Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

# THE KINETIC PRESSURES OF THE GYRO-ROTOR EIGEN SHAFT BEARINGS AND ROTATORS

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ABSTRACT

In our previous published paper the gyro-rotor was analyzed as a shaft-disc system with coupled rotations. The disc is eccentric and shaft is supported on both sides, at first side with a hinged fixed bearing and at other side with cylindrical sliding bearing on the support. The axes of a shaft self rotation and shaft support was with a cross section. In this paper, we presented special case when the support shaft is vertical and the gyro-rotor shaft of self rotation is horizontal, but they are without intersection. A system of non-linear differential equation is determined for such gyro-rotor dynamics. When the angular velocity of support shaft axis is constant, the motion of gyro-rotor was presented by means of phase trajectories and that is done for different cases of disk eccentricity and angle of skew disk inclination. Some numerical analysis of obtained analytical expressions is performed through Math Cad and corresponding graphs visualization of the non-linear kinetic parameters. From obtained analytical expressions for kinetic pressures to the gyro-rotor shaft bearings four vector components are separated. A pure kinematical vector rotator which depends on angular velocity and angular acceleration of the gyro-rotor shaft self rotation is defined and its properties are analyzed.

## INTRODUCTION

Numerous engineering systems and machines include many elements which rotate around axes. Such elements we usually call as rotors. Some rotors rotate around fixed axes but some rotate around moveable axes. The rotors are the basic working parts and sub-systems in many machines so that the problem of rotor vibrations has existed for a long time. The Vertical Gyro is a two-degree of freedom attitude gyro. It provides electrical outputs of the vehicle's pitch and roll angles, which are supplied to various systems including artificial horizons, autopilots, antenna stabilizers, and weapon delivery systems. Pickoff sensors such as potentiometers, resolvers or synchros are mounted on the gimbals and provide instantaneous pitch and roll output signals as the vehicle maneuvers.

The dynamic of such element motion is very old engineering problem beside that it is actually nowadays. Numerous applications of the gyro-rotor system dynamics are reason for numerous investigations of the non-linear dynamics of gyro- rotors as well non-linear phenomena appeared in this dynamics.

There are many research results and discoveries of new non-linear phenomena and of stationary and no stationary vibration regimes with different kinetic parameters of the dynamical system. But, many researches pay attention to this problem again. There are new numerical and experimental methods that help us to discover the properties of non-linear dynamics

Elementary model of the gyro-rotor was presented as a theoretical example in the Reference [1] by Andronov, Vitt and Haykin. This example is mass particle motion along rotate circle around vertical axis through center of circle and along circle vertical diameter. Monograph [2] by Gerard I. and Daniel J. contain basic of the elementary stability and bifurcation theory necessary for investigation non-linear dynamics and its kinetic parameter properties. Also, the monograph [3] by Guckenheimer and Holmes related to non-linear oscillations, dynamical systems and bifurcations of

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fields contain numerous fundamental theorems useful for application for investigation gyro-rotor dynamics.

University books [13] and [14] by Rašković give us a basic knowledge necessary for use in the descriptions of the gyro-rotor models and possible comparison by simplest models of the gyro-rotor dynamics and corresponding system of the forces with active of reactive sources, or gyroscopic effects. It is necessary to point out a Reference [15] by Stoker which contains an example with nonlinear dynamics mass particle in the turbulent damping very useful for applications in the investigation of the gyro-rotor dynamics.

Series published References [4-11] by Hedrih (Stevanović) present new results concerning non-linear dynamics of a heavy material particle along circle which rotates and optimal control in such system dynamics. In the Reference [5] series of the theorems of trigger of coupled singularities are defined with corresponding proofs. The optimal control in non-linear mechanical systems with trigger of the coupled singularities is contained in References [6] and [7]. Monograph [8] is related to the vector method of the heavy rotor kinetic parameter analysis and nonlinear dynamics and present series of the elementary examples with gyro-rotors non-linear phenomena presented by phase trajectory portraits with trigger of coupled singularities and homoclinic orbits in the form of number eight.

References [10] and [11] are related to the influence of the no ideal rough line with Coulomb's type friction and introduced non-linearity with alternation of the friction force directions.

Previous published paper [12] by authors of this paper is related to nonlinear dynamics of the heavy gyro-rotor with two skew rotating axes, and this paper present our new results in some area – investigation of the non-linear dynamics and kinetic parameter properties of the gyro-rotors.

