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CHAOS CAUSED BY HYSTERESIS IN 2-DOF ROTOR VIBRATIONS

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ABSTRACT

2-dof non-linear dynamics of the rotor suspended in a magneto-hydrodynamic field in the case of rigid magnetic materials is studied. Hysteresis is considered using the Bouc-Wen hysteretic model. It was shown that hysteresis may generate chaotic vibrations of the rotor under certain conditions. Influence of hysteretic dissipation on chaos occurring is investigated using an approach based on the analysis of wandering trajectories. The regions of chaotic behavior of the rotor are obtained in various control parameter planes: amplitude of external excitation versus dynamic oil film action characteristics, amplitude of external excitation versus magnetic control parameters, versus hysteretic dissipation as well as versus frequency of external excitation. The amplitude level contours of the horizontal and vertical vibrations of the rotor are obtained.

INTRODUCTION

In [1] 2-dof nonlinear dynamics of the rotor supported by the magneto-hydrodynamic bearing was studied. In the case of soft magnetic materials the analytical solutions were obtained by means of the method of multiple scales. In the non-resonant case the system exhibits linear properties. The perturbation solutions are in good agreement with the numerical solutions. The cases of primary resonances with and without internal resonance were investigated. The frequency-response curves were obtained. The saturation phenomenon was demonstrated. When the amplitude of the external excitation increases, after some critical value the energy pumping between various submotions of the rotor occurs. A comparison of the analytical and numerical solutions based on the approximate harmonic analysis was made. The amplitude level contours of the horizontal and vertical vibrations of the rotor were obtained. When hysteretic dissipation is increased the amplitude level of the rotor vibrations is decreased and resonance peaks correspond to the regions with lower frequencies of external excitation.

The next step is studying of conditions for chaotic vibrations of the rotor in various control parameter planes. As it was demonstrated in [2], systems with hysteresis may reveal an unexpected behaviour. On the one hand, hysteresis as any dissipation promotes to stabilization of motion and may restrain the occurrence of chaos. On the other hand, it may be a cause of chaotic vibrations in the system. 1-dof Bouc-Wen oscillator is linear in absence of hysteresis (when δ=1). Adding hysteretic dissipation leads to chaotic responses occurring in this system. Fig. 1 represents the region where chaotic behaviour of the 1-dof Bouc-Wen oscillator is possible in the amplitude of external harmonic excitation versus hysteretic dissipation plane – (F, δ) plane. Chaotic responses are not observed until $\delta=\delta_{cr}$, when the influence of the nonlinear terms becomes critical. This demonstrates the *generating effect* of the hysteretic dissipation on chaos occurring in the hysteretic system. Some information about chaotic responses of the 1-dof Bouc-Wen hysteretic oscillator is contained also in [3].

The present work has confirmed the ability of hysteresis to generate chaotic vibrations of the rotor.

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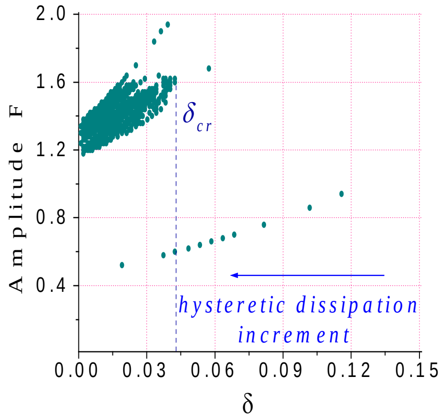


Fig. 1 The influence of hysteretic dissipation on chaos occurring in the case of 1-dof Bouc-Wen hysteretic model

