

## PPF VERSUS SATURATION CONTROL FOR A STRONGLY NONLINEAR BEAM STRUCTURE

**M. Bochenski**<sup>\*1</sup>,

**J. Warminski**<sup>\*</sup>,

**W. Jarzyna**<sup>\*</sup>,

**P. Filipek**<sup>\*</sup>,

**M. Augustyniak**<sup>\*</sup>

Lublin University of  
Technology, Lublin,  
Poland, \* Department of  
Applied Mechanics,  
Faculty of Mechanical  
Engineering,

\* Department of Electrical  
Drive Systems and  
Electrical Machines,  
Faculty of Electrical  
Engineering and  
Computer Science

---

### ABSTRACT

---

This paper presents numerical results of vibration suppression of a strongly nonlinear beam structure. Coupling of a nonlinear plant with PPF and saturation controllers (NSC) is tested. Influence of variation of frequency and amplitude of excitation for the system response and controllers' effectiveness is presented. Differences between two control strategies are shown.

---

### INTRODUCTION

Coupling of two vibrating subsystems give possibilities for energy transfer from one to another. By a selection of parameters one subsystem may play a role of the vibration absorber. To get such a phenomenon, absorber's frequency must be properly tuned to excitation frequency and structural parameters of the main system. This absorption effect supports control strategies, then the absorber is used as a controller and the main structure is a plant. Depending on the tuning method few types of control strategies can be distinguished. In the Positive Position Feedback (PPF) the natural frequencies of the subsystems are tuned in one-to-one ratio [1] [1] PPF method is characterized by a linear form of coupling realized by feedback loop with displacements multiply by constant gains only. The second, Nonlinear Saturation Control strategy (NSC), is based on tuning of the subsystems' natural frequencies in two-to-one ratio and then coupling the controller and the plant by a quadratic form [3] [4] Due to the nonlinear coupling the system is more complicated and needs more attention in studies. A multiple gain and two displacement (plant and controller) are used in the NSC method .

Usually controllers are designed to reduce vibration for frequency of excitation equal to natural frequency of the main system (the plant). The plant is treated as a linear model. In this case, near the resonance zone the response of the linear system achieve the biggest value, which is to be suppressed. However, appearance of nonlinearities in the plant model leads to significant changes in shape of the plant's resonant curve and additional interaction between plant and controller may appear.

This work is focused on comparison of effectiveness of PPF and NSC control strategies taking into account a strongly nonlinear model of a plant. Influence of variation of frequency and amplitude of excitation is tested.

---

<sup>1</sup> Marcin Bochenski. Email: [m.bochenski@pollub.pl](mailto:m.bochenski@pollub.pl)

## 1. MODEL OF THE STRUCTURE

The system taken for analysis consists of a composite beam with an embedded Macro Fiber Composite (MFC) actuator, which allows execute large flexural oscillations (Fig.1). A model of the plant is based on Euler – Bernoulli beam theory with an additional nonlinear curvature component. Horizontal beam (a mechanical system) is connected by the actuator and the sensor with the controller (an electrical system). External excitation is represented by harmonic vertical motion of a beam's support (direction  $x$  in Fig.1).

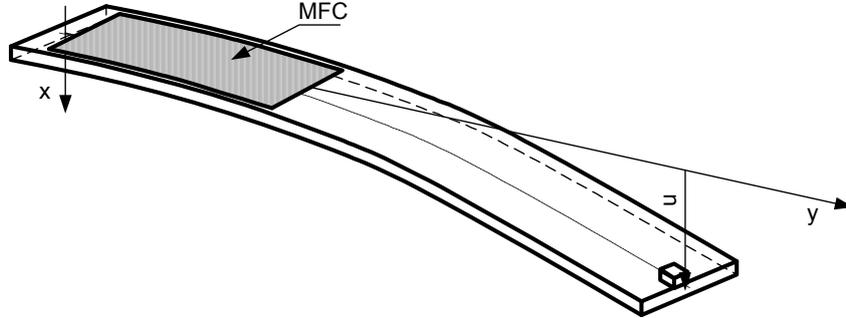


Fig. 1 Model of the system

Details of a mathematical model derivation are presented in [5]. The final equations which describe the nonlinear beam coupled with PPF (1) and NSC (2) control algorithms take form:

PPF system

$$\begin{cases} \ddot{u} + 2\mu\omega_s\dot{u} + \omega_s^2u + \beta u^3 - \delta(u\dot{u}^2 + u^2\ddot{u}) = f \cos(\Omega t) + \gamma v \\ \ddot{v} + 2\zeta\omega_c\dot{v} + \omega_c^2v = \alpha u \end{cases} \quad (1)$$

NSC system

$$\begin{cases} \ddot{u} + 2\mu\omega_s\dot{u} + \omega_s^2u + \beta u^3 - \delta(u\dot{u}^2 + u^2\ddot{u}) = f \cos(\Omega t) + \gamma v^2 \\ \ddot{v} + 2\zeta\omega_c\dot{v} + \omega_c^2v = \alpha uv \end{cases} \quad (2)$$

where  $u$  means the displacement of the beam's tip,  $v$  – denotes the controller's voltage,  $f$ ,  $\Omega$  – amplitude and frequency of excitation,  $\mu$ ,  $\zeta$  – damping ratio,  $\omega_s$  and  $\omega_c$  – natural frequencies of the main system (plant) and the controller, respectively. Feedback loop gains are denote  $\alpha$  and  $\gamma$ .

