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STABILITY OF WHIRL VIBRATIONS OF DRILL STRING BOTTOM ASSEMBLY

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ABSTRACT

Current methods for analysis of extraordinary behavior of the drill strings, as a rule, concern their critical states related to essential change of their motion modes and contact interaction with borehole wall.
The bending vibrations of bottom hole assembly under action of a friction moment applied to the bit is considered. The analysis of vibration self-excitation mechanism is performed. It is shown that the generated moment is not conservative and it is the main reason of the system dynamic instability. The modes of bottom hole assembly whirling are constructed for different values of characteristic parameters.

INTRODUCTION

Today, approximately 90% of all the energy consumed by mankind is accounted for by fossil hydrocarbon fuels of which oil and gas are the major ones and whose prices are skyrocketing due to their imminent depletion. Nevertheless, reconnaissance of new oil and gas reserves and progressively increasing rate of their extraction continue. As this takes place, the principal technological component of these processes is the drilling of new oil and gas bores. Even now their depths achieve several kilometers, but the problem of extraction of oil and gas from deeper tectonic levels continues to be urgent.

Rotor drilling of bore-wells realized through application of a torque M_z to the top edge of the drill string (DS) and a vertical reaction force R on the drill bit (Fig.1) can be accompanied by occurrence of some dynamic phenomena exerting essential influence on the whole working process. Among these is excitation of whirling vibrations caused by non-conservative bending of the bit shaft by contact forces.

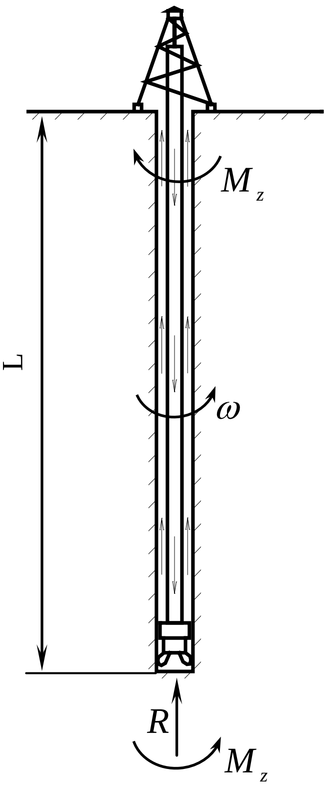

Fig. 1. Design scheme of a drill string

The theoretical simulation of dynamic behavior of the DS in the drilling possess essential analytical and computational difficulties stemming from the system dependence on complicated combination of dynamic and quasi-static force factors acting on the DS in its working [1-3].

But the principal obstacle arising in attempting to analyze the dynamic bending of the DS is associated with the necessity of integrating differential equations of their vibrations in large ranges of the DS length. As the DS is equivalent to human hair by the condition of geometrical similarity, it is very flexible. For this reason equations describing its bending

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possess the so called calculation stiffness and the majority of methods of their solving are poorly convergent. So usually the problems of the DS torsion vibrations are stated, though it is accented that integrated models of the DS bending dynamics should be elaborated [4].

The first step of dynamic analysis of a complicated structure consists in study of its free vibrations. It is associated with the statement of the Sturm-Liouville boundary problem for the equations of the rotary DS dynamics. In solution of this problem, the eigen values should be calculated and eigen modes should be built for the whole length of the DS. Previously, it was not stated and solved owing to essential theoretical and calculation difficulties.

Of critical importance is also the DS rotation with the resulting generation of centrifugal and Coriolis' inertia forces. Owing to large length of the DS, frequently these forces are the main reason of instability onset and they exclude the possibility of the system vibration with one common phase, as it occurs in gyroscopic systems. These effects are completely understood in the theory of rotating shafts [5], but in the DS they are realized in more complicated forms because proceed in combination with other mechanical effects.

In service, washing liquid (mud) required to remove the crushed particles of the destructed rock moves down inside the DS. Notice, that vibrations and quasi-static stability of rectilinear and curvilinear (spiral) tubes under action of heterogeneous flows of liquids are considered in references [6].

