Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

STOCHASTIC SYSTEMS UNDER PERIODIC AND WHITE NOISE EXTERNAL EXCITATIONS, AND THE ALTERNATIVE CLASSIFICATION FOR THE PDE SOLUTIONS

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ABSTRACT

It is shown that wide class of equations in partial derivatives (PDEs) is equivalent to a system of functional linear algebraic equations. It permits to construct exact and approximate solutions and to determine the solution character of evolution with respect to "limit attracting solution" according to eigenvalues of the matrix corresponding to the equation under consideration. K.A.Volosov proposes the alternative classification for PDE solutions on eigen values.

INTRODUCTION

The new important property of wide class PDE was found by K.A.Volosov [1-5] {see also www.aplsmath.ru}. One considers now a simple case of two independent variables x, y. For an arbitrary transformation of the variables, namely, $x = x(\xi,\delta),\ y = y(\xi,\delta)$, it is possible to present all PDEs of the second order, or more, as $AX = b$, that is as a system of linear algebraic equations with respect to derivatives of the initial variables $x(\xi,\delta), y(\xi,\delta)$ on the new variables $\xi,\ \delta : x'_\xi, x'_\delta, y'_\xi, y'_\delta$. This algebraic system has the unique solution. The same presentation is possible for a case of three and more independent variables $x, y, t, ...,$

In the present paper, we suggest this new approach to obtain closed formulae for exact solutions of the Kolmogorov-Fokker-Planck (KFP) equation. New identity is obtained which follows from conditions of the obtained algebraic system solvability. Eigenvalues are calculated in obvious form.

In this case these eigenvalues are functions of independent variables, but we use the classical terminology as in each specific point these ones are numbers. As far as there are only few exact solutions of the KFP eqauation (1), we refer to the analogy with quasilinear parabolic equations (7) which are well studied by many investigators who found around one hundred of exact families of solutions. We can calculate the pointed out eigenvalues for these exact solutions. Based on this comparison some important conclusion is made and it is proposed some hypothesis on the nature of the solutions evolution to so-called "attractive limits" solution of the same equation. This hypothesis by our opinion can be extended to evolution of the Cauchy problem solutions for the non-stationary KFP equation of the form (1) [6].

1. ANALYSIS OF THE KFP EQUATION (with Sinitsyn S.O.)

One considers a stochastic system under periodic and white noise external excitations: $x' + \alpha\ x = \lambda\ Cos(y(t)),\ y' = \omega + \xi_o,$

where ξ_o is the Gaussian white noise. Then one considers the KFP equation which follows from this system

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$$p_t^{'} - \alpha(x\ p)_x^{'} + \lambda\, Cos(y)\ p_x^{'} + \omega\ p_y^{'} - \varepsilon\ p_{yy}^{''} = 0, \qquad (1)$$

where $p = p(x, y, t)$ is the density function of probability and α , ε, λ, ω are constants.

The proposed algorithm works in assumption that all used functions are continuously differentiable ones.

One considers for a beginning a case when in (1) $p = p(x, y)$. It is introduced the following arbitrary transformation of variables:

$$p(x, y)\Big|_{x=x(\xi,\delta), y=y(\xi,\delta)} = U(\xi, \delta).$$

We note that $\det J = x_\xi^{'}\ y_\delta^{'} - x_\delta^{'} y_\xi^{'} \neq 0.$

One introduces the following relation:

$$\frac{\partial p}{\partial x}\Big|_{x=x(\xi,\delta), y=y(\xi,\delta)} = Y(\xi, \delta),\ \ \frac{\partial p}{\partial y}\Big|_{x=x(\xi,\delta), y=y(\xi,\delta)} = T(\xi, \delta).$$

One obtains from here the following formulas:

$$\frac{\partial U}{\partial \xi}\frac{\partial y}{\partial \delta} - \frac{\partial U}{\partial \delta}\frac{\partial y}{\partial \xi} = Y(\xi, \delta)\det J,\ \ -\frac{\partial U}{\partial \xi}\frac{\partial x}{\partial \delta} + \frac{\partial U}{\partial \delta}\frac{\partial x}{\partial \xi} = T(\xi, \delta)\det J. \qquad (2)$$

