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NUMERICAL MODAL ANALYSIS OF SANDWICH PLATES PARTIALLY DAMAGED DUE TO IMPACTS

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ABSTRACT

Dynamic responses of sandwich composite plates containing impact-induced damage are studied. A finite element formulation of the problem is developed by using the high-order sandwich plate theory. The finite element code ABAQUS is used to create a finite element model of the sandwich plate accounting for geometrical imperfections, stiffness changes and intermediate contact of detached plate parts caused by impact damage and to fulfil the free and forced vibration analyses.

INTRODUCTION

Dynamic responses of sandwich plates, which may offer the potential of 20-40 % weight saving over their metal counterparts of the same bending stiffness, are of primary importance for the design of aircrafts and spacecrafts. A good understanding of the free vibration behavior of such structural elements is essential toward a reliable prediction of their dynamic response to time-dependent external excitations, prevention of the occurrence of the resonance, and for optimal design from the vibrational point of view. Analytical, numerical and experimental studies of the dynamic response of sandwich plates have received a good deal of attention and a variety of references can be found in the free literature, e.g. [1-3]. Modal analysis being the normal issue for sandwich plates becomes a problem requiring enhanced attention when there are some imperfections within the sandwich plates such as cracks, partially damaged parts and so on. The presence of damages will affect the dynamic response of the plates and, hence, need to be studied as a single task.

One of unavoidable damages of structural components of aircrafts and spacecrafts during their in-service life is a flaw inflicted by impacts. Several works treating the dynamic flexural behavior of sandwich beams and plates have confirmed the substantial susceptibility of the sandwich structures to damage caused by the low-velocity impact of foreign objects [4]. It has been shown that the impact-induced damage greatly affects the load capacity of the structural components, causing them to fail at lower loads than expected and modifies their vibration characteristics resulting in a hazard that a construction may resonate at other working frequencies than it was initially found.

Analytical approaches for studying mechanical behaviors of sandwich structures containing the damaged core and facesheet and the imperfect core-to-facesheet interface are extremely difficult and are mostly confined by one- and two-dimensional models with through-width damaged region. A damaged beam (or plate) is being divided into separate regions, namely undamaged and damaged ones, which obey continuity conditions on their boundaries, so-called the split spanwise theory [5]. Moreover, contact-impact conditions have to be accounted for the debonded parts in the damaged region. Because of the complexity to solve this problem analytically for sandwich plates with an arbitrary form of the impacted site the finite element method (FEM) is usually utilized.

The main focus of this paper is on the study of the dynamic response of sandwich plates cored with honeycomb and polymer foams that were previously impacted. To perform the modal analysis and simulate the dynamic response, a finite element model of sandwich plates containing impact-induced damage is developed. The effects of the different sizes of the post-impact zone including local geometry perturbations and stiffness degradation on the dynamic properties are analyzed.

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1. IMPACT REGION PROPERTIES

In general, low-velocity impacts with a blunt object within sandwich plates produce a permanent indentation in the facesheet accompanied with substantial core crushing damage beneath and around the impacted site and partial interface debonding (or cavity) between the facesheet and the core in the damaged area [4]. In the cases of the barely visible level of impact damage, when the facesheet remains a little damaged, the core crushing and debonding occur only. The key geometrical parameters of the representative cross-section of a sandwich specimen impacted are shown on the Fig. 1 and include the peak depth of the residual facesheet indentation δ_i, the peak depth associated with core fracture δ_c and planar dimension of damaged facesheet radius R_i, and planar dimension of the crushed core R_c.

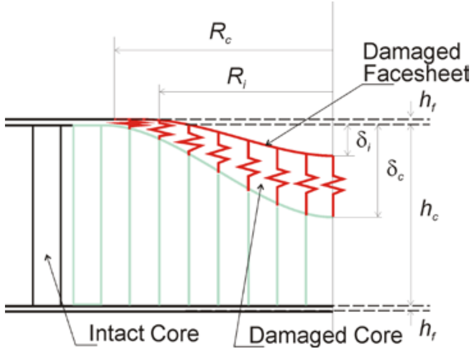

Fig. 1. Impact-damaged region.