## 1. THE MODEL OF THE GYRO-ROTOR SYSTEM AND BASIC EQUATIONS

In this paper we presented eccentric disc (eccentricity is $e$), with mass $m$ and radius $r$, which is inclined to the axes of its own rotation by the angle $\beta$ (see Figure 1.). The shaft is supported on both sides, on the first side with a hinged rigid bearing and at other side by cylindrical sliding bearing. In special case when the support shaft is vertical and the gyro-rotor shaft is horizontal, but they are without intersection between corresponding their axes. The normal distance between axes is $a$. The angle of own rotation around moveable horizontal axis oriented by the unit vector $\vec{n}_1$ is $\varphi_1$ and the angular velocity is $\omega_1$. The angle of rotation around the vertical shaft support axis oriented by the unit vector $\vec{n}_2$ is $\varphi_2$ and the angular velocity is $\omega_2$. The angular velocity of rotor is $\vec{\omega}_1 = \omega_1\vec{n}_1 + \omega_2\vec{n}_2 = \dot{\varphi}_1\vec{n}_1 + \dot{\varphi}_2\vec{n}_2$. The angles $\varphi_1$ and $\varphi_2$ are generalized coordinates in case when, we investigate system with two degrees of freedom. In this case $\varphi_1$ is independent generalized coordinate, and coordinate $\varphi_2$ is rheonomic coordinate with kinematical excitation, programmed by forced support rotation by constant or changeable angular velocity. When the angular velocity of shaft support axis is constant, that is $\varphi_2 = \omega_2 t + \varphi_{20}$, $\dot{\varphi}_2 = \omega_2 = const$, $\dot{\omega}_2 = 0$ (in this case the angle $\varphi_2$ is a rheonomic coordinate defined by previous time dependent function), and system is with two degree of mobility, but with one degree of freedom. For that case the differential equation of the gyro-rotor system rotation can be written in a following form (see Ref. [12]):

$$\dot{\omega}_1 + \Omega^2(\lambda - \cos\varphi_1)\sin\varphi_1 - \Omega^2\Psi\cos\varphi_1 = 0 \qquad (1)$$

where we use the following notation:

$$\Omega^2 = \frac{\varepsilon\sin^2\beta - 1}{\varepsilon\sin^2\beta + 1}\omega_2^2 \qquad \lambda = \frac{g(\varepsilon-1)\sin\beta}{(\varepsilon\sin^2\beta - 1)e\omega_2^2} \qquad (2)$$

$$\Psi = \frac{8ea}{(\varepsilon\sin^2\beta - 1)r^2}\sin\beta \qquad \varepsilon = 1 + 4\left(\frac{e}{r}\right)^2 \qquad (3)$$

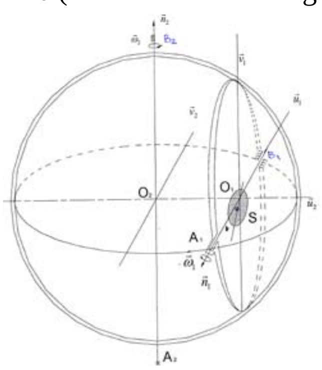
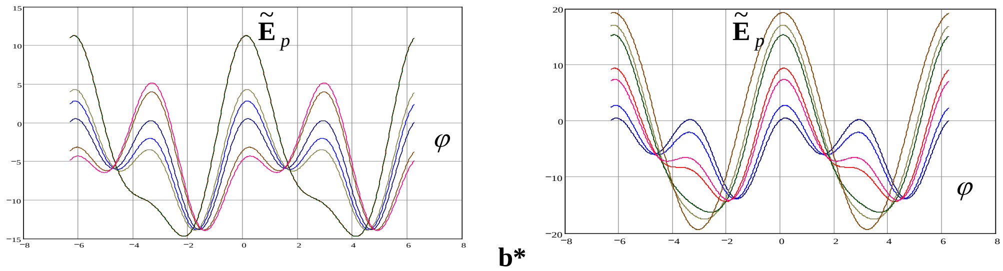