Equation (1) in new variables takes the form

$$\omega T + \varepsilon(x_\delta^{'}\, T_\xi^{'} - x_\xi^{'}\, T_\delta^{'}) / \det J + \lambda Y\, Cos(y(\xi, \delta)) - \alpha\, x(\xi, \delta)Y - \alpha U(\xi, \delta) = 0. \qquad (3)$$

As $p(x, y)$ is the continuously differentiable function, it must be realized a condition of equality of mixed derivatives,

$p_{x\,y}^{''}(x(\xi, \delta), y(\xi, \delta)) = p_{y\,x}^{''}(x(\xi, \delta), y(\xi, \delta))$, in the variables ξ, δ. It can write this equality in the form:

$$-\frac{\partial Y}{\partial \xi}\frac{\partial x}{\partial \delta} + \frac{\partial Y}{\partial \delta}\frac{\partial x}{\partial \xi} - \frac{\partial T}{\partial \xi}\frac{\partial y}{\partial \delta} + \frac{\partial T}{\partial \delta}\frac{\partial y}{\partial \xi} = 0 \qquad (4)$$

The system of equations (2)-(4) will be analyzed by two stages.

Theorem 1.

The implicit system of linear algebraic equations (2)-(4), $AX = b$,with regards to the derivatives $X_1 = x_\xi^{'}, X_2 = x_\delta^{'}, X_3 = y_\xi^{'}, X_4 = y_\delta^{'}$ has the next unique solution:

$$x_\xi^{'} = g_1(\xi, \delta), x_\delta^{'} = g_2(\xi, \delta), t_\xi^{'} = g_3(\xi, \delta), t_\delta^{'} = g_4(\xi, \delta) \qquad (5)$$

It is possible to calculated the functions_ $g_i(\xi, \delta), i = 1,...,4$ in obvious form, for example, $g_1(\xi, \delta) = [\varepsilon T\, T_\xi^{'} - (\omega T - \alpha U + \lambda\ Cos(y(\xi, \delta))Y - \alpha\, x(\xi, \delta)Y)U_\xi^{'}](-U_\xi^{'}\, T_\delta^{'} + U_\delta^{'}\, T_\xi^{'}) / P_1(\xi, \delta).$

Matrix A has the following form:

$$A = \begin{bmatrix} -T\,U_\xi^{'} & T\,U_\delta^{'} & -Y\,U_\xi^{'} & Y\,U_\delta^{'} \\ -T_\xi^{'} & T_\delta^{'} & -Y_\xi^{'} & Y_\delta^{'} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}.$$

Vectors X, b have the forms:

$$X = (X_4, X_3, X_2, X_1)^\tau, b = (0,0,0, b_4),$$

where

$$b_4 = g_1(\xi, \delta)\, P_1(\xi, \delta),\ \ a_{33} = [\alpha U + [\alpha\, x(\xi, \delta) - \lambda\ Cos(y(\xi, \delta))]Y]U_\xi^{'} + T(-\omega\, U_\xi^{'} + \varepsilon\, T_\xi^{'}),$$

$$a_{34} = [-\alpha U + [-\alpha\, x(\xi, \delta) - \lambda\ Cos(y(\xi, \delta))]Y]U_\delta^{'} + T(\omega\, U_\delta^{'} - \varepsilon\, T_\delta^{'}), \qquad a_{44} = P_1(\xi, \delta).$$

$$P_1(\xi, \delta) = [Y[\alpha U + [\alpha\, x(\xi, \delta) - \lambda\, Cos(y(\xi, \delta))]Y][U_\delta^{'}\, T_\xi^{'} - T_\delta^{'}\, U_\xi^{'}] + T^2\, [Y_\delta^{'}\, (\varepsilon\, T_\xi^{'} - \omega U_\xi^{'}) +$$
$$[-\varepsilon\, T_\delta^{'} + \omega U_\delta^{'}]Y_\xi^{'}] + T\, [\alpha\, U(Y_\delta^{'}\, U_\xi^{'} - U_\delta^{'}\, Y_\xi^{'}) + Y[[\omega T_\delta^{'} + (\alpha\, x(\xi, \delta) - \lambda\, Cos(y(\xi, \delta)))\, Y_\delta^{'}]U_\xi^{'} -$$
$$[U_\delta^{'}\, (\,\omega T_\xi^{'} + (\alpha\ \ x(\xi, \delta) - \lambda\, Cos(y(\xi, \delta)))\, Y_\delta^{'}))]].$$