## 2. NUMERICAL RESULTS

Based on equations (1) and (2) numerical models of the system in Simulink software are prepared. They are tuned according to the natural frequency of the plant:  $\omega_s = 3.0631$ , then for PPF system  $\omega_c = \omega_s$ , for NSC system  $\omega_c = 0.5\omega_s$ . Simulations are performed for the excitation frequency range  $\Omega$  from 2.9 to 3.3 Hz and for two levels of amplitudes of excitation. To make interpretation of the results more convenient, analytical resonance curves for the plant response without control are additionally shown in Figs.2-4. Blue rhombus correspond to the maximal vibration amplitude of the plant with PPF control, while the red triangles with NSC control. As we can observe in Fig.2, low level of excitation ( $f = 0.03$ ) leads to almost linear plant behavior but for  $f = 0.07$  (Fig.3) influence of nonlinear terms is clearly observed. In this case maximum of the resonance curve is placed very far from the natural frequency of the beam, out of the analyzed frequency range.

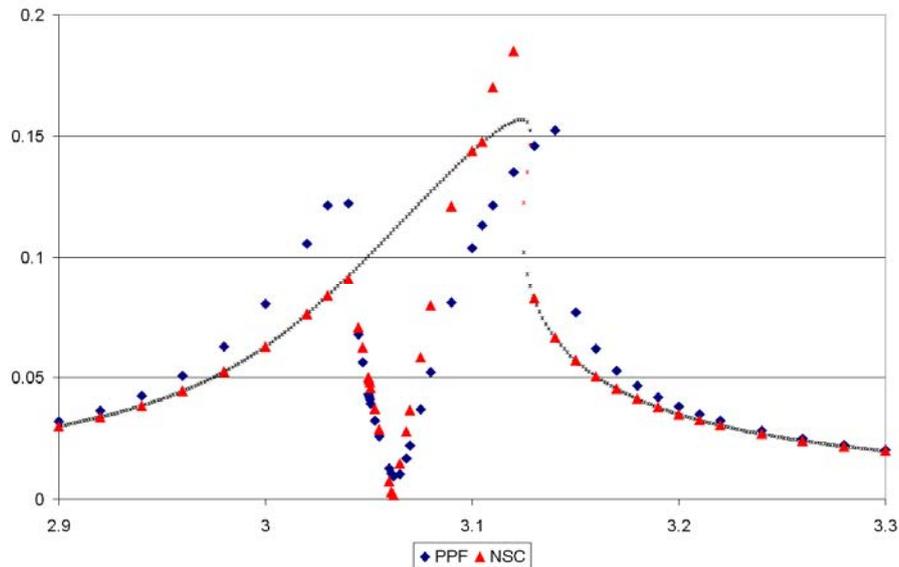


Fig. 2 The frequency response curve,  $f=0.03$

When frequency of excitation is close to the beam's natural frequency both systems show similar vibrations suppression level. However, close to the lower and upper limits of tested frequencies significant differences between analyzed controllers are observed. NSC system is not active in this area. Response of the controlled system agree to resonance curve for the no – control plant. Influence of the controller for beam's behavior is not observed. In the same area (for both amplitudes of excitation) negative effect of PPF algorithm occurs. Operation of this controller results in higher amplitude of vibration than for no – control system. For higher level of excitation ( $f = 0.07$ ) between 3.1 and 3.2 Hz significant growth of the plant response is observed (Fig.3). For both, PPF and NSC systems, beat vibrations in this region are present. However extent of this area is more wide for PPF system.