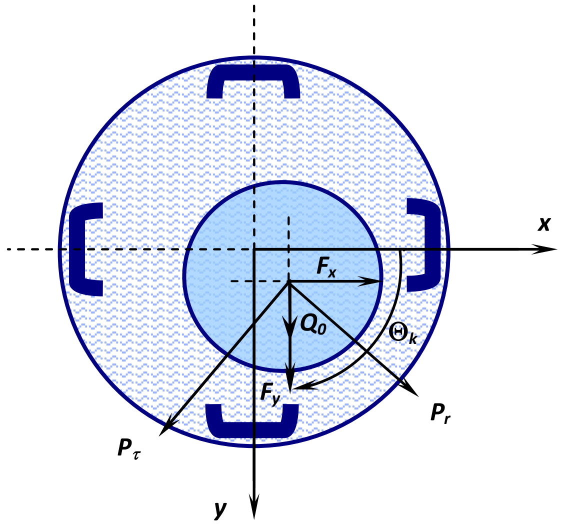


Fig. 2 The cross-section diagram of a rotor symmetrically supported on the magneto-hydrodynamic bearing

1. MATHEMATICAL MODEL

Consider a uniform symmetric rigid rotor (Fig. 2) which is supported by a magneto-hydrodynamic bearing (MHDB) system. The four-pole legs are symmetrically placed in the stator. F_k is the electromagnetic force produced by the kth opposed pair of electromagnet coils. This force, controlled by electric currents $i_k = i_0 \pm \Delta i_k$, can be expressed in the form

$$F_k = -\frac{2\mu_0 A N^2 i_0}{\left(2\delta + l/\mu^*\right)^2}\Delta i_k,$$

where i_0 denotes bias current in the actuators electric circuits, μ_0 is the magnetic permeability of vacuum, A is the core cross-section area, N is the number of windings of the electromagnet, δ is the air gap in the central position of the rotor with reference to the bearing sleeve, l is the total length of the magnetic path, the constant value $\mu^* = B_s/(\mu_0 H_s)$ denotes the magnetic permeability of the core material; the values of the magnetic induction B_s and magnetizing force H_s define the magnetic saturation level. θ_k is the angle between axis x and the kth magnetic actuator. Q_0 is the vertical rotor load identified with its weight, (P_r ,P_τ) are the radial and tangential components of the dynamic oil-film action. Equations of motion of the rotor are represented in the dimensionless form [1, 4]

$$\ddot{x} = P_r(\rho,\dot{\rho},\dot{\phi})\cos\phi - P_\tau(\rho,\dot{\phi})\sin\phi + F_x,$$
$$\ddot{y} = P_r(\rho,\dot{\rho},\dot{\phi})\sin\phi + P_\tau(\rho,\dot{\phi})\cos\phi + F_y + Q_0 + Q\sin\Omega t,$$
$$P_r(\rho,\dot{\rho},\dot{\phi}) = -2C\left\{\frac{\rho^2(1-2\dot{\phi})}{p(\rho)q(\rho)} + \frac{\rho\dot{\rho}}{p(\rho)} + \frac{2\dot{\rho}}{\sqrt{p(\rho)}}arctg\sqrt{\frac{1+\rho}{1-\rho}}\right\},\ P_\tau(\rho,\dot{\phi}) = \pi C\frac{\rho(1-2\dot{\phi})}{q(\rho)\sqrt{p(\rho)}}.$$

Here

$$x = \rho\cos\phi,\ y = \rho\sin\phi,\ \dot{\phi} = \frac{\dot{y}x - \dot{x}y}{\rho^2},\ \dot{\rho} = \frac{x\dot{x} + y\dot{y}}{\rho},\ \rho = \sqrt{x^2+y^2},\ \cos\phi = \frac{x}{\sqrt{x^2+y^2}},\ \sin\phi = \frac{y}{\sqrt{x^2+y^2}};$$

the magnetic control forces are expressed as follows $F_x = -\gamma\dot{x} - \lambda(x - x_0)$, $F_y = -\gamma\dot{y} - \lambda(y - y_0)$, where (x_0, y_0) are the coordinates of the rotor static equilibrium, γ and λ are the control parameters.