In this paper firstly we consider a hyper long (unbounded) DS and study analytically general regularities of its natural vibrations. Then bounded DSs are studied, their first frequencies are calculated and vibration modes are built. It is shown that they have complicated spiral shapes.

1. STATEMENT OF THE PROBLEM

To derive equations of vibration of a rotating drill string introduce the immovable coordinate system $OXYZ$ with its origin at the point of the DS suspension and axis OZ directed along the DS axis line. It is assumed that the DS rotates with constant angular velocity ω. As it is assumed in the theory of rotating shaft vibration, connect the rotating coordinate system $Oxyz$ with DS in such a way that the Oz axis coincides with the OZ axis and analyze the DS dynamics in this system. Let $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ be the unit vectors of this system.

Small vibrations of the DS are determined with the use of the functions of elastic displacements $u(z)$, $v(z)$ in the planes xOz, yOz, correspondingly. In the perturbed state of dynamic equilibrium these displacements are caused by action of internal longitudinal force $T(z)$, external torque M_z, distributed centrifugal inertia forces q_x^i, q_y^i of rotation and distributed centrifugal inertia forces q_x^m, q_y^m induced by motion of washing liquid (mud) inside the curved tube of the DS.

The force $T(z)$ is accountable to distributed gravity force with intensity

$$q_z = g(\rho_t - \rho_m)F_t \tag{1}$$

and vertical reaction R of the bit contact interaction with the rock medium. It is denoted in this formula: g = 9.81 m/s^2 – the acceleration of gravity; ρ – the density of the tube material; ρ_m – the mud density; F_t – the area of the tube wall cross-section.

At the deformed state of the DS, the distributed inertia forces q_x^ω, q_y^ω of the tube compound motion and the inertia forces q_x^m, q_y^m of the moving mud act on every element of the rotating tube, so the components of total inertia forces equal

$$q_x = q_x^\omega + q_x^m, \qquad q_y = q_y^\omega + q_y^m. \tag{2}$$

Vector $\mathbf{q}^{\omega} = q_x^{\omega}\mathbf{i} + q_y^{\omega}\mathbf{j}$ is calculated through the equality

$$\mathbf{q}^{\omega} = -(\rho F + \rho_m F_m)\mathbf{a}, \tag{3}$$

where F_m is the area of the tube bore cross-section; $\mathbf{a}$ is the absolute acceleration of the tube element.

The a value is calculated in the rotating coordinate system $Oxyz$, so the motion is compound and the Coriolis formula $\mathbf{a} = \mathbf{a}^e + \mathbf{a}^r + \mathbf{a}^c$ is used for the acceleration a determination. Here $\mathbf{a}^e$, $\mathbf{a}^r$, $\mathbf{a}^c$ are the bulk, the relative and the Coriolis accelerations calculated with the help of the formulas

$$\mathbf{a}^e = \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}), \qquad \mathbf{a}^r = \frac{d^2\mathbf{r}}{dt^2}, \qquad \mathbf{a}^c = 2\boldsymbol{\omega}\times\frac{d\mathbf{r}}{dt} \tag{4}$$

Analogously the equalities for the forces acting on the internal liquid flow moving with the velocity V are deduced [6]

$$q_x^m = V^2\rho_m F_m\frac{\partial^2 u}{\partial z^2} + 2V\rho_m F_m\frac{\partial^2 u}{\partial z\partial t}, \quad q_y^m = V^2\rho_m F_m\frac{\partial^2 v}{\partial z^2} + 2V\rho_m F_m\frac{\partial^2 v}{\partial z\partial t}. \tag{5}$$

To formulate constitutive equations describing vibration of the DS prestressed by longitudinal force T, reaction R and torque M_z, separate its element of length dz and consider equilibrium of internal moments with respect to the axes Oy, Ox of the rotating coordinate system $Oxyz$

$$dM_y - Q_x dz - Tdu - M_z d\left(\frac{dv}{dz}\right) = 0, \quad dM_x - Q_y dz - Tdv + M_z d\left(\frac{du}{dz}\right) = 0. \tag{6}$$

where $M_y = -EI\dfrac{d^2u}{dz^2}$, $M_x = -EI\dfrac{d^2v}{dz^2}$, Q_x, Q_y are the bending moments and shear forces oriented relative to the appropriate axes.