Nevertheless, a model of the impacted sandwich plate accounting for only the impact-based geometrical perturbations are not accurate. Because the size of the area impacted depends on the properties of the core material and the relationship between the properties of the core and of those the facesheets. Thereby, the actual damage state of the supporting core, core-to-facesheet interface and impacted facesheet should be taken into account. As a consequence, to predict mechanical responses of sandwich plates subjected to impact, an accurate estimation of transverse normal and shear stresses should be a major goal of a mathematical model that is being developed.

2. MATHEMATICAL STATEMENT OF PROBLEM

Following the splitting theory, each of the regions split may be separately considered from each other by using the assumptions one of the sandwich theories. The finite element model developed in the Section 3 of the paper is based on the high-order sandwich theory [6] that allows the accurate modeling of interlaminar normal and shear stresses. The theory briefly is only given herein.

Let's consider a rectangular sandwich plate as a three-layer structure with a core of uniform thickness h_c and with parallel facesheets of thicknesses h_f, where subscript f has the values 1 and 2 when referring to the top and bottom facesheets, respectively. The facesheets may in general be unequal and composite laminates, and are treated as being the first-order shear (FSDT) deformable plates. The core is assumed to be a fully three-dimensional, orthotropic solid body in which warping of a cross-section and changing of core thickness can be taken into consideration. This assumption primary relates to sandwich plates with continuous solid core, like foam cores, but the adopted approach can be used in predicting the behavior of sandwich plates with discontinuous cores, like a honeycomb structure, if appropriate smeared values of the core physical properties are used. A usual cross-section of the sandwich plate in the *x*-*z* plane is illustrated in Fig. 2 and shows displacements of the three layers and rotations of the facesheets in accordance with FSDT. Obviously, a similar view could be drawn related to the *y*-*z* plane. Consequently, the through-thickness behavior of the displacement fields in the facesheets may be expressed in terms of 10 fundamental quantities, namely u_{0f}, υ_{0f}, w_{0f}, φ_{0xf} and φ_{0yf} with f = 1,2 and '0' means reference axes of the principal layers. The through-thickness behavior of the core are expressed in terms of the 10 facesheet values, on applying the interface continuity conditions and two additional fundamental quantities u_{0c} and υ_{0c} that are the displacements at the core mid-plane in the *x*- and *y*-directions, respectively. Therefore, the displacements at a general point u, υ, w in each of the layers are expressed for the facesheets u_f, υ_f and w_f with f = 1,2 and for the core u_c, υ_c and w_c as functions of the 12 fundamental quantities.

The FE formulation presents the sandwich plate as an assembly of a number of finite elements.

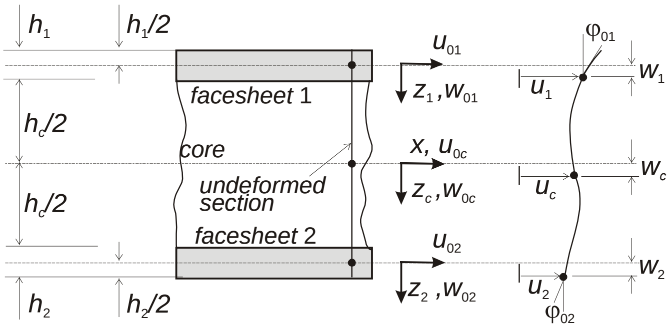

Fig. 2. Sandwich plate cross-section.

