## THE TRAJECTORIES OF SELF-REINFORCING SOLITARY WAVE IN THE GAS DISC OF GALAXIES

# Yuriy Turbal<sup>1</sup> ABSTRACT International University of Economics and Humanities named after academician Stepan Demianchuk, Rivne, Ukraine ABSTRACT

### INTRODUCTION

Problems, related to the study of solitons and their interactions lately are of interest in many areas of fundamental and applied scientific researches. Solitons are intensively studied in hydrodynamics, fiber optics, in magnets. Solitons may occur in proteins and DNA.

Note, that the study of various processes in the disks of galaxies using hydrodynamic models are carried out by A.M. Friedman [1]. In the Research Center "Kurchatov Institute" the processes in galaxies were studied under his leadership, in particular, hydrodynamic instability in the mechanisms of spiral density waves generation.

In this paper we consider structurally stable solitary waves in a medium, which is a gas component of galaxies. The assumption of the existence of such waves follows from the equivalence of shallow water equations and equations of gas dynamics of galaxies (see the [1]). But the existence of solitons in shallow water is a well known and experimentally verified fact. Note that in 2008, astronomers have recorded a soliton in space (the message of the European Space Agency ESA).

Solitons considered in this work are structurally stable density perturbation, localized in some small areas. Similar solitons considered in [3] as a weak asymptotic solution of equations of shallow water. In this paper such solitary waves are considered in the gas disk of galaxies. We study the trajectory of solitons.

### 1. SOLUTIONS OF GAS DYNAMICS EQUATIONS IN THE FORM OF SOLITARY WAVES

We consider the equation of gas dynamics of galaxies [2], written in polar coordinates for the case of the isentropic model and polytropic law ( $p = B\sigma^{\gamma_s}$ ) of the surface pressure and surface density:

$$\frac{\partial\sigma}{\partial t} + \sigma(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{r\partial\varphi}) + u\frac{\partial\sigma}{\partial r} + \frac{v\partial\sigma}{r\partial\varphi} = 0, \qquad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \varphi} - \frac{v^2}{r} = -\frac{\partial \Phi}{\partial r} - B\gamma_s \sigma^{\gamma_s - 2} \frac{\partial \sigma}{\partial r}, \qquad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \varphi} + \frac{uv}{r} = -\frac{1}{r} \frac{\partial \Phi}{\partial \varphi} - \sigma^{\gamma_s - 2} \frac{B\gamma_s}{r} \frac{\partial \sigma}{\partial \varphi}, \qquad (3)$$

<sup>&</sup>lt;sup>1</sup> Corresponding author. Email <u>turbaly@gmail.com</u>

where  $u(r, \varphi, t), v(r, \varphi, t)$  are the radial and azimuthal velocity components of gas, respectively,  $\sigma(r, \varphi, t)$  is the surface density of the gas disk,  $\Phi(r, \varphi, t)$  - is the gravitational potential, *B* is a positive constant. Note that the system (1) - (3) is quasi-linear.

Let  $u_0(r) \equiv 0$ ,  $v_0(r)$ ,  $\sigma_0(r)$ ,  $\Phi_0(r)$  are the equilibrium components of velocity, density and gravitational potential, respectively. We seek a particular solution of (1) - (3) (same as in [2]) as the sum of the equilibrium values, and some disturbances:  $\sigma(r,\phi,t) = \sigma_0(r) + \sigma_1(r,\phi,t)$ ,  $\Phi(r,\phi,t) = \Phi_0(r) + \Phi_1(r,\phi,t)$ ,  $u(r,\phi,t) = u_1(r,\phi,t)$ ,  $v(r,\phi,t) = v_0(r) + v_1(r,\phi,t)$ . (4)

