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AT THE VERY INSTANT WHEN THE AUTHOR CAME ACROSS AN INEXPERIENCED BEHAVIOR

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ABSTRACT

The appearance of the data the author accidentally came upon during hand-made analog computer (by three years his senior Mr. M. Abe using vacuum-tubes as his research project) experiments on the 27th of November, 1961 was like a broken egg with jagged edges. The original sheet of data was now kept at Brookheaven National Laboratory in New York (BNL Photography Division Negative No. 1-380-90). The data was eventually recognized as a chaotic attractor first obtained in an actual physical system. In this presentation the author would like to reproduce the unforgettable situation before the study of chaos began.

INTRODUCTION

In this presentation, periodically forced oscillatory phenomena are leading as a whole. The subject matters of reflections were nothing but the author's subjective accounts. Accordingly, he presumed to write proper nouns, and each subject was restricted within the possible inspections by references and/or survived materials.

2. SYNCHRONIZATION PHENOMENA

When a periodic force is applied, or a periodic signal is injected to self-oscillatory systems, the behavior of the systems is synchronized with the external signal depending on a frequency and an amplitude of the external force. Such effects are well known as synchronization phenomena. And a region of (control) parameters (frequency and amplitude of external signal) is called synchronization regime. Self-oscillatory systems generate respective fixed oscillations whose (angular) frequencies and amplitudes are maintained constants which depend on system structures and parameter values of consstituent elements.

When control parameters are given outside of synchronization regimes, asynchronous beat oscillations appear. It is well-known that the mechanism of synchronization is classified into two kinds, that is, frequency entrainment (pull-in) and (amplitude) quenching. Consequently, for intermediate values of external signals between the above mentioned two mechanisms, there may appear overlapped regime of both mechanisms in general, that is, coexisting attractors may be observed. The boundaries of different regimes are called bifurcation sets on the parameter plane.

Asynchronous beat oscillations observed in the periodically forced van der Pol's oscillator were represented by invariant simple closed curves of the mapping defined by using solutions of the equation. While among beat oscillations in general periodically forced self-oscillatory systems chaotic oscillations were subsisted. It was the author who first disclosed a chaotic oscillation in a periodically forced negative resistance oscillator. Since he met the data (like a broken egg), it rubbed him with the question "What are the possible steady states of a nonlinear system?" It seemed to give him intuitions that were shape of the attractor and movement of stroboscopic images on the attractor. In this section, Broken Egg (chaotic) Attractor and Local Bifurcation Sets are briefly explained. In both following Figures 1 and 2, items (a) were obtained in 1961, while items (b) were in 2006, truly 45 years was elapsed between these materials were obtained. The differential equation under study was

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the second order non-autonomous system with periodic external forcing term, that is,

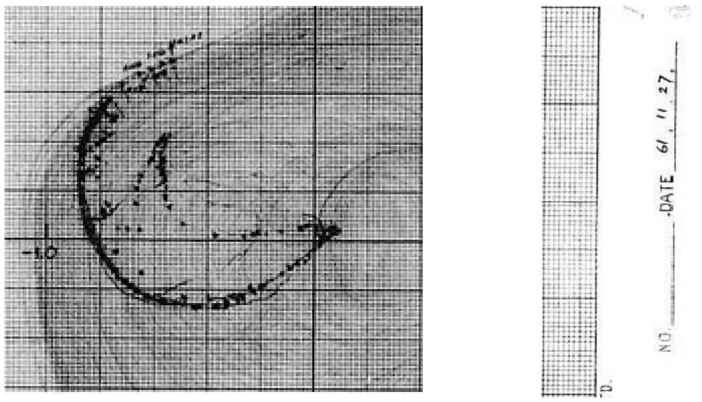
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} - \mu(1-\gamma x^2)\frac{\mathrm{d}x}{\mathrm{d}t} + x^3 = B\cos\nu t \tag{1}$$

It is to be remarked that the difference between periodically forced van der Pol's equation and the Eq. (1) is in the restoring term. This implies that the existence of nonlinear restoring term may be necessary condition for the existence of chaotic attractors in the second order non-autonomous periodic systems. Moreover, it is to be added that this statement is restricted to the comparatively small value of damping parameter μ (a conjecture).

