Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

ON ALGEBRAIC STRUCTURES INDUCED BY U-TURNS AND NON-INVERTIBLE TIME SUBSTITUTIONS

Valery N. Pilipchuk [1]
Wayne State University
Detroit, USA

ABSTRACT

In this work, we discuss such time substitutions that capture the basic temporal symmetries of dynamic processes in advance to facilitate any further use of the methods of dynamic analyses. For instance, if a particle makes a U-turn, then the corresponding substitution reverses the time direction exactly when the velocity changes its sign regardless other properties of the dynamics, namely – classes of smoothness, levels of unharmonicity, etc. From such a viewpoint, an oscillating process represents just a sequence of U-turns. In this case, the oscillating time substitutions fold time into bounded or half-bounded domains that promises quite essential advantages for both analytical and numerical approaches. Interestingly enough, transforming the temporal variable, brings the spatial coordinates into the specific set of complex elements – hyperbolic numbers – whose "imaginary number" squared is plus one. Such algebraic structures appeared to be known in mathematical literature in a very abstract way, regardless any dynamic problems or non-smooth functions, and often regarded to as a simple example of so-called Clifford's algebras; see [1] and references therein for introduction. Nevertheless, some geometrical interpretations from this theory hint on useful manipulations with dynamical systems when implementing non-smooth temporal substitutions [2]. In this work a series of illustrations and solutions is presented.

REFERENCES

[1] Antonuccio F. Hyperbolic numbers and the Dirac spinor, *http://arxiv.org/abs/hep-th/9812036v1*, 1998

[2] Pilipchuk V.N. *Nonlinear Dynamics: Between Linear and Impact Limits*, Springer – Verlag Berlin Heidelberg (in production), http://www.springer.com/engineering/book/978-3-642-12798-4, 2010.

[1] Valery N. Pilipchuk, Email: pilipchuk@wayne.edu