Let the approximation for displacement field vector of the each principal layer L referring to the core and the facesheets within the finite element be assumed as

$$\{u^L\} = [N^L]\{d^L\}, \tag{1}$$

where, in a contracted vector-matrix notation, that is traditional for FEM, $[N^L]$ is the matrix of the shape functions and $\{d^L\}$ is the nodal displacement vector of the element. Then, in the facesheets the components of the strain tensor can be obtained by using geometrically either linear or nonlinear strain-displacement equations of elasticity in conjunction with displacement fields defined early

$$\{\varepsilon^f\} = [\partial^f]\{u^f\} \text{ with } f = 1,2, \tag{2}$$

where $[\partial^f]$ is the matrix consisting of differential operators. In general, each of the two facesheets may be of composite laminated of arbitrary lay-up, which exhibits anisotropic mechanical properties, coupling between in-plane and out-of-plane behaviors, and through-thickness shearing. Consequently, the stress-strain relationships at a general point for the lth laminate layer are

$$\{\sigma^f\}_l = [Q]_l\{\varepsilon^f\}, \tag{3}$$

where Q_{rs} with $r, s = 1,2,4,5,6$ are the stiffness coefficients used usually in the laminate theory [2]. Therefore, the stress resultants of the laminated composite plate can be found as

$$\{\sigma^f\} = [D^f]\{\varepsilon^f\} \tag{4}$$

The strain tensor components of the core are obtained on the basis of the 3D elasticity theory

$$\{\varepsilon^c\} = [\partial^c]\{u^c\}. \tag{5}$$

The core is assumed an ortothropic homogeneous body, then, the stress-strain relationships are

$$\{\sigma^c\} = [D^c]\{\varepsilon^c\}, \tag{6}$$

where D^c_{rs} with $r, s = 1,...,6$ are the elastic stiffness constants.

The equation of motion can be derived using Hamilton's principle

$$\delta\int_{t_1}^{t_2}(T - \Pi)dt = 0\,, \tag{7}$$

where Π denotes the potential energy that consists of the strain energy and the work done by loadings $\{p\}$ applied to the outer surfaces and T is the kinetic energy. The strain energy is a sum of contributions of the two facesheets and the core. Consequently, we can write the variation of Π as

$$\delta\Pi = \sum_{i=1}^{3}\int_{A^i}\int_{z_i}^{z_{i+1}}\{\delta\varepsilon^i\}^T\{\sigma^i\}dzdA - \sum_{i=1}^{2}\int_{A^i}\{\delta u^i\}^T\{p\}dA \tag{8}$$

Using the discretization of the displacements (1) and substituting (2)-(6) into (8) yields

$$\delta\Pi = \sum_{i=1}^{3}\int_{A^i}\int_{z_i}^{z_{i+1}}\{\delta d^i\}^T\left([B]^T[D][B]\{d^i\}\right)dzdA - \sum_{i=1}^{2}\int_{A^i}\{\delta d^i\}^T[B]^T\{p\}dA\,, \tag{9}$$

where the matricies $[B]$ and $[D]$ consist of blocks corresponding to the matrices of the core and the fasesheets, and $[B] = [\partial][N]$ for the each principal layer.

Integrating and summarizing the terms in (9) we can finally obtain

$$\delta\Pi = \{\delta d^e\}(\left[K^e\right]\{d^e\} - \{f^e\}), \tag{10}$$

where $\{d^e\}$, $\left[K^e\right]$ and $\{f^e\}$ are the nodal displacement vector, the stiffness matrix and the loading vector of the *e*th element, respectively.

In the same way the variation of the kinetic energy within the sandwich FE is given by

$$\delta T = \sum_{i=1}^{3} \int_{A^i} \int_{z_i}^{z_{i+1}} \{\delta u^i\}^T \rho^i \{\ddot{u}^i\} dz dA, \tag{10}$$

where ρ^i is the mass density of the *i*th principal layer. Then,

$$\delta T = \sum_{i=1}^{3} \int_{A^k} \int_{z_i}^{z_{i+1}} \{\delta d^i\}^T [N]^T \rho^i [N] \{\ddot{d}^i\} dz dA \tag{11}$$

and after the manipulations mentioned above we find

$$\delta T = \{\delta d^e\}\left[M^e\right]\{\ddot{d}^e\}, \tag{12}$$

where $\{\ddot{d}^e\}$ and $\left[M^e\right]$ are the nodal acceleration vector and the mass matrix of the *e*th element.