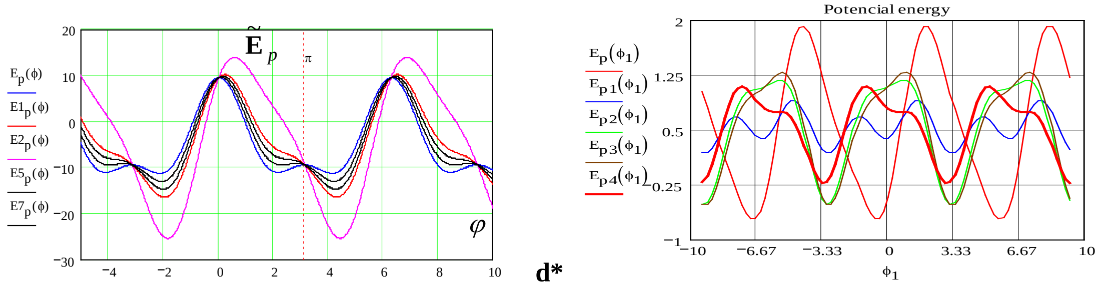
*Fig. 1. Gyro-rotor*

The motion of gyro-rotor was presented by means of phase trajectories and that is done for different cases of disk eccentricity and angle of skew. For that reason it is necessary to find first integral of the differential equation (1). After integration of the differential equation (10 the non-linear equation of the phase trajectories of the gyro rotor dynamics with the initial conditions $t_0 = 0$, $\varphi_1(t_0) = \varphi_{10}$, $\dot{\varphi}_1(t_0) = \dot{\varphi}_{10}$ is obtained in a form:

$$\dot{\varphi}_1^2 = \dot{\varphi}_{10}^2 + 2\Omega^2\left[\lambda(\cos\varphi_1 - \cos\varphi_{10}) + \frac{1}{2}\left(\cos^2\varphi_{10} - \cos^2\varphi_1\right) + \Psi(\sin\varphi_1 - \sin\varphi_{10})\right] \tag{4}$$

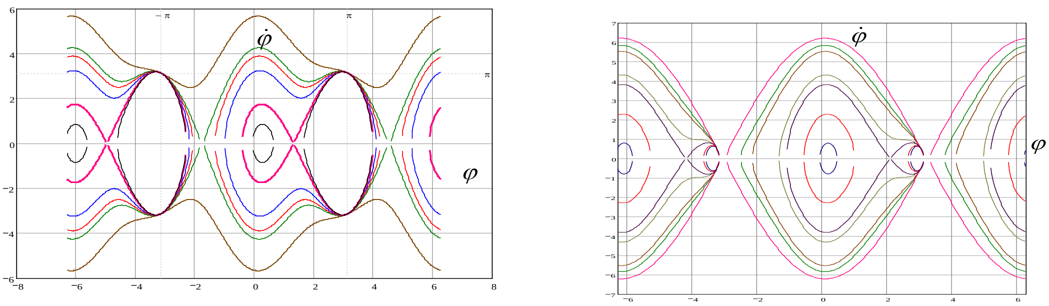



**a*** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = \pi[rad/\sec]$ **b*** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = 0$




**C*** **d***

*Fig. 2. The transformation of the graphical presentation of the potential energy analog of the heavy gyro rotor with rotating axis that are without intersection for different values (d*) of the eccentricity $e$ and (a*, b* and c*) of the angle $\beta$ of disk inclination to the proper shaft axis rotation.*




**a*** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = \pi[rad/\sec]$ **b*** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = 0$

*Fig. 3. Transformation of a phase trajectory of the heavy gyro-rotor with rotating axis that are without intersection for different values of disk inclination angle β to the axis of self rotation and for two different initial conditions:*

**(a*)** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = \pi[rad/\sec]$ and **(b*)** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = 0$

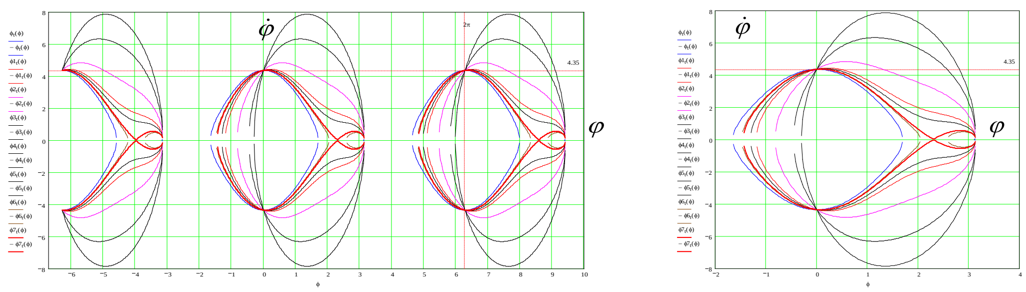



*Fig. 4. Transformation of a phase trajectory presentation of the heavy gyro-rotor with rotating axis that are without intersection for different values of normal distance between axes and for a corresponding initial condition.*