The vector symbol τ means a conjugation. The eigenvalues can be wrote of the form

$$\lambda_1 = a_{33},\ \lambda_2 = P_1(\xi,\delta), \quad \lambda_3 = \frac{1}{2}[M - \sqrt{D}],\ \lambda_4 = \frac{1}{2}[M + \sqrt{D}],$$
$$M = T_\delta^{'} - T\,U_\xi^{'}, \quad D = (T_\delta^{'})^2 + 2T\,T_\delta^{'}\,U_\xi^{'} + T[T\,(U_\delta^{'})^2 - 4\,U_\delta^{'}\,T_\xi^{'}]. \ \Diamond$$

At the second stage, we consider the new first-order system (5) with respect to the functions $x(\xi,\delta), y(\xi,\delta)$. It is well known that the solvability of a system of this type is verified by calculating the second mixed derivatives of the functions $x = x(\xi,\delta), y = y(\xi,\delta)$ on the arguments ξ and $\delta : x_{\xi\delta}^{''} = x_{\delta\xi}^{''},\ y_{\xi\delta}^{''} = y_{\delta\xi}^{''}$ [3, p.83], and [4, p.5].

Example. It is considered the more general equation KFP (1), where the second term is changed for $\alpha\,(m(x)\,p)_x^{'}$, where the last function is arbitrary twice continuously differentiated function. For concrete calculations the following function is selected: $m(x) = (\exp(\beta\,x) - 1)/(\exp(\beta\,x) + 1)$. If the solvability condition is satisfied, we can found the exact solution for the equation (1) with $p = p(x, y, t)$, having the parameter σ. It is existed a passage to the limit by this parameter, to a stationary solution (obtained for $p(x, y)$) having the some fixed value of this parameter. Corresponding formulae, which are analogical to (2)-(4) can be found in [3, p.89], [4, p.12].

Theorem 2

Let us solution of the equation (1) is the following:

$p(x, y, t) = \exp[\alpha\,t + y\,\omega/(2\,\varepsilon) - t\,\omega^2/(4\,\varepsilon)]W(x, y, t)$, which follows from the above presented condition of solvability, where the function $W(x, y, t)$ is a solution of the equation

$$W_t^{'} + \alpha\,W - \alpha\,(m(x)\,W)_x^{'} + \lambda\,Cos(y)\,W_x^{'} - \varepsilon\,W_{yy}^{''} = 0.$$

The exact solution of this equation has the form $Cos(y) - H(x, \exp(-t\,\sigma)W(x, y, t)) = 0,$ here the function $H(x,\eta), \quad \eta = \exp(-t\,\sigma)W(x, y, t),$ is a solution of the equation

$$H_x^{'} - \varepsilon\,H_{\eta\eta}^{''}\,(H^2 - 1)/[(\alpha\,m(x) - \lambda\,H)(H_\mu^{'})^2] +$$
$$[\varepsilon\,H + \eta\,H_\eta^{'}[\alpha + \sigma - \alpha\,m^{'}(x)]]/(\alpha\,m(x) - \lambda\,H) = 0. \ \Diamond \qquad (6)$$

We can determine the same solution of the equation (6) in the converge power series. The function $H(x,\eta)$ can be wrote of the form:

$H(x,\eta) = C_o(x) + C_1(x)\eta + C_2(x)\eta^2 + C_3(x)\eta^3 + C_4(x)\eta^4 + O(\eta^5)$, where $\eta < 1$.

Terms up to fourth degree are saved. Then we can use exact formulae for solutions of the algebraic equation of the fourth order. So, returning to initial variables, one has the explicit approximate solution, using the KFP equation exact solution.