In the case of rigid magnetic materials the hysteretic properties of system described can be considered using the Bouc-Wen hysteretic model. It was shown [2] that this modeling mechanism for energy dissipation was sufficiently accurate to model loops of various shapes in accordance with a real experiment, reflecting the behavior of hysteretic systems from very different fields. The hysteretic model of the rotor–MHDB system is as follows

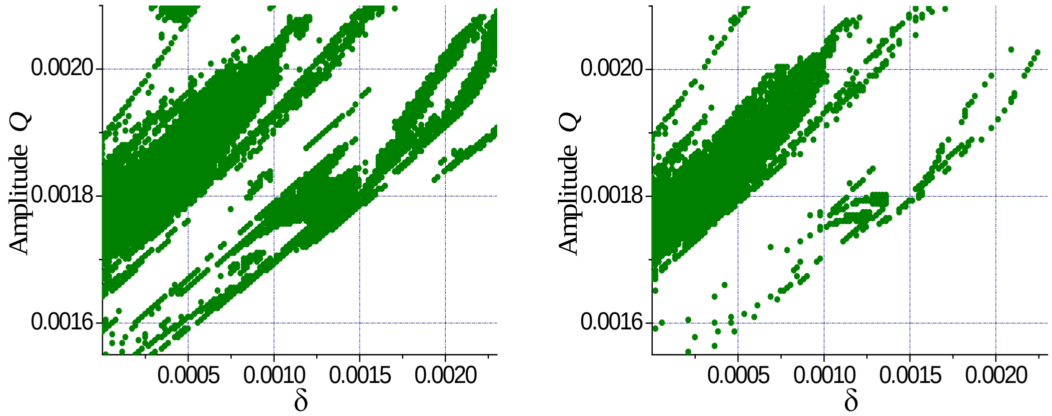
$$
\begin{aligned}
\ddot{x} &= P_r(\rho,\dot{\rho},\dot{\phi})\cos\phi - P_\tau(\rho,\dot{\phi})\sin\phi - \gamma_m\dot{x} - \lambda_m[\delta(x-x_0)+(1-\delta)z_1],\\
\ddot{y} &= P_r(\rho,\dot{\rho},\dot{\phi})\sin\phi + P_\tau(\rho,\dot{\phi})\cos\phi - \gamma_m\dot{y} - \lambda_m[\delta(y-y_0)+(1-\delta)z_2] + Q_0 + Q\sin\Omega t,\\
\dot{z}_1 &= \left[k_z - (\gamma+\beta\,sgn(\dot{x})\,sgn(z_1))\,|z_1|^n\right]\dot{x},\\
\dot{z}_2 &= \left[k_z - (\gamma+\beta\,sgn(\dot{y})\,sgn(z_2))\,|z_2|^n\right]\dot{y}.
\end{aligned}
\qquad (1)
$$

Here z_1 and z_2 are the hysteretic forces. The case $\delta=0$ corresponds to maximal hysteretic dissipation and $\delta=1$ corresponds to the absence of hysteretic forces in the system, parameters (k_z, β, n)$\in R^+$ and $\gamma\in R$ govern the shape of the hysteresis loops.

