CREEP-DAMAGE BEHAVIOUR OF THIN SHELLS SUBJECTED TO CYCLIC LOADING

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ABSTRACT

The paper presents the method of solution for cyclic creep problems of thin shell structures. The non-symmetrical loading and geometry are considered. The method of solution is based on the combination of asymptotic methods and averaging on the period of cyclic loading. The variational problem statement had been done and the FEM home-made code was used for numerical simulation of thin shell structures. The long-term strength in cyclic creep conditions of the flue pipe of jet engine was studied numerically and the distributions of displacements, forces and damage parameter were obtained.

INTRODUCTION

Structural thin shell elements are common in modern high-temperature technique. Operation of gas turbine engines, pipes, blocks of power machines and chambers of engines, heat exchangers, reactor equipment etc under the joint action of the quasi-static and cyclic loading is accompanied by development of irreversible creep strains and damage accumulation. Creep of materials under cyclic loading is attributed to cyclic creep, but depending on the frequency and level of loading the different types of creep and damage accumulation are observed. Thus, under cyclic loading with a frequency $f \geq 1 .. 3$ Hz the rate of creep does not depend on the frequency of cyclic processes, and the fracture occurs due to creep mechanisms. Such creep phenomenon by the classification of S.Taira and R. Ohtani [1] is called the dynamic creep. The number of cycles to failure in this case, as a rule, exceeds N $= 10^5$ cycles. In conditions of low-cycle creep, when N $< 10^5$, stress periods are a lot more (seconds or hours).

In connection with the special requirements for durability and reliability of structures, the significant results in creep-damage calculations are currently obtained [2-4]. However, the description of the stress-strain state of structures subjected to cyclic loading with the joint action of loads with different periods, remain poorly understood. The methods for estimation of an influence of mono- and polyharmonic loading with frequencies $f \geq 1 .. 3$ Hz on creep -damage processes in plates and shells were discussed in [2-4]. This paper contains the problem statement and methods for solving problems of creep and damage accumulation in thin shells under combined cyclic loading with very different periods.

1. CREEP AND DAMAGE IN THE CASE OF COMBINED CYCLIC LOADING

Let us consider the combined cyclic loading $\sigma = \sigma_0 + \sigma_1 + \sigma_2$ with simultaneous action of a constant stress σ_0, slowly changing stress σ_1 with the cyclic frequency f_1 of the cycle period T and stress σ_2 which is rapidly changing with the frequency f_2 (exceeding 1 Hz).

In general, stress σ_1 is determined by the parameters of the operating cycle (e.g. flight cycle for aircraft engine) with the slowly increasing and decreasing amplitude. Within such a cycle the stress in structural elements are usually accompanied by rapidly changing cyclical stress (e.g., caused by vibrations) which leads to the development of dynamic creep. This paper discusses the combined loading, which activates the creep-damage mechanisms are corresponding to the combined action of the dynamic and low-cycle creep.

Thus, the stress law for the combined cyclic loading can be written in the following form:

$$\sigma = \sigma^0 + \sigma^1 + \sigma^2 = \sigma^0\left(1 + \sum_{k=1}^{\infty} M_k \sin\left(\frac{2\pi k}{T}t + \beta_k\right) + A\sin\left(\frac{2\pi}{T_2}t\right)\right) \tag{1}$$

where $A = \dfrac{\sigma^a}{\sigma^0}$; $M_k = \dfrac{\sigma^{ak}}{\sigma^0}$ - are the amplitude coefficients in dynamic and cyclic creep processes correspondently, $\sigma^0 \neq 0$

Let us regard the Bailey-Norton flow rule and Kachanov-Rabotnov damage equation for single stress state:

$$\dot{c} = B\frac{(\sigma)^n}{(1-\omega)^k} \; ; \; \dot{\omega} = D\frac{(\sigma)^\gamma}{(1-\omega)^l} \; , \; \omega(0) = 0, \quad \omega(t_*) = \omega_*, \tag{2}$$

where $c(t)$, $\omega(t)$ are irreversible creep strain and damage parameter; ω_* is the damage parameter's value in the moment of the finishing of hidden damage accumulation process t_*.

To describe the processes of creep and damage accumulation for the combined loading the technique of asymptotic expansions and averaging on the period proposed in [2] was applied.