The area in which we consider a system (1) - (3) has the form:  $G = \{(r, \varphi) : r_0 \le r \le R\}$ . From (1)-(3) we get the system:

$$\begin{aligned} \frac{\partial \sigma_{1}}{\partial t} + \frac{v_{0}}{r} \frac{\partial \sigma_{1}}{\partial \varphi} + \sigma_{0} \left(\frac{\partial u_{1}}{\partial r} + \frac{u_{1}}{r} + \frac{\partial v_{1}}{r \partial \varphi}\right) + u_{1} \frac{\partial \sigma_{0}}{\partial r} + \frac{1}{r} \left(\sigma_{1}u_{1} + u_{1}r \frac{\partial \sigma_{1}}{\partial r} + \sigma_{1}r \frac{\partial u_{1}}{\partial r} + \sigma_{1} \frac{\partial v_{1}}{\partial \varphi} + v_{1} \frac{\partial \sigma_{1}}{\partial \varphi}\right) &= 0, \\ \frac{\partial u_{1}}{\partial t} + \frac{v_{0}}{r} \frac{\partial u_{1}}{\partial \varphi} - \frac{2v_{0}v_{1}}{r} &= -\frac{\partial \Phi_{1}}{\partial r} - B\gamma_{s}(\gamma_{s} - 2)\sigma_{1}\sigma_{0}^{\gamma_{s}-3} \frac{\partial \sigma_{0}}{\partial r} - B\gamma_{s}\sigma_{0}^{\gamma_{s}-2} \frac{\partial \sigma_{1}}{\partial r} - \\ -B\gamma_{s}(\gamma_{s} - 2)\sigma_{1}\sigma_{0}^{\gamma_{s}-3} \frac{\partial \sigma_{1}}{\partial r} + \frac{v_{1}^{2}}{r} - u_{1} \frac{\partial u_{1}}{\partial r} - \frac{v_{1}}{r} \frac{\partial u_{1}}{\partial \varphi}, \end{aligned}$$

$$(5) \\ \frac{\partial v_{1}}{\partial t} + u_{1} \frac{\partial v_{0}}{\partial r} + \frac{v_{0}}{r} \frac{\partial v_{1}}{\partial \varphi} + \frac{u_{1}v_{0}}{r} &= -\frac{1}{r} \frac{\partial \Phi_{1}}{\partial \varphi} - \frac{B\gamma_{s}}{r} \sigma_{0}^{\gamma_{s}-2} \frac{\partial \sigma_{1}}{\partial \varphi} - \frac{B\gamma_{s}}{r} (\gamma_{s} - 2)\sigma_{0}^{\gamma_{s}-3}\sigma_{1} \frac{\partial \sigma_{1}}{\partial \varphi} - \\ -u_{1} \frac{\partial v_{1}}{\partial r} - \frac{v_{1}}{r} \frac{\partial v_{1}}{\partial \varphi} - \frac{u_{1}v_{1}}{r}. \end{aligned}$$

For convenience, we introduce the vector-function perturbations

 $f_1(r,\phi,t) = (u_1(r,\phi,t), v_1(r,\phi,t), \sigma_1(r,\phi,t))$ , that will be sought in the form:

$$f_1(r,\varphi,t) = \varepsilon_0 \psi_f(t) \omega(r,\varphi,t,\varepsilon) .$$
(6)

where  $\psi_f(t) = (\psi_u(t), \psi_v(t), \psi_\sigma(t)), \quad \psi_u(t), \quad \psi_v(t), \quad \psi_\sigma(t), \quad \tilde{r}(t), \quad \tilde{\varphi}(t) \text{ are some functions,}$  $\tilde{r}(t) \ge 0, \quad \psi_\sigma(t) \ge 0, \quad \varepsilon \text{ is a small parameter }, \quad \varepsilon_0 \text{ is some constant,}$ 

$$\omega(r,\varphi,t,\varepsilon) = \exp\{-\frac{g(r-\tilde{r}(t)) + g(\varphi - \tilde{\varphi}(t))}{\varepsilon}\},\tag{7}$$

g(x) is a nonnegative pair function, which has the properties:

- 1.  $g(x) \ge 0$ ,  $x \in (-\infty, +\infty)$
- 2. g(0) = 0
- 3. g(-x) = g(x)

4. There are constants,  $\alpha_1, \tilde{\alpha}_1, \alpha_2, \tilde{\alpha}_2, c_1 > 0, c_2 > 0, 0 < \alpha_1, \tilde{\alpha}_1 < 1, 0 < \alpha_2, \tilde{\alpha}_2 < 1, \varepsilon > 0$  such that  $\tilde{c}_1 g^{\tilde{\alpha}_1}(x) \le |g'(x)| \le c_1 g^{\alpha_1}(x), |g''(x)| \le c_2 g^{\alpha_2}(x)$  in the field  $\{x : g(x) \le -\varepsilon \ln \varepsilon\}$ .

It is obvious, that such function exists (for example,  $g(x) = x^4$ ). It is evident from relation (7) that the perturbation is a solitary wave. The point of maximum of the wave moves along the trajectory, which is described in polar coordinates by the functions  $\tilde{r}(t)$  and  $\tilde{\varphi}(t)$ .

Differentiating (7), we obtain:

$$\frac{\partial f_1}{\partial r} = -\varepsilon_0 \frac{g'(r-\tilde{r}(t))}{\varepsilon} \psi_f(t) \omega(r,\varphi,t,\varepsilon), \\ \frac{\partial f_1}{\partial \varphi} = -\varepsilon_0 \frac{g'(\varphi-\tilde{\varphi}(t))}{\varepsilon} \psi_f(t) \omega(r,\varphi,t,\varepsilon), \\ \frac{\partial f_1}{\partial t} = \varepsilon_0 \psi_f'(t) \omega(r,\varphi,t,\varepsilon) + \frac{1}{\varepsilon} \varepsilon_0 \psi_f(t) \omega(r,\varphi,t,\varepsilon) [\tilde{\varphi}'(t)g'(\varphi-\tilde{\varphi}(t)) + \tilde{r}'(t)g'(r-\tilde{r}(t))].$$
(8)

The remaining arguments consist of the substitution of relations (7) - (8) into (5) and the allocation of appropriate conditions.

From the first equation (5) we obtain the following system:

$$\begin{cases} \psi_{\sigma}'(t) + \sigma_{0} \frac{\psi_{u}(t)}{r} + \psi_{u}(t) \frac{\partial \sigma_{0}}{\partial r} + \frac{\psi_{v}(t)}{r} \frac{\partial \sigma_{0}}{\partial \varphi} + \frac{1}{r} \varepsilon_{0} \psi_{\sigma}(t) \psi_{u}(t) \omega(r, \varphi, t, \varepsilon) = 0 \\ \tilde{\varphi}'(t) - \frac{v_{0}}{r} - \psi_{v}(t) \frac{\sigma_{0}}{r \psi_{\sigma}(t)} - \frac{2\varepsilon_{0}}{r} \psi_{v}(t) \omega(r, \varphi, t, \varepsilon) = 0 \\ \tilde{r}'(t) - \sigma_{0} \psi_{u}(t) / \psi_{\sigma}(t) - 2\varepsilon_{0} \psi_{u}(t) \omega(r, \varphi, t, \varepsilon) = 0 \end{cases}$$
(9)

The second and third of equations (5) can be written as:  