As the analyzed system is conservative it is the energy integral. For that case we can separate part of expressions in the equation (4) in the following form:

$$\tilde{\mathbf{E}}_p = \Omega^2(\beta,\varepsilon,\omega_2)\left[\lambda(\beta,\varepsilon,e,\omega_2)(\cos\varphi_1 - \cos\varphi_{10}) + \frac{1}{2}\left(\cos^2\varphi_{10} - \cos^2\varphi_1\right) + \Psi(\beta,e,a,r)(\sin\varphi_1 - \sin\varphi_{10})\right] \tag{5}$$

as a analog to the potential energy in this rheonomic system. The analog to the potential energy exchange curves for different values of the system parameters (the eccentricity $e$ and the angle $\beta$ of the disk inclination) are given on Fig.2.

In Figure 3 a transformation of a phase trajectory of the heavy gyro-rotor with rotating axis that are without intersection for different values of disk inclination angle β to the axis of self rotation and for two different initial conditions: **(a*)** $\varphi_0 = \pi[rad]$; $\dot{\varphi}_0 = \pi[rad/\sec]$ and **(b*)** $\varphi_0 = \pi[rad]$, $\dot{\varphi}_0 = 0$ is presented. In Figure 4 a transformation of a phase trajectory presentation of the heavy gyro-rotor with rotating axis that are without intersection for different values of normal distance between axes and for a corresponding initial condition is presented.

## 2. THE KINETIC PRESURES ON SHAFT BEARINGS OF THE GYRO-ROTOR

The shafts and axis are supported by bears so they are subjected to static and kinematics forces. Bearing force analysis of mechanisms is an important field in which mechanical engineers study a motion in order to design mechanisms to perform useful tasks. The forces whose nature is static have constant intensity but those with kinetics nature are changeable. So, the kinetic pressures on bearings can be very changeable in intensity and could involve some damages. The task is minimizing kinetic components.

An analytical formulation of forces in a form of four components is obtained by using two theorems: the theorem of linear momentum derivative and the theorem of angular momentum. By application of the two theorems we can write:

$$\frac{d\vec{K}}{dt} = \sum_i \vec{F}_i \qquad \text{and} \qquad \frac{d\vec{L}_O}{dt} = \vec{M}_O\left(\vec{F}_i\right) + m\vec{\rho}_C \times \left(\vec{v}_O \times \vec{\omega} - \vec{a}_O\right) \tag{6}$$

By solving these vector equations, we get of bearing forces in a form of four components. We separate some new unit vectors, also, as orientation of the kinetic pressure components applied to bearings in the following forms;:

$$F_{B1}^{kin} = \frac{1}{2\ell}\left(J_{vn}\omega_2 \cos\varphi_1 - J_n\dot{\varphi}_1\right)\omega_2 \qquad F_{A1}^{kin} = \frac{1}{2\ell}\left(J_{vn}\omega_2 \cos\varphi_1 + J_n\dot{\varphi}_1\right)\omega_2 \tag{7}$$

$$F_{B2}^{kin} = \frac{1}{2\ell}\left(J_u - J_v\right)\omega_2\dot{\varphi}_1 \qquad F_{A2}^{kin} = -\frac{1}{2\ell}\left(J_u - J_v\right)\omega_2\dot{\varphi}_1 \tag{8}$$

$$F_{B3}^{kin} = \frac{1}{2}m\omega_2^2(a + e\sin\beta\cos\varphi_1 + 2e\frac{a}{\ell}\cos\beta) \quad F_{A3}^{kin} = \frac{1}{2}m\omega_2^2(a + e\sin\beta\cos\varphi_1 - 2e\frac{a}{\ell}\cos\beta) \tag{9}$$

$$F_{B4}^{kin} = \frac{1}{2}\left(me\sin\beta - \frac{1}{\ell}J_{vn}\right)\sqrt{\ddot{\varphi}_1^2 + \dot{\varphi}^4} \qquad F_{A4}^{kin} = \frac{1}{2}\left(me\sin\beta + \frac{1}{2\ell}J_{vn}\right)\sqrt{\ddot{\varphi}_1^2 + \dot{\varphi}^4} \tag{10}$$