2.1. Broken Egg Attractor

In this section, Broken Egg Attractor is briefly explained. Figure 1 embraces many implications. The points S and D in Fig. 1 (*b*) are fixed points of the mapping or the transformation T of the (x, y)–plane into itself, defined by using the solutions of Eq. (1), not to mention that the variable y is the derivative of x with respect to t. The former S represents fundamental harmonic (entrained) oscillation with period $2\pi/\nu$, and is called completely le fixedstab point or simply sink. The latter D is

(*a*)



(*b*)

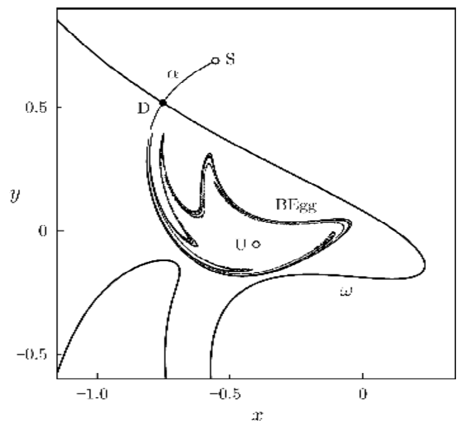


Fig. 1. (*a*) Partial output of an analog simulation of the equation (1) with μ = 0.2, γ = 8, and B = 0.35 obtained on 27 November 1961 is shown. A continuous trajectory is drawn lightly on the (x, y)-plane and points in the stroboscopic observation at phase zero are given by heavy dots; five dots near the top are fixed points for a sequence of values at ν = 1.01, 1.012, 1.014, 1.016 and 1.018, the remaining points are on the chaotic attractor at ν = 1.02 (a few points represent transient state after node-saddle bifurcation). (*b*) Phase portrait obtained by digital simulation. Parameters are ν = 0.99, and B = 0.35 (Note: In the early 1960, we had been using alphabet "v" in-stead of "x", as a variable of the equation, however, in this report these are united to "x").

called directly unstable fixed point or saddle, and corresponding periodic oscillation is not easily observed due to its instability. Another attractor was called Broken Egg Attractor "BEgg" according to its appearance. The point U inside the BEgg is called completely unstable fixed point or source. It represents the capability of self-excited component of the system.

It is known that saddle type fixed (or periodic) point has α-branch and ω-branch, which are determined uniquely. In the figure, ω-branch forms basin boundary between two attractors S and BEgg. The forcing parameters for Fig. 1(b) are given by ν = 0.99, and B = 0.35, just inside of the boundary of frequency entrainment (see Fig. 2(b)). When ν is increased outside the boundary of the entrainment, points S and D approach each other and coalescent on the boundary, then disappear (node-saddle or fold bifurcation). The entire (x, y)-plane becomes basin of BEgg chaotic attractor.

Figure 1(a) was obtained by analog computer experiments. This shows small numerical discrepancy between analog and digital experiments, which is inevitable. That is to say, this fact implies profound and important meaning concerning the concept of structural stability.

2.2. Local Bifurcation Sets

Figure 2 shows parameter regimes of external force on the (ν, B)-plane. The reasons why we paid attention to this limited part were as follows:

(1) In order to check appropriateness of the data obtained by applying averaging principle (only 2D autonomous systems could be attacked in those days), the aim of the author's master thesis was to make analog simulation of the van der Pol's equation with sinusoidal external force (non-autonomous periodic system) to confirm the validity of the data obtained from approximated averaged autonomous system. The plenty of data was almost calculated by Professor H. Shibayama.

(2) Through this simulation study, the author acquired many valuable fundamental phenomena and concepts of non-linear dynamics. Among them, in synchronization phenomena there are differences