2. CONDITIONS FOR CHAOTIC VIBRATIONS OF THE ROTOR

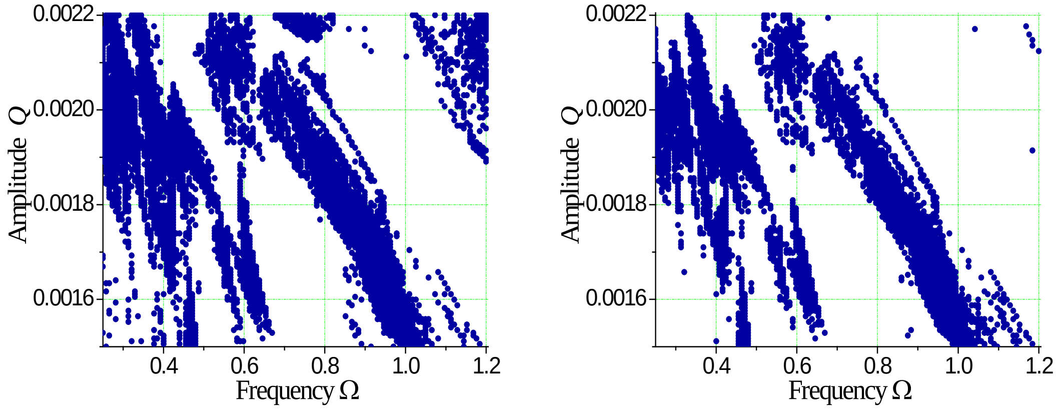
Conditions for chaotic vibrations of the rotor have been found using the approach based on the analysis of the wandering trajectories. The description of the approach, its advantages over standard procedures and a comparison with other approaches can be found, for example, in [2, 5, 6].

Figure 3 displays the regions of rotor chaotic vibrations in (δ, Q) plane. In Fig. 4 chaotic regions for the horizontal and vertical vibrations of the rotor are depicted in the (Ω, Q) parametric plane.



(a) (b)

Fig. 3 The influence of hysteretic dissipation δ on chaos occurring in horizontal (a) and vertical (b) vibrations of the rotor (1): C=0.2, γ_m=0, λ_m=500, k_z=0.000055, γ=15, β=0.25, n=1.0, Ω=0.87, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$



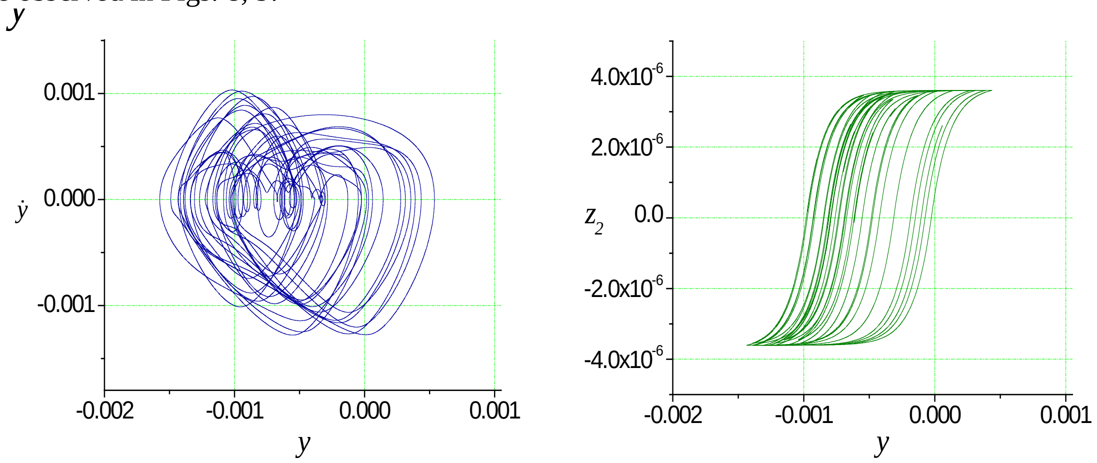
(a) (b)

Fig. 4 Chaotic regions for the horizontal (a) and vertical (b) vibrations of the rotor (1) in the (Ω, Q) parametric plane at δ=0.0001, C=0.2, γ_m=0, λ_m=500, k_z=0.000055, γ=15, β=0.25, n=1.0, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$

All domains have complex structure. There are a number of scattered points, streaks and islets here. Such structure is characteristic of domains where chaotic vibrations are possible. For each aggregate of control parameters there is some critical value of the hysteretic dissipation (1-δ_{cr}) that, if (1-δ)<(1-δ_{cr}), chaos is not observed in the system considered. Figure 5 shows the phase portrait and hysteretic loop of chaotic motion of the rotor. Parameters of motion correspond to the parameters of chaotic regions depicted in Fig. 3 and Fig. 4. The phase portrait and hysteretic loop of the periodic rotor motion that also agree well with the regions of regular/irregular behaviour of the rotor (Fig. 4) are shown in Fig. 6.