The first components (7) are directed in line with unit vector $\vec{w}_1 = \vec{u}_1 \sin\varphi_1 + \vec{v}_1 \cos\varphi_1$, the second components (8) are directed in line with unit vector $\vec{w}_2 = \vec{u}_1 \sin\varphi_1 - \vec{v}_1 \cos\varphi_1$. These components are depending on angular velocity $\omega_2$ and angular velocity $\omega_1$**,** the body disk mass distribution, the body mass axial inertia moment for the rotating axis, $J_n$, the body mass axial inertia moment for the axes normal on rotating axis, $J_u$ and $J_v$, and the deviational moment of the body mass for a couple of normal axis oriented by the unit vectors $\vec{n}_1$ and $\vec{v}_1$, $J_{nv}$.These are periodical components with period of 2π and with extreme values, too.

The third components (9) are depending on the body mass, the disk eccentricity, ε, distance between two axes, $a$, the angle of disk inclination, β, and they are proportional to square angular velocity $\omega_2$**.** These components are directed in line with unit vector $\vec{w}_3 = -\vec{u}_1 \cos\varphi_1 + \vec{v}_1 \sin\varphi_1$.

The fourth components (10) are directed in line with vector named rotator. The intensity of these components depends on the deviational moment of the body gyro-rotor (disk) mass for a couple of normal axis oriented by the unit vectors $\vec{n}_1$ and $\vec{v}_1$, $J_{nv}$ the body gyro-rotor mass *m*, eccentricity *e* and the angle of disk inclination, β.

## 3. THE ROTATOR

In the expressions of the kinetic pressure components (10) to bearings of shaft self rotation, there are intensity as multiplication by the member with constant intensity (this means that its intensity depends only on mass and geometrical characteristics of rotor) and multiplied by a member depending only of kinematical parameters, angular velocity and angular acceleration of self rotation of gyro-rotor. That kinetic pressure component is directed is in line with the vector which is named rotator [8]. The rotator is pure kinematics vector and it rotates and increases by angular velocity and angular acceleration of the gyro-rotor rotation around self shaft of self rotation. Its intensity dependences on angular velocity and angular acceleration, that is,

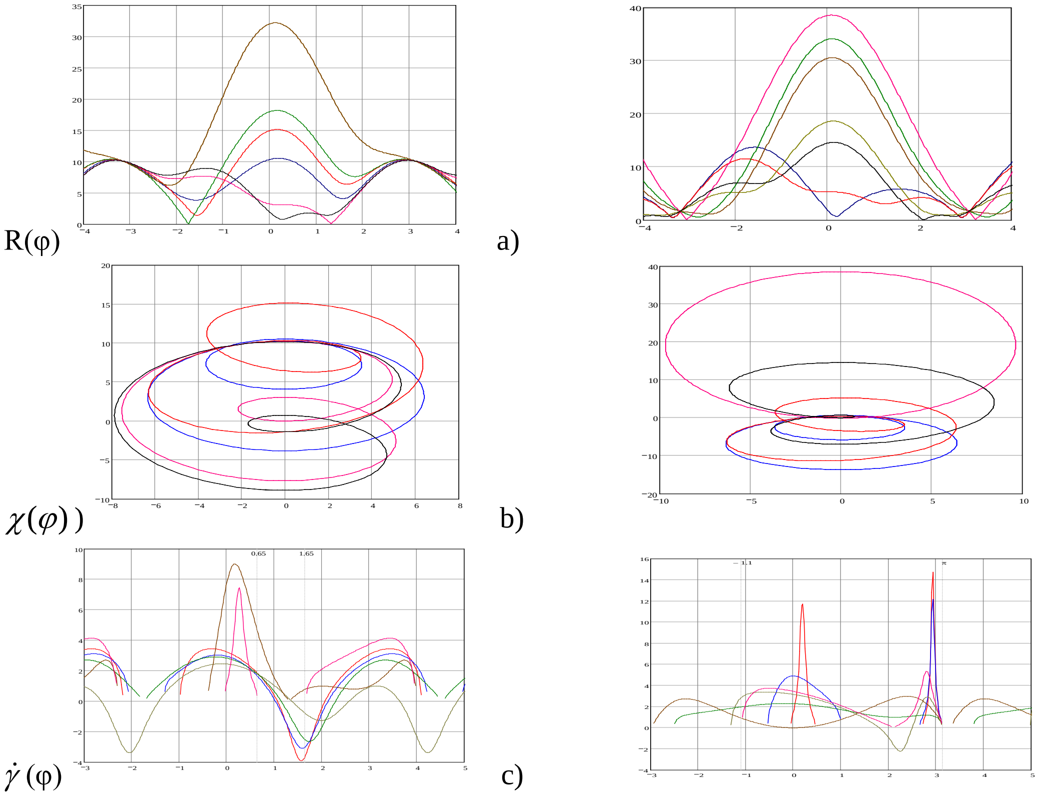
$$\Re = \left|\vec{\Re}(\varphi_1)\right| = \sqrt{\ddot{\varphi}^2 + \dot{\varphi}^4} \tag{11}$$