Rearranging Eqs (1)–(6), one gains the equations determining the dynamic behavior of the rotating DS

$$\begin{gathered}
EI\frac{\partial^4 u}{\partial z^4} - \frac{\partial}{\partial z}\left(T\frac{\partial u}{\partial z}\right) - \frac{\partial^2}{\partial z^2}\left(M_z\frac{\partial v}{\partial z}\right) - (\rho F + \rho_m F_m)\omega^2 u - \\
-2(\rho F + \rho_m F_m)\omega\frac{\partial v}{\partial t} + V^2\rho_m F_m\frac{\partial^2 u}{\partial z^2} + 2V\rho_m F_m\frac{\partial^2 u}{\partial z\partial t} + (\rho F + \rho_m F_m)\frac{\partial^2 u}{\partial t^2} = 0, \\
EI\frac{\partial^4 v}{\partial z^4} - \frac{\partial}{\partial z}\left(T\frac{\partial v}{\partial z}\right) + \frac{\partial^2}{\partial z^2}\left(M_z\frac{\partial u}{\partial z}\right) - (\rho F + \rho_m F_m)\omega^2 v + \\
+2(\rho F + \rho_m F_m)\omega\frac{\partial u}{\partial t} + V^2\rho_m F_m\frac{\partial^2 v}{\partial z^2} + 2V\rho_m F_m\frac{\partial^2 v}{\partial z\partial t} + (\rho F + \rho_m F_m)\frac{\partial^2 v}{\partial t^2} = 0.
\end{gathered} \tag{7}$$

In application of system (7) to analysis of the DS dynamics it is usual to state the boundary value problem. However the problem about free vibration of infinite twisted rotary rod should be considered also with the aim to establish the most general regularities of the DS behavior.

2. THE SPIRAL STRUCTURE OF THE RUNNING BENDING WAVES

The problem of harmonic vibrations of an unbounded twisted rotary tube rod with internal flows of liquid is multiparametric and for this reason it is difficult to be analyzed. For this reason firstly consider the simplified case $T = const$, $M_z = const$ for the sake of separation of the phenomenon of free spiral wave propagation.

It can be shown that system (7) does not admit any solution in the form of stationary or running waves with nodal points, therefore we shall construct its solution in the mode of running cylindrical spiral waves

$$u(z,t) = A\cos(kz - ct), \quad v(z,t) = B\sin(kz - ct), \tag{8}$$

where k is the wave number; c the cyclic frequency.

Substituting Eqs (8) into reduced system (7) and excluding the summands containing the multiplier ρ_m, one obtains the homogeneous system of algebraic equations

$$\begin{aligned} &(EIk^4 + Tk^2 - \rho F\omega^2 - \rho Fc^2)A + (M_z k^3 + 2\rho F\omega c)B = 0 \\ &(M_z k^3 + 2\rho F\omega c)A + (EIk^4 + Tk^2 - \rho F\omega^2 - \rho Fc^2)B = 0 \end{aligned}. \tag{9}$$

It generates the characteristic equation

$$(EIk^4 + Tk^2 - \rho F\omega^2 - \rho Fc^2)^2 - (M_z k^3 + 2\rho F\omega c)^2 = 0, \tag{10}$$

connecting the wave number k and the cyclic frequency c. This equation has four roots

$$c_{1,2} = \omega \pm \frac{k}{\sqrt{\rho F}}\sqrt{EIk^2 + M_z k + T}, \quad c_{3,4} = -\omega \pm \frac{k}{\sqrt{\rho F}}\sqrt{EIk^2 - M_z k + T} \tag{11}$$

corresponding to dextral $(A/B = 1)$ and sinistral $(A/B = -1)$ spiral forms (Fig.2).