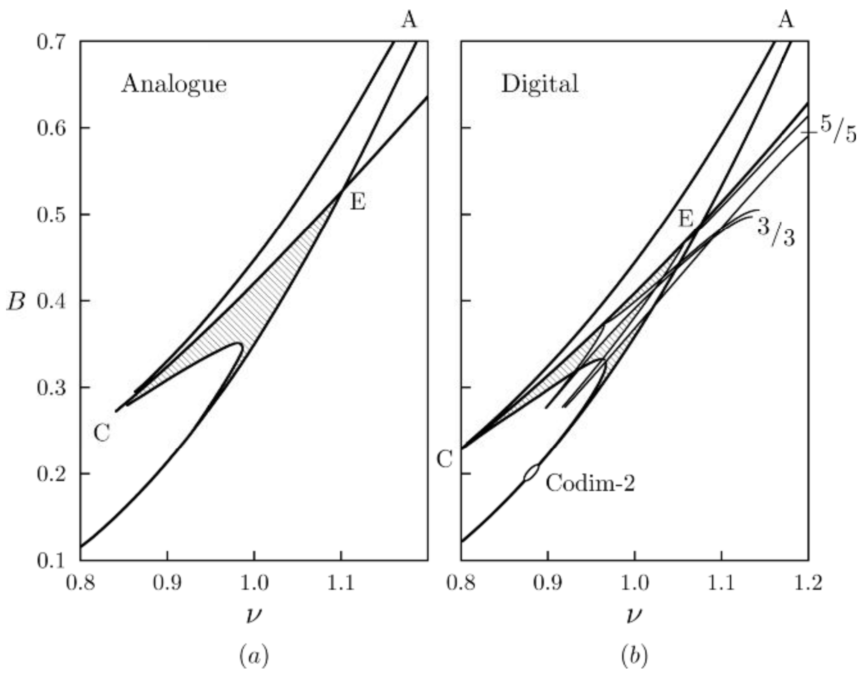


Fig. 2. (a) Partial bifurcation sets obtained by an analog simulation of the equation (1) with system parameters μ = 0.2, and γ = 8. In the triangular region ACE, two point attractors coexist, and the shaded regime represents region of attractor representing beat oscillations (in addition to entrained oscillations), intrudes into the region of harmonic entrainment. (b) The same by a digital simulation.

between frequency entrainment (pull-in) and quenching mechanisms. These were well-known typical mechanisms of synchronization.

(3) As the van der Pol's equation has linear restoring term, asynchronous beat oscillations were all quasi- or almost-periodic oscillations. That is, no chaotic oscillations were observed.

(4) The author's master thesis was summarized in Fig. 12.7, p. 298 of Professor Hayashi's Book, "Nonlinear Oscillations in Physical Systems" McGraw Hill (1964).

(5) Theoretically, there are two synchronous oscillations in rather restricted regime on the (ν, B)-plane. However, the regime was extremely small, therefore we couldn't make simulation experiments. In other words, overlapping region of entrainment and quenching is too narrow to practice simulation experiments by analog computer.

(6) Difference of phase portraits between "Entrainment" and "Quenching" occupied the author's interest in those years, hence a negative resistance oscillator was sought out, which had a wide problematic region i.e., entrainment and quenching regions overlap widely. This made analog experiment possible (see triangular regions ACE in Fig. 2(a) and 2(b)).

From the above reasons the readers can see why periodically forced negative resistance oscillator represented by Eq. (1) was taken up for study.

Small quantitative differences are observed between bifurcation sets of Figs. 2(a) and 2(b). This fact is due to the principle between analog and digital simulations. Moreover, these differences cannot be avoided anyway. It should be noted that regions indicated by 3/3 and 5/5 harmonic oscillations represent outer boundary of corresponding oscillations (i.e., hysteresis phenomena were neglected) and parallel to so-called windows. In other words, depending on the practical point of view, digital simulation results reveal impractical subtle aspects of the phenomena included in the corresponding equation (of course, this description is just the author's personal view. Appropriate concept of structural stability, or robustness should be established urgently, in order to avoid misunderstanding between virtual and real phenomena prevailing even among researchers of nonlinear dynamics).

3. Atmosphere at the Very Instant

When the author gives a presentation at the Conference, he will show Video Animation corresponding to Fig. 1. Unfortunately it cannot be shown here. Instead, Analog Computer Block Diagram is given in Fig. 3. Time scale was set at $a = 2$, this implies computer time $2\pi/\nu$ corresponds to $4\pi/\nu$ [sec]. System and control parameters used were given in the caption of Fig. 1. Following Fig. 4 showed some members of C. Hayashi's Laboratory in front of the analog computer.

4. Descendent Unsettled Problems from Experimental Studies

In this section, previous to mention unsettled problems, results concerning analog and digital simulation experiments are summarized, yet the descriptions are just author's examinations of the experimental results.

4.1. Summaries of Experimental Studies

(1) When the control parameter is given inside the chaotic regimes, closure of α-branch of some D or I type (saddle) periodic point represents chaotic attractor. Because in this case, α-branch and ω-branch of every saddle point (fixed or periodic) cross each other and homoclinic structure is formed. In other words, hence all α-branches in the chaotic attractor seemed to be connected through heteroclinic connections, or prolongation of an α-branch rises from every periodic (saddle) point inevitably crosses ω-branch from the same saddle point or group.

(2) Let us pay attention to the totality of α- and ω-branches, almost all homoclinic and heteroclinic

points are transversal, however, non existence of special type doubly asymptotic points seems to be negative.

(3) Let us pay attention to some arc on an ω-branch, the edges of which are nearest homoclinic points along certain α-branch, there exist multiple Cantor set like homoclinic points on the arc. However, on the reverse arc exchanged α with ω, homoclinic points seem to spread densely over on the arc on an α-branch like rational numbers on the real axis, otherwise transitive property of chaotic attractor becomes doubtful. It goes without saying that the transitive property of chaotic attractors observed in the second order non-autonomous periodic systems is not yet agreed (proven).