Asymptotic expansions on the small parameter $\mu = T / t$ allow to present the processes in two time scales, the 'slow' t and the 'fast' ξ, $\xi = \tau/T$, $\tau = t/\mu$, in the following form :

$$c \cong c^0(t) + \mu c^1(\xi), \quad \omega \cong \omega^0(t) + \mu \omega^1(\xi), \tag{3}$$

where $c^0(t)$, $\omega^0(t)$ are the functions which correspond to basic 'slow' creep and damage process as well as we have for 'fast' periodic processes the functions $c^1(\xi)$, $\omega^1(t, \xi)$. Considering that the creep and damage due to creep depend only on the slow time, after averaging over the period we have: $\langle c^1(\xi) \rangle \cong 0$, $\langle \omega^1(\xi) \rangle \cong 0$, and we can escape from 'fast' time' ξ in the expansions (3).

In this case of cyclic combined loading the creep-damage equations are accepted as follows:

$$\dot{c}_{ij} = Bg_n K_n \frac{3(\sigma_i^0)^{n-1}}{2(1-\omega)^k} S_{ij}^0, \dot{\omega} = Dg_r K_r \frac{(\sigma_e^{\omega 0})^r}{(1-\omega)^l} \; , \; \omega(0) = 0, \quad \omega(t_*) = 1, \tag{4}$$

where $\quad g_n = \int_0^1 \left(1 + \sum_{k=1}^{\infty} M_k \sin(2\pi k\xi + \beta_k)\right)^n d\xi$, $K_n = \int_0^1 (1 + A_n \sin(2\pi\xi))^n d\xi$, $A_n = \dfrac{A}{g_n^{1/n}}$;

$$g_r = \int_0^1 \left(1 + \sum_{k=1}^{\infty} M_k \sin(2\pi k\xi + \beta_k)\right)^r d\xi, \; K_r = \int_0^1 (1 + A_r \sin(2\pi\xi))^r d\xi, \; A_r = \frac{A}{g_r^{1/r}};$$

are the functions of the stress cycle asymmetry coefficients: $A = \dfrac{\sigma_e^a}{\sigma_e^0}, M_k = \dfrac{\sigma_e^{ak}}{\sigma_e^0}$; B, D, n, r, k, l are the constants in creep-damage laws, determined for fixed temperature T^0 by creep and long term strength curves; S_{ij}^0 are the components of stress tenzor σ_{ij}^0.

2. PROBLEM STATEMENT FOR CREEP OF CYCLICALLY LOADED THIN SHELLS

Let us formulate the problem by use of described in [2-4] approach, under which the original problem is reduced to solving two related initial-boundary problems. The first of them corresponds to the problem of forced vibrations of elastic shells under harmonic loading. The second one, which describes the creep under a static component of the load jointly with the state equations (4). These problems are connected by calculated amplitude stress cycle asymmetry coefficients.

Let us consider a shell of revolution with arbitrary genaratrix in non-axisymmetric stress-strain state in creep conditions. Due to using of FEM, let us cover the surface of the shell by the set of conical surfaces, using piecewise linear approximation of the generatrix.

For common used designations of displacements $\boldsymbol{u}$ (u, v, w), curvature variations χ, strains $\boldsymbol{\varepsilon}$ etc the geometrical relations can be written:

$$\varepsilon_{11} = \varepsilon_{11}^m + z\chi_{11};\ \varepsilon_{22} = \varepsilon_{22}^m + z\chi_{22};\ \varepsilon_{12} = \varepsilon_{12}^m + 2z\chi_{12};$$

$$\varepsilon_{11}^m = \frac{\partial u}{\partial s};\ \varepsilon_{22}^m = \frac{\partial v}{r\partial s} + \frac{u}{r}\cos\alpha + \frac{w}{r}\sin\alpha;\ \varepsilon_{12}^m = \frac{\partial u}{r\partial\varphi} + \frac{\partial v}{\partial s} - \frac{v}{r}\cos\alpha; \quad (5)$$

$$\chi_{11} = \frac{\partial^2 w}{\partial s^2},\ \chi_{22} = \frac{\partial^2 w}{r^2\partial\varphi^2} + \frac{\cos\alpha}{r}\frac{\partial w}{\partial s} - \frac{\sin\alpha}{r^2}\frac{\partial v}{\partial\varphi};\ \chi_{12} = 2\frac{\partial^2 w}{r\partial s\partial\varphi} - \frac{\partial w}{r^2\partial\varphi}\cos\alpha + \frac{v}{r^2}\sin\alpha\cos\alpha - \frac{\partial v}{\partial s}\frac{\sin\alpha}{r},$$

where α is an angle between the axis of revolution and the generatrix; r is a distance from axis of revolution to shell middle surface.