Finally, note that if we put $\sigma = \omega^2/(4\varepsilon) - \alpha$, then the obtained solution of the non-stationary KFP equation transforms to the stationary solution of this equation with the coefficient $p(x, y)$. It can be wrote the ODEs system to determine coefficients $C_i(x),\ i = 0{,}4$. Zero conditions at the infinity are used for this system. By using the obtained explicit approximate formulae, for $x = 0$ we numerically construct a function having zero conditions at infinity to determine coefficients $C_i(x)$. Then the iteration process is formed with additional traditional non-local condition of normalization.

2. ON CONNECTION EIGENVALUES AND CHARACTER OF EVOLUTION OF THE SOLUTIONS OF THE NONLINEAR AND LINEAR PARABOLIC EQUATIONS

(with Volosova A.K., Vdovina E.K.)

Remark 1. It is not simple to construct solution of the equation (1). Problems for the equation (1) are badly studied. On the contrary, problems for the equation (7) are well studied; it investigated during a long time. Hundred families of solutions of the equation (7) and of equations similar to it can be found in papers by G.I.Barenblatt, L.D.Landau, A.N.Kolmogorov, I.G.Petrovskii, I.S.Piskunov, R.Fischer, Ya.B.Zeldovich, A.S.Kalashnikov, A.D.Polyanin, V.F.Zaitev, V.N.Denisov, E.M.Vorob'ev, V.P.Maslov and many others. References on publications by these authors can be found in [3].

K.A.Volosov made the following mathematical experiment. Using formulae for eigenvalues of the equation (7) matrix A, it is possible to calculate they on these exact solutions. As a result, we have alternative classification for the PDE solutions on the eigenvalues.

In papers [1-5] the proposed method with arbitrary transformation of variables is described for the following equation:

$$Z'_t - (K(Z)Z'_x)'_x + F(Z) = 0 \quad (7)$$

One uses the arbitrary transformation of variables of the form: $Z(x,t)|_{x=x(\xi,\delta),t=t(\xi,\delta)} = U(\xi,\delta)$.

We note that the determinant, $detJ = x'_\xi t'_\delta - t'_\xi x'_\delta \neq 0$, is nonzero. The inverse transformation of variable exists, at least locally: $\xi = \xi(x,t), \delta = \delta(x,t)$. The derivatives of the old independent variables on the new variables are determined as follows:

$$\frac{\partial x}{\partial \xi} = detJ\frac{\partial \delta}{\partial t},\ \frac{\partial t}{\partial \xi} = -detJ\frac{\partial \delta}{\partial x},\ \frac{\partial x}{\partial \delta} = -detJ\frac{\partial \xi}{\partial t},\ \frac{\partial t}{\partial \delta} = detJ\frac{\partial \xi}{\partial x}.$$

Let us introduce the following relation:

$K(Z)\frac{\partial Z}{\partial x}\Big|_{x=x(\xi,\delta),t=t(\xi,\delta)} = Y(\xi,\delta),\ K(Z)\frac{\partial Z}{\partial t}\Big|_{x=x(\xi,\delta),t=t(\xi,\delta)} = T(\xi,\delta).$ We obtain the formulae:

$$K(U(\xi,\delta))\left(\frac{\partial U}{\partial \xi}\frac{\partial t}{\partial \delta} - \frac{\partial U}{\partial \delta}\frac{\partial t}{\partial \xi}\right) = Y(\xi,\delta)\det J,$$

$$K(U(\xi,\delta))\left(-\frac{\partial U}{\partial \xi}\frac{\partial x}{\partial \delta} + \frac{\partial U}{\partial \delta}\frac{\partial x}{\partial \xi}\right) = T(\xi,\delta)\det J \quad (8)$$

The equation (7) takes the form:

$$T(\xi,\delta) - K(U)\left(\frac{\partial Y}{\partial \xi}\frac{\partial t}{\partial \delta} - \frac{\partial Y}{\partial \delta}\frac{\partial t}{\partial \xi}\right)/\det J + K(U)F(U) = 0 \quad (9)$$