The global stiffness and mass matricies are obtained by the standard assembly procedure of FEM, thus, the motion equation of the elastodynamic problem without damping is

$$[M]\{\ddot{d}\} + [K]\{d\} = \{f\}, \tag{13}$$

were $\{d\}$, $[K]$, $[M]$ and $\{f\}$ are the corresponding global vectors and matricies.

If there is no debonding, displacements and interlaminar stresses are continuous across the interface of any of two adjacent layers. Otherwise, such continuity does not exist. One can assume that in the damaged region the debonded surfaces may longitudinally slip one with respect to the other, being in contact vertically, or can be a contact free. The stiffness and mass matricies of the damaged region do not differ from those for the undamaged one but the contact-impact conditions of detached parts plus a local damage of the core and the facesheet should be added. The contact behaviour can be accounted for through the compatibility conditions at the core–face interface, which govern the displacement and stress fields of the core as following: in the case of a contact, debonded surfaces are free of core shear stresses and have full compatibility for the displacements between the core and the facesheets; if contact does not exist, debonded surfaces are free of the core shear and normal stresses and there is a jump for displacements between the core and the facesheets. Moreover, as a result of impact both the core and the facesheet can be locally damaged. It will impair their ability to transfer stresses through the small damaged zone. Therefore, the stiffnesses in both the facesheet and the core should be reduced (or even to be zero in extremely case) throughout the damaged zone.

3. FINITE ELEMENT MODELLING

Dynamic characteristics such as natural frequencies and mode shapes of both intact and damaged by low velocity impact sandwich plates were calculated using the commercial FE code ABAQUS/Standard v.6.6. In accordance with the possibilities of this engineering software the free vibration analysis was performed using the linear perturbation load step, where the Lanczos or the subspace iteration methods for eigenvalues extraction were used. The direct method based on the direct solution of the steady-state dynamic equations projected onto a subspace of modes was utilized to calculate harmonic responses of the plate excited at an external harmonic concentrated force.

The 6- and 8-node general-purpose reduced integrated continuum shell elements and 6- and 8-node with incompatible mode linear solid elements were applied to discretize the facesheets and the core of sandwich plates, respectively. The general mesh was subdivided into three different zones: fine meshed impacted region, the next zone surrounding the impacted region with gradually decreased mesh density, and coarse meshed the undamaged zone. The connection between the impacted facesheet and the remained part of the sandwich plate was simulated by imposing multi-point constrains in general nodes. The shell elements selected allow to avoid the inconsistency between the displacement fields of the core and the facesheet because can correctly transfer the moment/rotation at their reference surface. The core-to-facesheet debond was modeled by removing the displacement restrictions and, thus, double nodes appear in this zone. To prevent a physically unreal penetration of the debonded parts and to simulate their contact conditions, the spring elements SPRING2 were introduced between the double nodes. This element had zero stiffness in tension and very big stiffness in compression, if the relative displacement between the nodes goes to zero. Finally, the parts of the core and the facesheet damaged due to impact were modeled by reducing gradually the stiffness of the finite elements belonging the damaged area. For this purpose the initial stiffness coefficients of the corresponding elements were multiplied by appropriate reduction factors.

4. NUMERICAL RESULTS AND DISCUSSIONS

4.1. Test calculations

For verification of the proposed FE model test studies were firstly carried out. A simply supported foam cored sandwich beam with rectangular cross-section damaged at the middle span is used for this purpose. The numerical results of the first six natural frequencies of the damaged sandwich beam found with ABAQUS' model were compared with those analytical results given in the work of Schwartz-Givli et al. [7]. The close results were obtained and they are listed in Table 1.

Table 1. Mode frequencies of the damaged sandwich beam with foam core (Hz).