$$\frac{\partial \Phi_{1}}{\partial r} = -\varepsilon_{0} \alpha(r, \varphi, t, \varepsilon) (\psi_{u}'(t) - \frac{2v_{0}\psi_{v}(t)}{r} - \frac{3}{2r} BD(\gamma_{s} - 1)\sigma_{0}^{\gamma_{s} - 2}\psi_{\sigma}(t) + \\
+ \frac{\varepsilon_{0}\psi^{2}v(t)\alpha(r, \varphi, t, \varepsilon)}{r} + B\gamma_{s}(\gamma_{s} - 2)\psi_{\sigma}(t)\sigma_{0}^{\gamma_{s} - 3}\frac{\partial\sigma_{0}}{\partial r}) - \\
-\psi_{u}(t) \frac{g'(r - \tilde{r}(t))}{\varepsilon} \varepsilon_{0}\alpha(r, \varphi, t, \varepsilon)(\tilde{r}'(t) - B\gamma_{s}\sigma_{0}^{\gamma_{s} - 2}\psi_{\sigma}(t)/\psi_{u}(t) - B\gamma_{s}(\gamma_{s} - 2)\sigma_{0}^{\gamma_{s} - 3}\varepsilon_{0}\psi_{\sigma}^{2}(t)/\psi_{u}(t)\alpha(r, \varphi, t, \varepsilon) - \\
-\varepsilon_{0}\psi_{u}(t)\alpha(r, \varphi, t, \varepsilon)) - \psi_{u}(t)\frac{g'(\varphi - \tilde{\varphi}(t))}{\varepsilon} \varepsilon_{0}\alpha(r, \varphi, t, \varepsilon)(\tilde{\varphi}(t) - \frac{v_{0}}{r} - \frac{\psi_{v}(t)}{r} \varepsilon_{0}\alpha(r, \varphi, t, \varepsilon)) \\
\frac{1}{r}\frac{\partial\Phi_{1}}{\partial\varphi} = -\varepsilon_{0}\omega(r, \varphi, t, \varepsilon)(\psi_{v}'(t) + \psi_{u}(t)\frac{\partialv_{0}}{\partial r} + \frac{\psi_{u}(t)v_{0}}{r} + \frac{\psi_{v}(t)}{r}\frac{\partialv_{0}}{\partial\varphi} + \frac{\psi_{u}(t)\varepsilon_{0}\psi_{v}(t)\alpha(r, \varphi, t, \varepsilon)}{r} + \\
+ \frac{B\gamma_{s}}{r}(\gamma_{s} - 2)\psi_{\sigma}(t)\sigma_{0}^{\gamma_{s} - 3}\frac{\partial\sigma_{0}}{\partial\varphi}) - \varepsilon_{0}\omega(r, \varphi, t, \varepsilon)\frac{g'(r - \tilde{r}(t))}{r}\psi_{\sigma}(t)/\psi_{v}(t)(\tilde{r}'(t) - \psi_{u}(t)\varepsilon_{0}\omega(r, \varphi, t, \varepsilon)) - \\
-\frac{g'(\varphi - \tilde{\varphi}(t))}{\varepsilon}\varepsilon_{0}\omega(r, \varphi, t, \varepsilon)\psi_{v}(t)(\tilde{\varphi}'(t) - \frac{v_{0}}{r} - \frac{B\gamma_{s}}{r}\sigma_{0}^{\gamma_{s} - 2}\psi_{\sigma}(t)/\psi_{v}(t) - \\
-\frac{B\gamma_{s}}{r}(\gamma_{s} - 2)\sigma_{0}^{\gamma_{s} - 3}\varepsilon_{0}\psi_{\sigma}^{2}(t)/\psi_{v}(t)\omega(r, \varphi, t, \varepsilon) - \frac{1}{r}\varepsilon_{0}\psi_{v}(t)\omega(r, \varphi, t, \varepsilon))$$
(10)

### **2 SYSTEMS OF EQUATIONS THAT DETERMINE THE TRAJECTORY OF WAVES**

**Proposition 1.** Let  $\psi_u(t) = 0$ ,  $\psi_v(t) = 0$ ,  $\Phi_1(r, \phi, t) = 0$ . Then the nonzero structure-stable perturbation of the surface density in the form (4) can be existed in the region where the surface density is constant (for the isentropic model). The trajectory of perturbations coincides with the trajectories of the gas. In the region

$$\begin{aligned} G_{\varepsilon}^{\alpha} &= \{ (r, \varphi, t) : g(r - \tilde{r}(t)) + g(\varphi - \tilde{\varphi}(t)) < \varepsilon^{1/\alpha_1}, t \in [0, T] \} \text{ the relation} \\ \sigma_0(\tilde{r}(t)) + (\gamma_s - 2)\varepsilon_0 \psi_{\sigma}(t) = O(\varepsilon^{1/\alpha_1 - 1}) . \end{aligned}$$