Figures 5.a* show the dependence on the vector rotator intensity in the function of the elongation and for different values of the initial parameters h of the energy. The rotator is different from zero so the dynamic pressures on the bearings are different from zero, too. The smallest values of the rotator are corresponding to the position of the unstable static equilibrium position, while the greatest values of the rotator are corresponding to the position of the stable static equilibrium position.

Figures 5.b* show the rotator trajectories. There are some shapes of trajectories and their shapes depend on parameters of the system. The parametric equations of rotator trajectories are:

$$u_{\Re}(\varphi_1) = \Omega^2\left[-(\lambda - \cos\varphi_1)\sin\varphi_1 + \psi\cos\varphi_1\right] \tag{12}$$

$$v_{\Re}(\varphi_1) = 2\Omega^2\left(\lambda\cos\varphi_1 - \frac{1}{2}\cos^2\varphi_1 + \psi\sin\varphi_1\right) + h$$




$\varphi_0 = \pi[rad];\ \dot{\varphi}_0 = \pi[rad/\sec]$ $\qquad$ $\varphi_0 = \pi[rad];\ \dot{\varphi}_0 = 0$

*Fig. 5. Vector rotator of the heavy gyro rotor: a)the intensity portrait; b) the hodograph; c) the angular velocity for different values of angle β and for different initial conditions*

The angle that rotator form with axis $\vec{u}_1$ is determined by express: $tg\gamma = \frac{\dot{\varphi}^2}{\ddot{\varphi}}$. The derivative by time is:

$$\dot{\gamma} = \frac{\dot{\varphi}\left(2\ddot{\varphi}^2 - \dddot{\varphi}\right)}{\ddot{\varphi}^2 + \dot{\varphi}^4} = \frac{\dot{\varphi}\left(2\ddot{\varphi}^2 - \dddot{\varphi}\right)}{\Re^2} \tag{13}$$

and its graphical presentation is shown on Figure 5.c*.

## CONCLUSIONS

By use analytical expressions of the kinetic components of the kinetic pressures to the gyro-rotor shaft bearings through MathCad program numerous visualizations are presented through characteristic graphs and qualitatively analyzed. Special attentions are focused to the vector rotators, as well as to the absolute and relative angular velocities of the rotation of the kinetic components of the kinetic pressures to the gyro-rotor self rotation shaft bearings.

From obtained analytical expressions for kinetic pressures to the gyro-rotor shaft bearings four vector components are separated. One component of the kinetic pressures to the gyro-rotor shaft bearings of self rotation is caused by deviation properties of the gyro-rotor mass distribution around self rotation shaft axis and is expressed as product between deviation mass inertia moment according shaft axis of self rotation and pure kinematical vector rotator which depends on angular velocity and angular acceleration of the gyro-rotor shaft self rotation. Three other components of the kinetic pressures to the gyro-rotor shaft bearings are functions of the both angular coordinates and angular velocities of the gyro-rotor system dynamics as well as of the gyro-rotor mass distributions and deviational properties.

***Acknowledgment.*** *Parts of this research were supported by the Ministry of Sciences and Environmental Protection of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON144002 and Faculty of Mechanical Engineering University of Nis.*

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