Since Z is the continuously differentiable function, one has that $\frac{\partial}{\partial t}Z'_x = \frac{\partial}{\partial x}Z'_t$ in variables ξ, δ, or

$$-\frac{\partial x}{\partial \delta}\frac{\partial}{\partial \xi}\left[\frac{Y}{K(U)}\right] + \frac{\partial x}{\partial \xi}\frac{\partial}{\partial \delta}\left[\frac{Y}{K(U)}\right] - \frac{\partial t}{\partial \delta}\frac{\partial}{\partial \xi}\left[\frac{T}{K(U)}\right] + \frac{\partial t}{\partial \xi}\frac{\partial}{\partial \delta}\left[\frac{T}{K(U)}\right] = 0. \quad (10)$$

The system (8)-(10) will be analyzed in two stages. At the first stage, we consider the system (8)-(10) as a nonlinear algebraic equation system with respect to the derivatives $x'_\xi, x'_\delta, t'_\xi, t'_\delta$.

Theorem 3.

The implicit linear algebraic equation system (8)-(10) $AX = b$ with regards to the derivatives $X_1 = x'_\xi, X_2 = x'_\delta, X_3 = t'_\xi, X_4 = t'_\delta$, has the unique solution

$$x'_\xi = \Psi_1, x'_\delta = \Psi_2, t'_\xi = \Psi_3, t'_\delta = \Psi_4 \quad (11)$$

where functions $\Psi_i, i = 1,...,4$ are presented in [1-5], and the denominator in (11) is the following: $P_1(\xi,\delta) = FK[(TY'_\xi - YT'_\xi)U'_\delta + (YT'_\delta - TY'_\xi)U'_\xi] + TY[-U'_\delta T'_\xi + U'_\xi T'_\delta] + Y^2[Y'_\delta T'_\xi - T'_\delta Y'_\xi] + T^2[U'_\delta Y'_\xi - U'_\xi Y'_\delta]$. Matrix A has the form:

$$A = \begin{bmatrix} YU'_\delta & -YU'_\xi & TU'_\delta & -TU'_\xi \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}.$$

Vectors X, b are the following:

$X = (X_1, X_2, X_3, X_4)^\tau, b = (0,0,0,b_4)$, where

$$a_{21} = -K(U)Y'_\delta + YK'(U)U'_\delta, \quad a_{22} = K(U)Y'_\xi - YK'(U)U'_\xi,$$

$$a_{23} = -K(U)T'_\delta + TK'(U)U'_\delta, \quad b_4 = \quad K(U)[-YY'_\delta + (FK(U) + T)U'_\delta][U'_\delta Y'_\xi - Y'_\delta U'_\xi],$$

Vector symbol τ means a conjugation. The eigenvalues have the following form:

$$\lambda_1 = a_{33},\ \lambda_2 = P_1(\xi,\delta), \quad \lambda_3 = \frac{1}{2}[M - \sqrt{D}],\ \lambda_4 = \frac{1}{2}[M + \sqrt{D}],$$

$$M = KY'_\xi + Y(U'_\delta - K'(U)U'_\xi), \quad D = 4YK(Y'_\delta U'_\xi - U'_\delta Y'_\xi) + [KY'_\xi + Y(U'_\delta - K'(U)U'_\xi)^2].$$

It is proved that two conditions of solvability of the new system (11) of arbitrary functions U, Y, T always have the common multiplier [1]- [5].

K.A.Volosov with collaborators analyzed more than one hundred known, exact or approximate solutions, and calculated for them eigenvalues indicated at the Theorem 4. The astonishing regularities are obtained; see a lot of examples in [6]. It was formulated a problem of connection of the eigen values with a character of evolution and stabilization of solutions of the mixed problems for the equation (7). Analysis of calculated eigenvalues for many known solutions permits to select three cases of mixed problems [6].

The necessary conditions presented in the theorem 4 are strongly connected with an existence of the special solution $\Omega(x,t)$ of the mixed problem (with initial and boundary conditions) formulated for the equation (7). This solution is called the "limit attracting solution". Three cases are selected below. Note that a proof of the theorem 4 is obtained by the induction method.