**Proposition 2.** Let  $\psi_v(t) \neq 0$ ,  $\psi_u(t) \neq 0$ ,  $\Phi_1(r, \phi, t) = 0$ . Then there is a disturbance of the surface density of the form (4) for the case  $\gamma_s = 3$  and thus the following equations:

$$\sigma_{0}(\tilde{r}(t),\tilde{\varphi}(t),t) + \varepsilon_{0}\psi_{\sigma}(t)\omega(\tilde{r}(t),\tilde{\varphi}(t),t,\varepsilon) = 0$$

$$\begin{cases} \frac{\partial\sigma_{0}}{\partial t} + \frac{v_{0}\partial\sigma_{0}}{r\partial\varphi} = 0 \\ -\frac{v_{0}^{2}}{r} = -\frac{\partial\Phi_{0}}{\partial r} - B\gamma_{s}\sigma_{0} \quad \frac{\partial\sigma_{0}}{\partial r} + \frac{3}{2r}DB\sigma_{0}^{2} \\ \frac{\partial v_{0}}{\partial t} + \frac{v_{0}}{r}\frac{\partial v_{0}}{\partial\varphi} = -\sigma_{0} \quad \frac{3B}{r}\frac{\partial\sigma_{0}}{\partial\varphi} \end{cases}$$
(13)

$$\begin{cases} \psi_{u}'(t) - \frac{2v_{0}\psi_{v}(t)}{\widetilde{r}(t)} + \frac{3}{\widetilde{r}(t)}BD\sigma^{2}_{0}(\widetilde{r}(t),\widetilde{\varphi}(t))/\varepsilon_{0} + \frac{\varepsilon_{0}\psi^{2}_{v}(t)}{\widetilde{r}(t)} - 3B\frac{\partial\sigma_{0}}{\partial r}\sigma_{0}(\widetilde{r}(t),\widetilde{\varphi}(t))/\varepsilon_{0} = 0\\ \psi_{v}'(t) + \psi_{u}(t)\frac{\partial v_{0}}{\partial r} + \frac{\psi_{v}(t)}{\widetilde{r}(t)}\frac{\partial v_{0}}{\partial \varphi} + \frac{\psi_{u}(t)v_{0}}{\widetilde{r}(t)} + \frac{\psi_{u}(t)\varepsilon_{0}\psi_{v}(t)}{\widetilde{r}(t)} = 0\\ \widetilde{\varphi}'(t) - \frac{v_{0}}{\widetilde{r}(t)} - \frac{\psi_{v}(t)}{\widetilde{r}(t)}\varepsilon_{0} = 0\\ \widetilde{r}'(t) - \psi_{u}(t)\varepsilon_{0} = 0 \end{cases}$$
(14)

**Proposition 3.** Let  $\psi_{\nu}(t) = 0$ ,  $\psi_{\mu}(t) \neq 0$ ,  $\Phi_{1}(r, \phi, t) = 0$ . Then there is a disturbance of the surface density of the form (4) for the case  $\gamma_{s} = 3$  and thus the following equations: (12), (13),