The cited reasonings permit one to make important conclusions. Firstly, only spiral bending waves can propagate in rotating twisted rods. In the second place, four different values c_i of cyclic frequency correspond to every value of the spiral pitch. Two of them conform to dextral spiral and other two are consistent with the sinistral one. The directions of their propagation and characters of their dispersive curves $c_i = c_i(k)$ are determined by correlations between the bending stiffness EI of the rod, value M_z of the torque and value and sign of the longitudinal force T.

Phase velocities v_i of the wave propagation in the rotary coordinate system $Oxyz$ are determined by the equalities

$$\begin{aligned} v_{1,2} &= \frac{c_{1,2}}{k} = \frac{\omega}{k} \pm \sqrt{\left(EIk^2 + M_z k + T\right)/\rho F}, \\ v_{3,4} &= \frac{c_{3,4}}{k} = -\frac{\omega}{k} \pm \sqrt{\left(EIk^2 - M_z k + T\right)/\rho F}. \end{aligned} \tag{13}$$

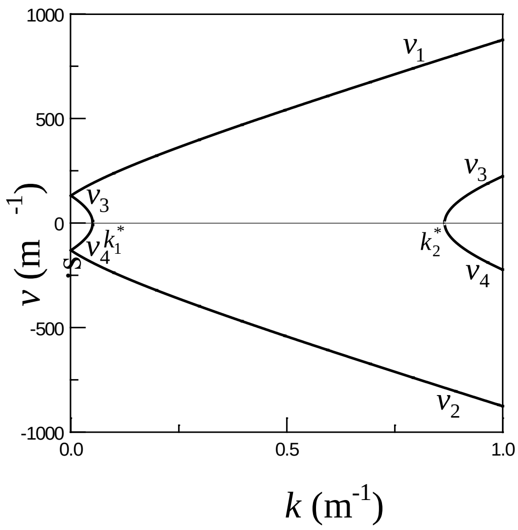


Fig. 3. Phase velocities of spiral waves at $\omega = 0$

By the way of example the diagrams

$v_i = v_i(k)$ $(i = \overline{1,4})$ (Fig. 3) are plotted for the waves in the shapes of dextral $(i = 1,2)$ and sinistral $(i = 3,4)$ spirals, propagating in the steel tube rod of $35.5\, sm$ in external diameter, $1.4\, sm$ in thickness, at given values of the parameters $EI = 4.586 \cdot 10^7\ Pa \cdot m^4$, $\rho F = 117\ kg/m$, $M_z = 4.2 \cdot 10^7\ N \cdot m$, $T = 2 \cdot 10^6\ N$. As the ω value exists only as an summand in Eqs (14), (15), the calculations are performed for $\omega = 0$.

The gained results permit one to anticipate that the free and forced vibrations, as well as quasi-static bifurcation buckling of the DS subjected to action of torque, longitudinal force and rotary inertia forces may occur only with formation of spiral (regular or irregular) modes.

3. FREE BENDING VIBRATIONS OF HYPER DEEP DRILL STRINGS

System (7) can be used for analysis of free vibrations of a bounded DS. Consider that it is pinned at its ends and boundary conditions

$$u(0) = v(0) = 0, \quad u''_{zz}(0) = v''_{zz}(0) = 0, \qquad u(L) = v(L) = 0, \quad u''_{zz}(L) = v''_{zz}(L) = 0 \tag{13}$$

are satisfied.

Then a periodic solution of homogeneous system (7), (13) can be constructed with the help of substitution

$$u(z,t) = U_s(z)\sin ct + U_c(z)\cos ct, \qquad v(z,t) = V_s(z)\sin ct + V_c(z)\cos ct, \tag{14}$$

where c is the cyclic frequency of free vibration; U_s, U_c, V_s, V_c the unknown functions.