In a creep conditions the total strain at the shell point consists of elastic and irreversible parts: $\varepsilon_{ij} = e_{ij} + c_{ij}$, $i, j = 1,2$. So, let us write the physical law in the following form:

$$\sigma_{11} = \frac{E}{1-\nu^2}(\varepsilon_{11} + \nu\varepsilon_{22}) - \frac{E}{1-\nu^2}(c_{11} + \nu c_{22});\ \sigma_{22} = \frac{E}{1-\nu^2}(\varepsilon_{22} + \nu\varepsilon_{11}) - \frac{E}{1-\nu^2}(c_{22} + \nu c_{11});\ \sigma_{12} = G\varepsilon_{12} - Gc_{12},\ (6)$$

where E, G are the Young and shear modulus correspondently, ν is the Poisson ratio.

Substituting the expression (5) into (6), let us connect the membrane forces N_{ij}, bending and torsional moments M_{ij} with the geometrical unknowns:

$$N_{11} = \frac{Eh}{1-\nu^2}(\varepsilon_{11}^m + \nu\varepsilon_{22}^m) - N_{11}^c;\ N_{22} = \frac{Eh}{1-\nu^2}(\varepsilon_{22}^m + \nu\varepsilon_{11}^m) - N_{22}^c;\ S = Gh\varepsilon_{12}^m - S^c; \quad (7)$$

$$M_{11} = D(\chi_{11}^m + \nu\chi_{22}^m) - M_{11}^c;\ M_{22} = D(\chi_{22}^m + \nu\chi_{11}^m) - M_{22}^c;\ H = D(1-\nu)\chi_{12}^m - H^c.$$

Here
$$N_{11}^c = \frac{E}{1-\nu^2}\int_{-h/2}^{h/2}(c_{11} + \nu c_{22})dz;\ N_{22}^c = \frac{E}{1-\nu^2}\int_{-h/2}^{h/2}(c_{22} + \nu c_{11})dz;\ S^c = \frac{E}{2(1+\nu)}\int_{-h/2}^{h/2}c_{12}dz;$$

$$M_{11}^c = \frac{E}{1-\nu^2}\int_{-h/2}^{h/2}(c_{11} + \nu c_{22})zdz;\ M_{22}^c = \frac{E}{1-\nu^2}\int_{-h/2}^{h/2}(c_{22} + \nu c_{11})zdz;\ H^c = \frac{E}{2(1+\nu)}\int_{-h/2}^{h/2}c_{12}zdz$$

are the additional power factors, caused by irreversible creep strains of metal.

By use the Lagrange variational principle and equations (5) and (7), the variational equality is obtained:

$$\int_S (b_{ijkl}\varepsilon_{ij}^m\delta\varepsilon_{ij}^m - d_{ijkl}\chi_{kl}\delta\chi_{ij})dS - \int_S p\delta w\,dS - \int_S N_{ij}^c\delta\varepsilon_{ij}^m dS + \int_S M_{ij}^c\delta\chi_{ij}dS = 0, \quad (8)$$

here $\delta\varepsilon_{ij}^m$ and $\delta\chi_{ij}$ are the variations of the total strain components as well as curvature variation in the shell; p is the vector of loading; δw is the variation of normal displacements.

$$b_{ijkl} = \frac{Eh}{1-\nu^2}\left(\delta_{ik}\delta_{jl}\frac{1-\nu}{2} + \nu\delta_{ij}\delta_{kl}\right);\ d_{ijkl} = \frac{Eh^3}{12(1-\nu^2)}\left(\delta_{ik}\delta_{jl}\frac{1-\nu}{2} + \nu\delta_{ij}\delta_{kl}\right).$$