$$\begin{cases} \psi_{u}'(t) + \frac{3}{\widetilde{r}(t)} BD\sigma^{2}{}_{0}(\widetilde{r}(t),\widetilde{\varphi}(t))/\varepsilon_{0} - 3B\frac{\partial\sigma_{0}}{\partial r}\sigma_{0}(\widetilde{r}(t),\widetilde{\varphi}(t))/\varepsilon_{0} = 0, \\ \frac{\partial v_{0}}{\partial r} + \frac{v_{0}}{\widetilde{r}(t)} - \frac{3B}{\widetilde{r}(t)\varepsilon_{0}}\sigma_{0}(\widetilde{r}(t),\widetilde{\varphi}(t),t)\frac{\partial\sigma_{0}}{\partial\varphi} = 0, \\ \widetilde{\varphi}'(t) - \frac{v_{0}}{\widetilde{r}(t)} = 0, \\ \widetilde{\varphi}'(t) - \psi_{u}(t)\varepsilon_{0} = 0, \\ \psi_{\sigma}(t) = -\sigma_{0}(\widetilde{r}(t),\widetilde{\varphi}(t),t)/\varepsilon_{0} \end{cases}$$
(15)

**Proposition 4.** Let  $\Phi_1 = -B\gamma_s \sigma_0^{\gamma_s - 2} \varepsilon_0 \psi_\sigma(t) \omega(r, \varphi, t, \varepsilon)$ ,  $\psi_v(t) \neq 0$ ,  $\psi_u(t) \neq 0$ . Then there is a disturbance of the surface density of the form (4) for the case  $\gamma_s = 2$  and thus the following equations: (12), (13),

$$\begin{cases} \psi_{\sigma}'(t) + \sigma_{0} \frac{\psi_{u}(t)}{r} + \psi_{u}(t) \frac{\partial \sigma_{0}}{\partial r} + \frac{1}{r} \varepsilon_{0} \psi_{\sigma}(t) \psi_{u}(t) \omega(r, \varphi, t, \varepsilon) = 0 \\ \psi_{u}'(t) - \frac{2v_{0} \psi_{v}(t)}{r} - \frac{3}{2r} BD \psi_{\sigma}(t) + \frac{\varepsilon_{0} \psi^{2}_{v}(t) \omega(r, \varphi, t, \varepsilon)}{r} = 0 \\ \psi_{v}'(t) + \psi_{u}(t) \frac{\partial v_{0}}{\partial r} + \frac{\psi_{u}(t)v_{0}}{r} + \frac{\psi_{u}(t)\varepsilon_{0} \psi_{v}(t) \omega(r, \varphi, t, \varepsilon)}{r} = 0 \\ \widetilde{\varphi}'(t) - \frac{v_{0}}{r} - \frac{\psi_{v}(t)}{r} \varepsilon_{0} \omega(r, \varphi, t, \varepsilon) = 0 \\ \widetilde{r}'(t) - \psi_{u}(t)\varepsilon_{0} \omega(r, \varphi, t, \varepsilon) = 0 \end{cases}$$
(16)

**Proposition 5.** Let  $\psi_v(t) \neq 0$ ,  $\psi_u(t) \neq 0$ ,  $\Phi_1 = (\tilde{r}'(t)\psi_u(t) - B\gamma_s\sigma_0^{\gamma_s-2}\psi_\sigma(t))\varepsilon_0\omega(r,\varphi,t,\varepsilon)$ . Then there is a disturbance of the surface density of the form (4) for the case  $1 < \gamma_s < 2$  and thus the following equations: (12), (13),