Mode No	1	2	3	4	5	6
Analysis [7]	288.98	388.32	1093.2	1146.9	1771.3	1842.2
Present FEA	293.07	433.67	1093.1	1132.0	1769.9	2080.2

4.2. Free vibrations of impacted sandwich plates

The influence of the total planar size of the impacted region involving the core crushing, the face sheet damage and the core-to-face sheet debonding was further studied. For this purpose, a simply supported rectangular honeycomb sandwich plate of the total area 135×180 mm^2 with the facesheets thickness of 1 mm and the core thickness of 5 mm, containing a circular impacted zone with planar parameter R_c = 30 mm at the center was considered. The material properties of the plate are presented in Table 2. It is worthy to notice that the homogeneous properties of the honeycomb core were previously obtained basing on the unit cell approach by using the FEM.

Table 2. Material properties of impacted sandwich plates.

Components	Elastic constants
Honeycomb core	E_{11} = 0.461, MPa E_{22} = 0.461, MPa E_{33} = 1494, MPa G_{12} = 0.194, MPa G_{13} = 341.7, MPa G_{23} = 192.1, MPa ρ_c = 57.17, kgm^{-3}
Rohacell foam	E_c = 135, MPa G_c = 45, MPa ρ_c = 100, kgm^{-3}
CFRP facesheets	E_{11} = 140, GPa E_{22} = E_{33} = 10, Gpa G_{12} = G_{13} = 4.6, GPa G_{23} = 3.8, GPa ρ_f = 1650, kgm^{-3}
GFRP facesheets	E_{11} = E_{33} = 16500, MPa E_{22} = 3800, MPa G_{12} = G_{23} = 1800, MPa G_{13} = 6600, MPa ρ_f = 1650, kgm^{-3}

Calculations showed that the natural frequencies of the impact-damaged honeycomb sandwich plate are shifted from the intact one. This effect on the higher modes is greater than the lower ones. Also, this effect does not exhibit monotonous trends when a mode number increases. Moreover, the mode shapes of the impacted plate were also changed. Purely local modes and mixed modes that are combination of local and global mode shapes often occur. Also the numerical results showed that the natural frequencies decrease with increases of the impacted region size, R_c. Besides, the frequencies change more rapidly as a mode number increases. Although this trend of the frequencies changing can be violated due to local thickening phenomenon caused by debonding which in some cases made the frequencies of the damaged plate even higher than the intact one. To show the influence of other damage characteristics produced by impact, the sandwich plate with in-plan dimensions of 270×180 mm^2 consisting of 2.4 mm GFRP facesheets and 50 mm Rohacell™ WF51 foam core was analyzed. The mechanical properties of the constituent materials are shown in Table 2. In analyses it was assumed that if one of the parameters of the impacted region is being varied during calculations, other ones to be constant. The influence of the cavity depth, $\delta_c – \delta_i$, on vibration responses of the impacted sandwich plate was firstly studied. It was found that the values of the natural frequencies of the impacted plates slightly decrease, but their mode shapes curvatures slightly increase with the cavity depth increasing. This effect of the minor changing of the lower frequencies holds for the higher ones. The same minor influence of the residual facesheet indentation depth on the natural frequencies at the cavity depth equal to 10% of the facesheet thickness was obtained. The mode shapes had more visible changes with increasing of the facesheet indentation. Finally, substantial decreasing of the natural frequencies with increasing the facesheet degradation level was found.

