tunneling. We demonstrate analytically and numerically that in a system of two weakly coupled pendulums or oscillators an efficient irreversible transfer of vibration energy from one subsystem to another occurs when the coupled subsystems pass through the internal resonance. Nonlinear effects can substantially enhance the irreversible character of the transfer of the vibration energy. The revealed phenomena open up the possibility of designing the fundamentally new types of energy traps for the dynamic protection of various nano-, micro-, and macromechanical systems.

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