Let us use for this problem solution the FEM approach with 4-nodal finite element of conical shell [5]. The shape functions of third order are used. Using vector-matrix representation of relations (5-8), we finally obtain the variational equation in the following form :

$$\frac{1}{2}\left(\int_S \delta\varepsilon^T[E]\delta\varepsilon\,dS\right) - \int_S\left(\left(N_1\frac{\partial w}{\partial s}\right)_{,1} + \frac{1}{r}\left(S\frac{\partial w}{\partial\varphi}\right)_{,1} + \frac{1}{r}\left(S\frac{\partial w}{\partial s}\right)_{,2} + \frac{1}{r^2}\left(N_2\frac{\partial w}{\partial\varphi}\right)_{,2}\right)\delta w\,dS - \int_S \delta\{u\}^T\{p\}dS = 0, \quad (9)$$

where $[E]$ is the matrix of elasticity.

Equilibrium condition in a node leads to the summation of the components of internal and external forces on all elements containing this node. Hence, substituting in equation (9) the integration over the shell by a sum of integrals over finite elements, we obtain:

$$\sum_e \frac{1}{2}\int_{S^e}\left(\delta\{q\}^T[D]^T[E][D]\delta\{q\}\right)dS - \sum_e \int_{S^e}\left(\delta\{q\}^T[D]^T[P_m]\{c_m\}\right)dS - \sum_e \int_S\left(\delta\{q\}^T[B]^T\{p^p\}\right)dS - \sum_e \int_S\left(\delta\{q\}^T[B]^T\{p\}\right)dS = 0, \quad (10)$$

where $\{q\}$ is a vector of nodal displacements in the element e.

Thus, the use of FEM allows reduce the variational equality (10) to a system of linear algebraic equations

$$[K]\{\Delta\} = \{P^v\} + \{P^c\} + \{P^p\} + \{P^n\}, \quad (11)$$

where $[K]$ is a global stiffness matrix; $\{P^v\}$ is a vector of external nodal forces, $\{P^v\} = \sum_e \int_{S^e}[\Phi]^T\{p\}dS$; $\{P^c\}$ is a vector of nodal forces, caused by creep strains,

$\{P^c\} = \sum_e \int_{S^e}[B]^T[R]\{c\}dS$, $\{P^p\}$ is a vector of nodal forces caused by projection of generalized forces

on the shell's normal, $\{P^p\} = \sum_e \int_{S^e}[\Phi]^T\{p^p\}dS$; $\{P^n\}$ is a vector of nodal forces caused by non-linear

components of elastic strains, $\{P^n\} = \sum_e \int_{S^e}[B]^T[R]\{\varepsilon_n\}dS$.

To describe the processes of high temperature creep and the associated damage, which take place in shells, let us use the constitutive equations (4). As was shown, in order to use them we need to obtain the distributions of amplitude stresses. So, the problem of forced oscillations has to be solved.

In these problems it is necessary to determine the mass matrix of the system: $[M] = \sum_e \int_V [B]^T \rho [B]dS$.

Then the basic equation has to be following:

$$\left([K] - \Omega^2[M]\right)\{q_a^k\} = \{P_a^k\}, \quad (12)$$

where $\left([K] - \Omega^2[M]\right)$ is a matrix of 'dynamic stiffness' of the system; $\{q_a^k\}$ is a vector of amplitude values of nodal displacements. The components of the vector $\{P_a^k\}$ are determined by amplitude values of load's harmonic part: $p(t) = p_0 + p_a \sin(2\pi f_2 t)$.

The system (12) is solved relatively $\{q_a^k\}$ by the frontal method, and further the amplitude von Mises equivalent stresses are determined. The system of algebraic equations (12) is solved by Cholesky method.

The presented method for cyclic creep-damage simulation in thin shell structures is realized as application package for IBM-type computers.

3. ESTIMATION OF LONG-TERM STRENGTH OF AVIATION GAS TURBINE CORPS

Let us consider the results of numerical studies in the cyclic creep and damage in the flue pipe of gas turbine engine AI-20. By use of the developed software let us simulate it by the combination of cylindrical and conical shells. FE model consists of 650 elements is presented on the Fig. 1.