4.3. Forced vibrations of impacted sandwich plates

The forced vibration analysis of the honeycomb cantilever sandwich plate containing a post-impact circular damage at the center, as in the previous study, was carried out. The radius of the impacted site was varied from 5 to 60 mm. The cavity depth was taken as a constant equal to half of the facesheet thickness. A harmonic concentrated load with magnitude equal to 100 N was applied in the transverse direction on the free edge of the cantilever plate with a frequency range taken from 500 to 2000 Hz as forcing frequencies. Harmonic responses of the impacted sandwich plate were calculated at the point where the force was applied for all simulated damage states. The changes in harmonic responses versus sizes of the impacted site, as the deflection-frequency curves, at the forcing frequency defined are shown in Fig 3. The dominant harmonic response is obtained at 1512.6 Hz that corresponds to the third resonance frequency of the plate damaged by impact. From the calculated results, we can conclude that the harmonic response increases when the planar size of the impacted site increases that corresponds to the stiffness degradation due to the damage presence.

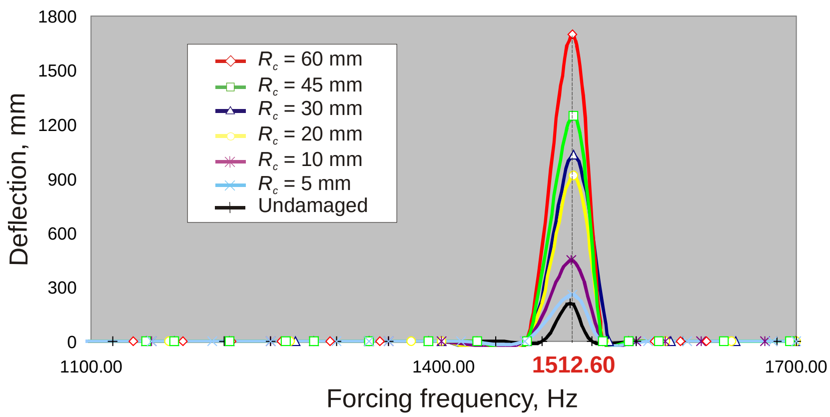

Fig. 3. Harmonic deflection responses vs radius of impacted site.

CONCLUSIONS

In sum the following conclusions from the viewpoint of sensitivity of dynamic characteristics to the presence of impact damage can be drawn. First, both the natural frequencies and the harmonic responses of sandwich plates subjected to low velocity impact are sensitive to the presence of the impact-induced damage. In doing so, the natural frequencies usually decrease due to loss in stiffness caused by damage, while harmonic responses increase because of that. Second, the higher natural frequencies and mode shapes are more sensitive to the impact damage presence. Third, natural frequencies and associated mode shapes are the most sensitive to the planar size of the impact domain and are poorly sensitive to the damage extended through the thickness and induced in the facesheet. Fourth, the displacement harmonic responses can be primary used for detection the impact damage.

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REFERENCES

[1] Lok T.S., Cheng Q.H. Free and forced vibration of simply supported orthotropic sandwich panel. *Computers and Structures*, 79, pp. 301-302, 2001.

[2] Nayak A.K., Shenoi R.A. Free vibration analysis of composites sandwich plates based on Reddy's high-order theory. *Composites*: *Part B*, 33, pp. 505-519, 2002.

[3] Kurpa L.V., Vensel E.S. Analysis of sandwich plates of arbitrary shape. *Mechanics of Advanced Materials and Structures*, 12(1), pp. 33-43, 2005.

[4] Scarponi C., Briotti G. and Renato B. Impact testing on composite laminates sandwich panels. *Journal of Composite Materials*, 30, pp. 1873-1911, 1996.

[5] Ramkumar R.L., Kulkarni S.V. and Pipes R.B. Free vibration frequencies of a delaminated beam. In: *34th Annual Technical Conference, Reinforced Plastic/Composite Institute*, pp. 1-5, 1979.

[6] Frostig Y., Baruch M. Localized load effects in high-order bending of sandwich panels. *Journal of Engineering Mechanics*, 122, pp. 1069-1076, 1996.

[7] Schwarts-Givil H., Rabinovich O., and Frostig Y. Free vibration of delaminated unidirectional sandwich panels with a transversely flexible core and general boundary conditions – A high order approach. *Journal of Sandwich Structures and Materials*, 10, pp. 99-131, 2008.