$$\begin{cases} \psi_{\sigma}'(t) + \sigma_{0} \frac{\psi_{u}(t)}{r} + \psi_{u}(t) \frac{\partial \sigma_{0}}{\partial r} + \frac{\psi_{v}(t)}{r} \frac{\partial \sigma_{0}}{\partial \varphi} + \frac{1}{r} \varepsilon_{0} \psi_{\sigma}(t) \psi_{u}(t) \omega(r, \varphi, t, \varepsilon) = 0 \\ \psi_{u}'(t) - \frac{2v_{0} \psi_{v}(t)}{r} - \frac{3}{2r} BD(\gamma_{s} - 1) \sigma_{0}^{\gamma_{s} - 2} \psi_{\sigma}(t) + \frac{\varepsilon_{0} \psi^{2}_{v}(t) \omega(r, \varphi, t, \varepsilon)}{r} = 0 \\ \psi_{v}'(t) + \psi_{u}(t) \frac{\partial v_{0}}{\partial r} + \frac{\psi_{u}(t) v_{0}}{r} + \frac{\psi_{v}(t)}{r} \frac{\partial v_{0}}{\partial \varphi} + \frac{\psi_{u}(t) \varepsilon_{0} \psi_{v}(t) \omega(r, \varphi, t, \varepsilon)}{r} = 0 \\ \psi_{v}'(t) - \frac{v_{0}}{r} - \frac{\psi_{v}(t)}{r} \varepsilon_{0} \omega(r, \varphi, t, \varepsilon) = 0 \\ \widetilde{r}'(t) - \varepsilon_{0} \psi_{u}(t) \omega(r, \varphi, t, \varepsilon) = 0 \\ B\gamma_{s}(\gamma_{s} - 2) \sigma_{0}^{\gamma_{s} - 3} \psi_{\sigma}^{2}(t) + \psi_{u}^{2}(t) = 0 \end{cases}$$
(17)

In the above statements obtained the general system of equations whose solutions give the trajectory of single waves. Note, that condition (12) means that the perturbation of the surface density must be negative.

# 3 THE BEHAVIOR OF WAVES IN THE REGIONS OF SURFACE DENSITY VARIATION

It is interesting to investigate the behavior of the wave as it passes through the region of increased or decreased surface density. For this study, we introduce a function  $\Delta(r, \varphi, t)$  which characterizes the density perturbation. Let  $\sigma_0(r, \varphi, t) = \tilde{\sigma}_0(r) + \Delta(r, \varphi, t)$ ,

 $\frac{\partial}{\partial r}\Delta(r,\varphi,t) = \vartheta(r,\varphi,t). \text{ Then } \frac{\partial\sigma_0}{\partial r} = \frac{\partial\tilde{\sigma}_0}{\partial r} + \vartheta(r,\varphi,t). \text{ By analyzing the corresponding}$ 

systems (15) - (17) the conclusion can be easily obtained that the solitary wave is deflected upward surface density. In the collision of two solitary waves the effect of repulsion can be expected.

In the following figures, we see the trajectory of the maximum single disturbance of the surface density for the case where the surface density is given:



 $a=-9, \mu = 0.01, \psi_{\mu}^{0} = 4, \psi_{\nu}^{0} = 150$ 

*ing.* 2 The trajectory of the wave, a=-9,  $\mu = 0.01$ ,  $\psi_u^0 = 44$ ,  $\psi_v^0 = 150$ 



Fig. 3 The trajectory of the wave, a=-1,  $\mu = 0.01$ ,  $\psi_{\mu}^{0} = 44$ ,  $\psi_{\nu}^{0} = 150$ 

Fig. 4 The trajectory of the wave, a=4,  $\mu = 0.01$ ,  $\psi_u^0 = 4$ ,  $\psi_v^0 = 70$ 

### CONCLUSIONS

Thus in this paper we consider the trajectory of solitary waves in a gas disk of the galaxy. Note, that the gas disk is a rotating system. The equations of gas dynamics of galaxies and shallow water equations are equivalent. This fact was considered by A.M. Friedman in [1]. We can assume that similar waves exist in shallow water, which is rotated.

### REFERENCES

- [1] Fridman A.M. Prediction and discovery of new structures in spiral galaxies *Physics-Uspekhi*, N.2, pp. 121-148, 2007 .
- [2] Morozova G., Hoperskov A.V. Physics of the galaxies disk www.astronet.ru .
- [3] Ekimov R.V. Construction of a weak asymptotic solution of the type of infinitely narrow-solitons for equations of shallow water *Proceedings of the School-Seminar*, Sudak, 2001.

[4] Turbal Y.V. On the partial solutions of equations of galaxies gas dynamics with oscillations like

 $\delta$  -solitone. Bulletin of the University of Kiev. Ser.: Physics and mathematics, N.1., pp. 67-72, 2009.