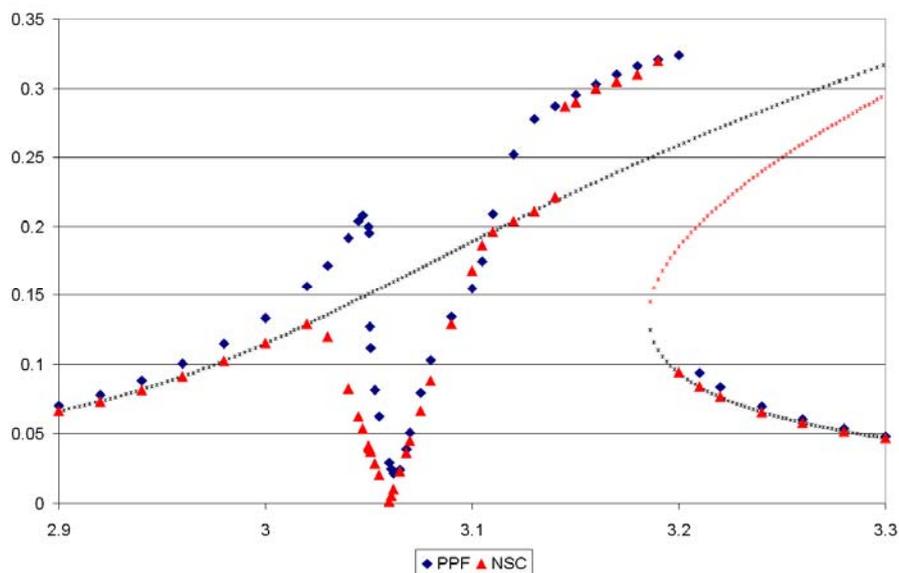


Fig. 3 The frequency response curve,  $f=0.07$

This behavior occurs also for lower amplitude of excitation ( $f = 0.03$ , Fig.2), but is not so strongly emphasized because nonlinear feature of the plant is weakly exposed. When frequency of excitation is tuned to the natural frequency of the beam both control methods work properly. The amplitude response curve takes “V” letter shape in this zone. Influence of amplitude of excitation on the resonance curve for PPF system is clearly visible in Fig.4. Beam's response for small and large amplitude of excitation there are also presented. As can be seen, comparing the resonance curves, the controller gives better vibration reduction and in a wider frequency range for low excitation level.

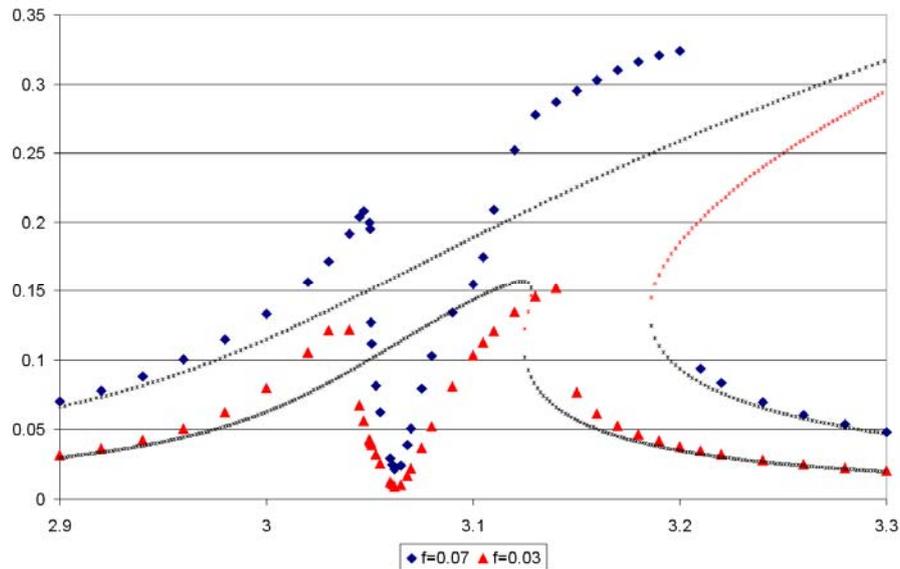


Fig. 4 The frequency response curve for PPF structure

For frequencies of excitation close to the beam's natural frequency, response of the PPF system grows along with the amplitude of excitation is increasing (Fig.4). The NSC system keeps beam's vibration on the same level despite of the change of amplitude of excitation. This feature results from occurrence of saturation phenomena [3] .

## CONCLUSIONS

On the basis of numerical simulation we may conclude that for a strongly nonlinear system NSC algorithm allows to obtain better vibration suppression than PPF controller. For both, PPF and NSC control strategies beat vibrations may occur, which lead to large amplitude plant's response. PPF system works very effectively only for weakly nonlinear plant and for frequency of excitation close to plant's natural frequency. Presented numerical results will be tested experimentally. Analysed systems will be equipped in additional module to measure current frequency related to the generated excitation.

## ACKNOWLEDGEMENT

Financial support of Structural Funds in the Operational Programme - Innovative Economy (IE OP) financed from the European Regional Development Fund - Project "Modern material technologies in aerospace industry", Nr POIG.01.01.02-00-015/08-00 is gratefully acknowledged.

## REFERENCES

- [1] Shan J., Liu H., Sun D., Slewing and vibration control of a single-link flexible manipulator by positive position feedback (PPF) *Mechatronics*, Vol. 15, pp. 487–503, 2005.
- [2] Kwak M. K., Heo S., Active vibration control of smart grid structure by multiinput and multioutput positive position feedback controller *Journal of Sound and Vibration*, Vol. 304, pp. 230–245, 2007.
- [3] Oueini S. S., Nayfeh A. H., and Pratt J. R., A Nonlinear Vibration Absorber for Flexible Structures *Nonlinear Dynamics*, Vol. 15, pp. 259–282, 1998.
- [4] Saguranrum S., Kunz D. L., Omar H. M., Numerical simulations of cantilever beam response with saturation control and full modal coupling *Computers and Structures*, Vol. 81, pp. 1499–1510, 2003.
- [5] Warminski J., Bochenski M., Jarzyna W., Filipek P., Augustyniak M., Active suppression of nonlinear composite beam vibrations by selected control algorithms. *Communications in Nonlinear Science and Numerical Simulation*, doi:10.1016/j.cnsns.2010.04.055, 2010.