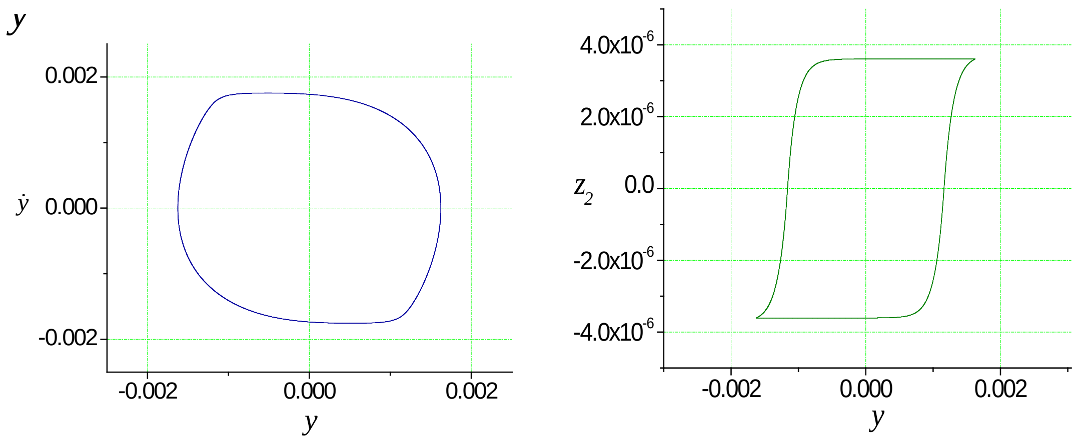
The influence of the dynamic oil-film action characteristics on chaos occurring in the rotor motion can be observed in Fig. 7. The restraining of chaotic regions with decreasing of hysteretic dissipation (1–δ) occurs.

The influence of the magnetic control parameters γ_m, λ_m on chaotic vibrations of the rotor can be observed in Figs. 8, 9.



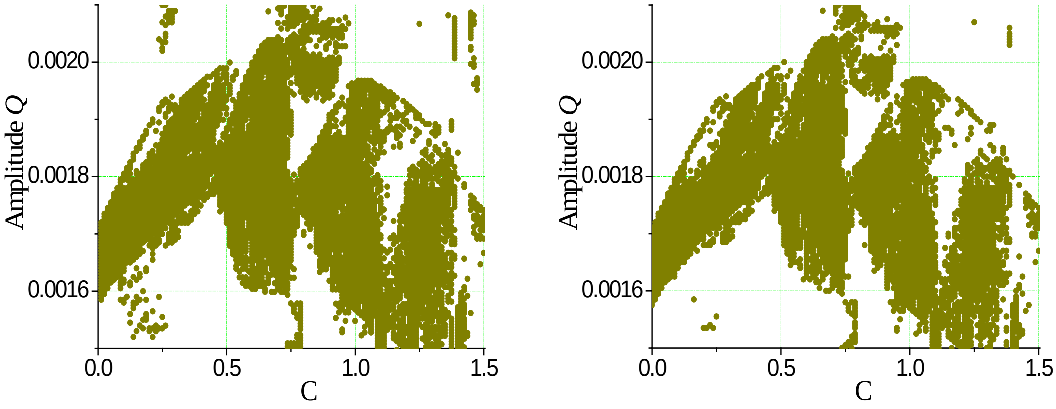
(a) (b)