Part 1. It exists a class of exact solutions of the mixed problems for concrete types of the equation (7) when in the presence of dissipation, and for the corresponding boundary conditions a solution of the problem tends to constant, may be to zero. It is a stabilization of the solution [7]. This result is correct as for linear equations or half-linear parabolic equations, as well for degenerate quasilinear parabolic equations of the form (7), but only in the region of the solution localization. In this case, from our point of view, the "limit attracting solution" is a constant $\Omega(x,t) \equiv$ constant, or, may be, $\Omega(x,t) \equiv 0$.See papers by L.K.Martinson, A.D.Polyanin, V.N.Denisov [7], R.O.Kershner.

Part 2. It exists a class of the mixed problems with initial and boundary conditions. Properties of solutions of these problems are determined by properties of the function $F(Z)$ in the equation (7). It is the famous problems by A.N.Kolmogorov, I.G.Petrovskii, I.S.Piskunov, R.Fischer and others. Solutions of such problems, as it was shown in different publications including publications by authors, tend to the "limit attracting" solutions, which are **waves** having the specific profile and velocity.

Part 3. If there is a stationary solution of the equation (7), that is a solution which is not depend on the independent variable t, then other solutions tend to the stationary one. In this case, from our point of view, this is the "limit attracting solution", $\Omega(x,t)$. The mixed problem with initial and boundary conditions for degenerate quasilinear parabolic equations has been investigated in [8].

By results of our investigation all three cases are united.

Plan of the analysis is the following. Formulae of the Theorem 3 are applied for the next trivial transformation of variables: $x(\xi,\delta) = \xi,\ t(\xi,\delta) = \delta$, where the Jacobian is equal to unit. This transformation is isomorphism, and the equation (7) pass to itself and solutions of the equation (7) pass to itself. Then by the exact solution, obtained in papers by other authors, or by the asymptotic properties of the solution, the eigenvalues and $Tr\ A$, that is a trek of the matrix A, can be calculated directly.

In all three cases we have as a result: three eigenvalues are equal to zero, and one eigenvalue is smaller than zero in region $\omega_1 \subset R^2$; or two eigenvalues smaller than zero in region $\omega_1 \subset R^2$.

By analogy with the dynamic systems theory we can stress that in all three cases the limit steady-states are of the knot type or of the saddle -knot type.

It is formulated the following theorem on evolution of solution of the equation in partial derivatives to the "limit attracting solution" and to propose the alternative classification for PDE solutions on the corresponding Eigen values.

Theorem 4. Let the conditions of the Theorem 3 are satisfied. Let unknown special solution $\Omega(x,t)$ of the mixed problem (with initial and boundary conditions) for concrete types of the equation (7) having the special properties as the "limit attracting solution". One assumes that in formulae of the Theorem 3 the transformation $x(\xi,\delta) = \xi,\ t(\xi,\delta) = \delta$ is made.

By necessity, the determinant $D \geq 0$, and eigenvalues $\lambda_2 \leq 0, \lambda_3 \leq 0$ in region $\omega_1 \subset R^2$; and a sign of $Tr\ A$ of the matrix A changes in a region of determination of the functions $\lambda_i, i = 1,...,4,$ then $\Omega(x,t)$ is exist, and it is realized the limit $Z(x,t) \to \Omega(x,t),$ for any values of $x,$ for $t \to \infty.$

Remark 2. We divide two following questions:

1. Which are necessary conditions of existence of the "limit attracting solution" for three problems described above?

2. How is the passage to the solution realized? In which functional spaces is it performed?

In the paper authors answer only for the first question.

In all three cases we have the difficult special steady point, namely, a saddle - knot takes place. In a region $\omega_1 \subset R^2$ the functions $\lambda_2(x,t), \lambda_3(x,t),$ depend on variables and change, but the special singular point type saves. For the localized solutions the theorem 4 works only in the localization area. The proposed theory can be extended to cases of many variables and to other PDEs and to equation KFP (1) too.

CONCLUSIONS

It is shown that wide class of equations in partial derivatives (PDEs) is equivalent to a system of functional linear algebraic equations.

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