(4) Bundle of solutions is formed by solution curves in (t, x, y)-space in which each solution curve is started from every point on the attractor. As every solution curve is unstable, an actual representative point cannot continue to move along sole solution curve, but wanders randomly among the solution curves in the bundle resulting from small uncertain factors in actual physical systems. Real small uncertain factors are regarded as noises or disturbances acting on a representative point and fluctuation of system elements, however, it cannot be distinguished between the one from the other in a real physical system. It is to be noted that in digital simulations there is no system fluctuation as far as introduced intentionally.

(5) Summarizing the above things, the author advocated corresponding oscillations as “randomly transitional oscillations or phenomena”.

As stated at the beginning of this section, all above statements have to be examined mathematically, i.e., every statement is just a conjecture by the author.

4.2. Unsettled Problems or Conjectures

Decomposition of chaotic attractors and establishment of reasonable concept of structural stability are primarily emerged. These are well known long-standing difficult problems.

(1) Every chaotic attractor observed in the second order non-autonomous periodic systems seems to be decomposed into following subsets: Fixed points, if any. Infinitely many n-periodic points or n-periodic groups, where n is positive integer. These are the sim plest minimal sets corresponding to

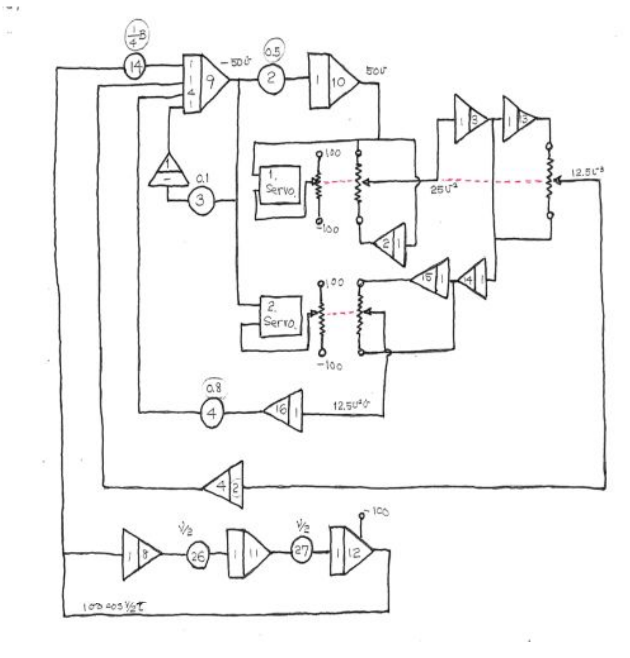


Fig. 3. Analog Computer Block Diagram for Eq. (1). Multipliers were made by using servo mechanism. This displayed that the Analog Computer was slow speed type.

the periodic solutions of the differential equation and are seemed to be distributed densely in the attractor. Infinitely many homoclinic points are also seemed to be distributed similarly.

(2) There exists no minimal set which is composed of invariant simple closed curve characterized by irrational rotation number. There seems no singular case agree with the theory of differential equations on the torus.

(3) There also exist sets of wandering points of higher orders. These are closely related to the (dis-) continuity properties of α-branches composing chaotic attractors.

(4) Summarizing the above items, every point on a chaotic attractor observed in the second order non-autonomous periodic systems is classified into a fixed point, a periodic point, a homoclonic point, or a higher order wandering point. This implies that there exist no minimal set differs from n-periodic group (includes fixed point, if any).

Based on the results of simulation studies, no proof is expected to be done for the above conjectures. Regarding to the concept of structural stability of chaotic attractors, the author's ability cannot reach up to propose the concept. He strongly hopes that appropriate concept will be established in the near future.

6. Conclusion

This report explained how was the "Origin of Experimental Chaos Research" since early 1960 and summarized the author's conjectures about chaotic attractors observed in the second order non-autonomous periodic systems.

References

Almost all contents of this report are stored up in the following Kyoto University Research Information Repository KURENAI (although not a little Japanese materials are included):

http://repository.kulib.kyoto-u.ac.jp/dspace/handle/2433/68907

http://repository.kulib.kyoto-u.ac.jp/dspace/handle/2433/24272

http://repository.kulib.kyoto-u.ac.jp/dspace/handle/2433/24257

http://repository.kulib.kyoto-u.ac.jp/dspace/handle/2433/71101

Fig. 4. In front of the Analog Computer from the left:
Kashimura (deceased), Nishikawa, Abe, Shibayama (deceased), Hashimoto and Ueda (Summer, 1960).