Fig. 5 Phase portrait (a) and hysteresis loop (b) of the rotor motion that agree with the chaotic regions in Figs. 3, 4. The parameters Ω=0.87, Q=0.00177, δ=0.0001, C=0.2, γ_m=0, λ_m=500, k_z=0.000055, γ=15, β=0.25, n=1.0, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$ *are fixed*



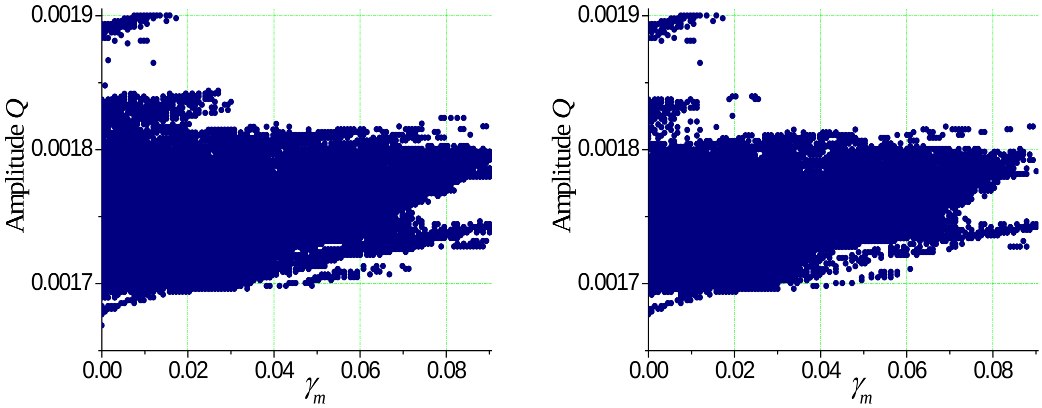
(a) (b)

Fig. 6 Phase portrait (a) and hysteresis loop (b) of the periodic rotor motion that agree with the regions of regular motion in Fig. 4. The parameters Ω=1.2, Q=0.0017, δ=0.0001, C=0.2, γ_m=0, λ_m=500, k_z=0.000055, γ=15, β=0.25, n=1.0, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$ *are fixed*



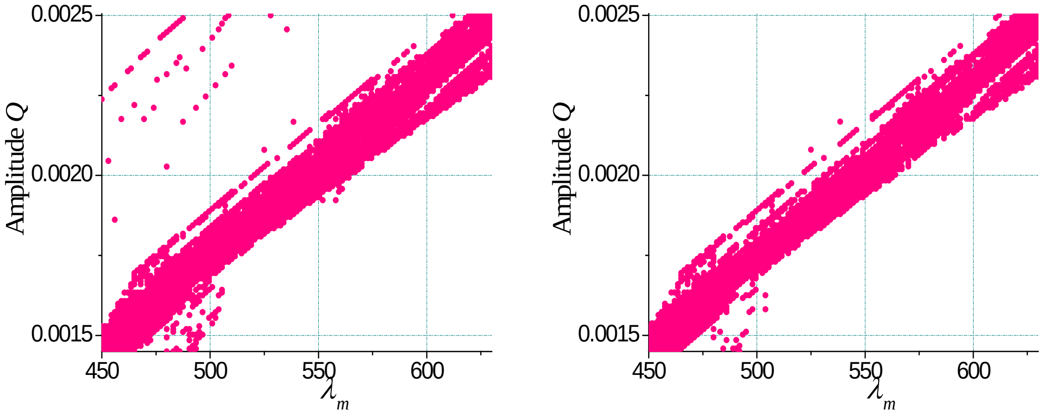
(a) (b)

Fig. 7 The influence of the dynamic oil-film action characteristics on chaos occurring in horizontal (a) and vertical (b) vibrations of the rotor (1): δ=0.000001, γ_m=0, λ_m=500, k_z=0.000055, γ=15, β=0.25, n=1.0, Ω=0.87, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$