Fig. 1 FE model of flue pipe of gas turbine engine AI-20

The working temperatures of flue pipes are 700-900°C [6]. Therefore, for the their manufacturing the high-temperature steels EI435 and EI437B are used. The flue pipe made from EI437B steel was studied. The material constants for constitutive equations (4), which were obtained after the processing of test data, are: $B=1.31\times10^{-6}\,MPa^{-n}/h$, $n=k=4.12$, $D=2.08\times10^{-5}\,MPa^{-r}/h$, $r=l=4.5$. Simulation was performed for following values: length is 2 m, initial diameter of burning zone is 0.4 m, the nozzle angle is 37°, diameter of cylindrical part of primary zone is 0.8 m; exit diameter of secondary part is 0.7 m with nozzle angle 7°. The height of walls is 0.001 m.

Two types of loading were considered. First one is connected with high frequency oscillations caused by fuel burning. The second type of loading is connected with plane evolution and acceleration.

Distribution of pressure in the combustion chamber of modern aircraft matches the form of the cycle, which is shown in Fig.2 [7].

Fig. 2 Typical flying cycle

So, to calculate the stress-strain state of GTE and its long term strength in creep conditions let us consider the joint action of static load p_0, cyclic load component similar to shown in Fig. 2 and harmonic loading with amplitude value p_a , which is caused by wall vibration in primary-combustion zone:

$$p(t)= p_0 + \sum_{k=1}^{\infty}\frac{2}{\pi k}\sin\left(\frac{2\pi k}{T}t\right)\left(p_1\left(1-\cos(\pi k)\right)+ (p_1-p_2)\left(\cos\left(\frac{33\pi k}{35}\right)-\cos\left(\frac{3\pi k}{70}\right)\right)+\right.$$

$$(p_3-p_4)\left(\cos\left(\frac{11\pi k}{35}\right)+\cos\left(\frac{18\pi k}{35}\right)+\cos\left(\frac{13\pi k}{70}\right)-\cos\left(\frac{13\pi k}{35}\right)-\cos\left(\frac{32\pi k}{35}\right)+\cos\left(\frac{22\pi k}{35}\right)-\cos\left(\frac{3\pi k}{5}\right)- \quad (13)$$

$$\left.-\cos\left(\frac{17\pi k}{70}\right)+\cos\left(\frac{4\pi k}{7}\right)-\cos\left(\frac{19\pi k}{35}\right)+ (p_4-p_3)\left(\cos\left(\frac{23\pi k}{35}\right)-\cos\left(\frac{29\pi k}{35}\right)\right)\right)+ p_a\sin(2\pi f_2 t)\cdot$$

The influence of the vibrations for secondary zone is negligible, so for it the last summand in (13) can be omitted.

The numerical simulation of long term strength of the considered flue pipe had been performed, the determined time to fracture is equal to 660 h. The results are presented on Figs.3 – 6. Curve 1 had been built for the initial time moment as well as curve 2 corresponds to the time 660 h, when the process of hidden damage accumulation was finished. Fig.3 and 4 contain the distribution of normal and axial displacements along the flue pipe. Fig 5 and 6 contain the distribution of axial and circumferential forces.

Fig. 4 Axial displacement along the flue

Fig. 3 Normal deflection along the flue pipe   pipe

Fig. 5 Axial force distribution   Fig. 6 Circumferential force distribution

The damage parameter's distribution on outer surface of flue pipe is presented in Fig. 7. Here the Fig 7,a contains the data for first 10 hours of damage accumulation, Fig. 7,b corresponds to $t=660\ h$.

a   b

a – 10 hours; b – fracture moment, t=660 hours
Fig. 7 Damage distribution on the outer surface of flue pipe

Deformation feature of the considered flue pipe is the fact, that irreversible normal and axial displacements are very small (0.1mm) that visually in operation cannot be noticed. However, the damage accumulation proceeds just due to creep mechanisms.

Thus, the result of numerical simulation of the cyclic creep in flue pipe of gas turbine engine is the place, where fracture occurs. This one corresponds to the burning zone. Analysis of the distribution of damage parameter shows that in the shell presents another area with its very large values - the region of transition between primary and secondary zones ($\omega = 0.58 – 0.68$). When some design parameters and values that characterize the load will be changed, it is very likely macro-crack occurrence and in this place.

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