Substituting Eqs (14) into system (7) and separating terms containing $\sin ct$ and $\cos ct$, one gains the system of four ordinary differential equations relative to the unknown variables $U_s(z), U_c(z), V_s(z), V_c(z)$.

To find frequencies c_i under prescribed values of T, M_z, ω, the item-by-item analysis is used. In doing so the constitutive system of ordinary differential is represented in the vector form

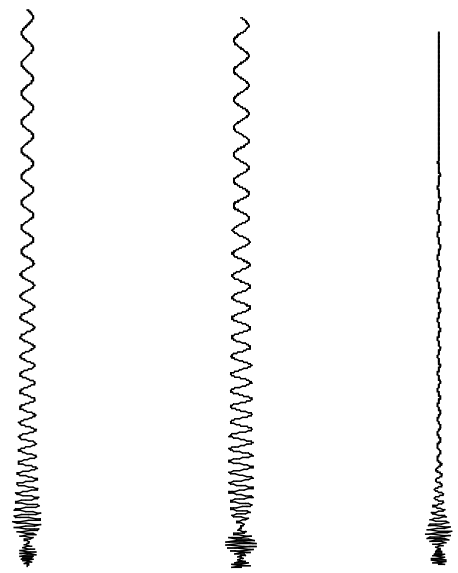
$$\frac{d\mathbf{w}}{dz} = F(z)\mathbf{w} + c^2 G\mathbf{w} + cH\mathbf{w}, \tag{15}$$

$$A\mathbf{w}(0) = 0, \qquad B\mathbf{w}(L) = 0. \tag{16}$$

Here $\mathbf{w}(z)$ is the 16-dimentional unknown vector combining the variables $U_s(z), U_c(z), V_s(z), V_c(z)$ and their derivatives; $F(z), G, H$ are the matrices of dimension 16×16; A, B are the constant matrices of dimension 8×16 constructed from boundary conditions (18).

Solution of system (16) is represented in the Cauchy form

$$\mathbf{w}(z) = W(z) \cdot \mathbf{C}, \tag{17}$$



c_1=0,02745 s^{-1} c_2=0,03769 s^{-1} c_3=0,09251 s^{-1}

Fig 4. Modes of free vibrations of a drill string 7000 m in length where $W(z)$ is the Cauchy matrix of dimension 16×16 of system (16) solutions with initial conditions $W(0) = E$, E is the unit matrix, $\mathbf{C}$ is the required constant 16-dimensional vector.

The elaborated techniques were used for analysis of free vibrations of the DSs prestressed by torque M_z and gravity force $T(z)$. The DS can rest by its lower end against the bore hole bottom, so compressive reactive force $T(L) = R = -1.6 \cdot 10^5 N$ can act on it. The influence of the internal flow of liquid was not taken into attention.

The vibration mode complication is retained also for the elongated DSs but it prevails in their lower segments, where the twisting of the harmonic curves is visible, their pitches are small and amplitudes are large (Fig.4 for the DS of 7000 m in length). With approaching to the DS middle part and further the harmonic twisting diminishes, the curves become nearly plane, their pitches enlarge and amplitudes decrease.

CONCLUSIONS

1. The problems about free vibrations of elongated drill strings are stated with allowance made for the additional disturbing factors of longitudinal non-uniform preloading, action of torque, inertia forces of rotation and internal flow of washing liquid. The constitutive equations are deduced, methods for their solving are elaborated.
2. Relying on the constructed equations, free vibrations of unbounded elastic rods subjected to action of the mentioned factors are studied. It is shown through analysis of the appropriate dispersion equation, that free bending vibrations of these rods can be realized only in the modes of running cylindrical spiral waves. As this takes place, four different values c_i of cyclic frequency correspond to everyone value of the spiral pitch. Two of them conform to a dextral spiral and other two are consistent with a sinistral one. All these waves propagate with different phase velocities along the positive and negative directions of the longitudinal axis.

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