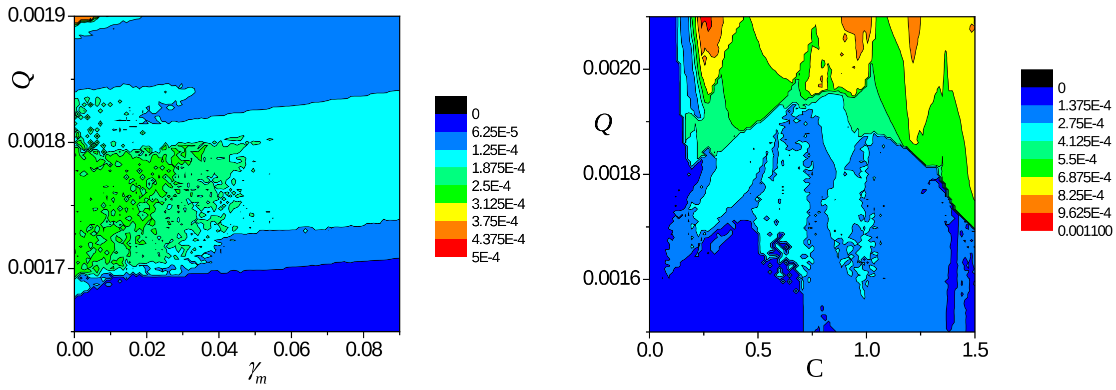
(a) (b)

Fig. 8 The influence of magnetic control parameter γ_m on chaos occurring in horizontal (a) and vertical (b) vibrations of the rotor (1): δ=0.000001, C=0.2, λ_m=500, k_z=0.000055, γ=15, β=0.25, n=1.0, Ω=0.87, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$



(a) (b)

Fig. 9 The influence of magnetic control parameter λ_m on chaos occurring in horizontal (a) and vertical (b) vibrations of the rotor (1): δ=0.000001, C=0.2, γ_m=0, k_z=0.000055, γ=15, β=0.25, n=1.0, Ω=0.87, Q_0=0, x_0=0, y_0=0, $x(0)=y(0)=10^{-8}$, $\dot{x}(0)=\dot{y}(0)=0$, $z_1(0)=z_2(0)=0$



(a) (b)

Fig. 10 The amplitude level contours of the rotor vibrations (a) in (γ_m, Q) parametric plane that correspond to Fig. 8; (b) in (C, Q) parametric plane that correspond to Fig. 7

In order to see if the rotor chaotic motion is accompanied by increasing of the amplitude of vibrations, the amplitude level contours of the horizontal and vertical vibrations of the rotor have been obtained. In Fig 10 (a) the amplitude level contours are presented in (γ_m, Q) parametric plane with the same parameters as in Fig. 8. Some "consonance" between the chaotic vibrations regions and the amplitude level contours is observed. The amplitudes of chaotic rotor vibrations are greater in comparison with the periodic vibrations. In Fig 10 (b) the amplitude level contours are presented in (C, Q) parametric plane with the same parameters as in Fig. 7. Although some "consonance" between the chaotic vibrations regions and the amplitude level contours is observed, in this case it can not be concluded that chaos leads to essential increasing of the rotor vibrations amplitude.

CONCLUSIONS

2-dof non-linear dynamics of the rotor suspended in a magneto-hydrodynamic field is studied. In the case of rigid magnetic materials, hysteresis was considered using the Bouc-Wen hysteretic model. It was shown, that hysteresis may be a cause of chaotic vibrations of the rotor. Using the approach based on the analysis of the wandering trajectories the regions of chaotic vibrations of the rotor were found in various control parameter planes: amplitude of external harmonic excitation versus hysteretic dissipation, versus frequency of external harmonic excitation, dynamic oil-film action characteristics as well as versus the magnetic control parameters. The amplitude level contours of the horizontal and vertical vibrations of the rotor were obtained. Phase portraits and hysteretic loops are in good agreement with the chaotic regions obtained.

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