

## HYSTERESIS PHENOMENON IN FERRO/ANTIFERRO LAYERED SYSTEM

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ABSTRACT

A simple model for description of certain unusual properties of exchange bias phenomenon is proposed. In our model a half-space of AFM with fixed magnetic configuration contacts with a FM film, which consists of only two magnetic layers. While the magnetic anisotropy is taken into account and anisotropy constants  $\beta_i$  are larger than certain critical value, the hysteresis loops are observed. The obtained analytical results describe some features which are observed in experiments

### INTRODUCTION

At present, due to their technological importance for data recording, investigation of complex layered magnetic systems and, first of all, the ones including contacting layers of ferromagnetic (FM) and antiferromagnetic (AFM), draw an increasing experimental attention. In 1956, an interesting phenomenon called **exchange bias** was found in such FM/AFM systems. In contrast to the bulk FM, where the hysteresis loop of the magnetization  $\vec{M} = \vec{M}(\vec{H})$  is symmetric with respect to the point  $H = 0$ , for the exchange biased systems it is shifted along the field:  $\vec{M}(\vec{H}) \neq -\vec{M}(-\vec{H})$ . In the case of a layered AFM with a non-compensated magnetic interface ( $\vec{M}|_s \neq 0$ ), the simplest explanation is the following. The boundary layer of AFM creates an effective field which acts through the interface on the FM-subsystem and breaks the symmetry of the problem. However, the last experimental works show that the phenomenon of exchange bias may be more complicated [3, 4]. In these experiments the inclined parts of the  $\vec{M} = \vec{M}(\vec{H})$  curves are observed. Their slopes are not caused by the kinetics of the magnetization reversal (by the finite field change velocity in the experiment) and can be associated with non-homogeneous states of the magnetic subsystems. Secondly, the shelves (horizontal plateaus with non-saturated magnetization) in the  $\vec{M} = \vec{M}(\vec{H})$  curves are observed, where the magnetization does not change with the change of the field in a certain domain of  $H$  values. Finally, the hysteresis loop is not symmetric with regard to the exchange bias field. Earlier there were suggestions that these features could correspond to the bulk non-homogeneous states similar to incomplete domain walls. Supporting this idea, in the previous works [5, 6] we studied this phenomenon in the framework of two simple models: (1) the “2-spin model”, where the FM-subsystem consists of only two magnetic layers (the simplest model which admits magnetic states inhomogeneous in the direction perpendicular to the interface) and (2) the “continuous model” of a FM-film with a finite number of layers treated in the continuous approximation. For both models the strong easy plane anisotropy of a magneto-dipole origin was considered, and the anisotropy in the easy plane was neglected. Furthermore, several observed phenomena, i.e., the appearance of the shelves, inclined parts of the magnetization curve and asymmetry of these curves in the exchange bias field were qualitatively explained. However, the presence of the hysteresis was beyond the scope of these papers, as one needs the considering of the easy-axis anisotropy to obtain the hysteresis behavior.

## 1. MODEL

Consider a FM/AFM system consisting of a magnetic hard AFM subsystem, in which all magnetic moments are fixed and do not rotate in the external field, and a FM subsystem consisting of two magnetic layers with the strong easy plane anisotropy. For the case of the FM-film with a finite thickness it is determined by the magneto - dipole interaction. The magnetic state is determined by the rotation angles of the magnetization vectors in the easy plane. In addition, a weak anisotropy in this plane is taken into account. It is also assumed that the external magnetic field is directed along the “easy” axis in the plane. The system state is assumed to be homogeneous along the interface of the media. The complete magnetic energy of the system includes the exchange interactions between the FM layers and with the first uncompensated AFM layer (across the interface), the energy of magnetic anisotropy in the easy plane as well as Zeeman energy:

$$E = -J_0 \cos \varphi_1 - J \cos(\varphi_1 - \varphi_2) - \frac{\beta_1}{2} \cos^2 \varphi_1 - \frac{\beta_2}{2} \cos^2 \varphi_2 - H(\cos \varphi_1 + \cos \varphi_2), \quad (1)$$

where the indices 1,2 correspond to the layer adjacent to the interface and the other FM layer (on the free boundary of the FM) respectively. The exchange interaction across the interface with constant  $J_0$  is assumed to be ferromagnetic while the anisotropy values in the ferromagnetic subsystem and on the interface ( $\beta_2$  and  $\beta_1$ ) may general be different. The possible equilibrium states are given by the following equations:

$$(H + J_0) \sin \varphi_1 + J \sin(\varphi_1 - \varphi_2) + \beta_1 \sin \varphi_1 \cos \varphi_1 = 0 \quad (2)$$

$$H \sin \varphi_2 + J \sin(\varphi_2 - \varphi_1) + \beta_2 \sin \varphi_2 \cos \varphi_2 = 0. \quad (3)$$

We start the study of this system for the simple model with  $\beta_1 = \beta_2$ . Even in this case in the presence of anisotropy the dependencies  $\varphi_i = \varphi_i(H)$  for the “canted” phase (with  $\varphi_i \neq 0, \pi$ ) cannot be found analytically. But a general picture of the magnetic structure of the FM-layer and the corresponding field dependences for different values of the parameters  $J$ ,  $J_0$  and  $\beta$  can be easily found. Firstly, we note that the system admits collinear structures with vectors  $\vec{M}_i$  parallel to each other and parallel (or antiparallel) to the direction of the magnetic field (which coincides with the easy axis of anisotropy and the vector of antiferromagnetism of the AFM-subsystem). Besides, the states with antiparallel directions of the vectors  $\vec{M}_i$  (that remain collinear with the field direction) are also possible. As it is shown in our previous works, the hysteresis loop for this case  $\beta_1 = \beta_2$  is antisymmetric with respect to the exchange bias field  $H = -J_0/2$ . Therefore, it is sufficient to consider the transformation of the parallel ( $\uparrow\uparrow$ ) and antiparallel ( $\uparrow\downarrow$ ) phases into the canted one.

## 2. THE TRANSFORMATION OF COLLINEAR STATE INTO THE CANTED PHASE.

In order to analyze the transformation of the collinear phase ( $\uparrow\uparrow$ ) with  $\varphi_1 = \varphi_2 = 0$  into the canted one, we must find the corresponding bifurcation point with respect to the field. In this limit we linearize Eqs. (1,2) with respect to the angles  $\varphi_1, \varphi_2 \ll 1$  and put the corresponding determinant to zero to obtain the nonzero solutions of the system of linear equations. This gives the bifurcation field

$$H_{\uparrow\uparrow} = \left( \sqrt{J_0^2 + 4J^2} - (J_0 + 2J) \right) / 2 - \beta. \quad (4)$$

It is marked in Fig.1 by the point (a).

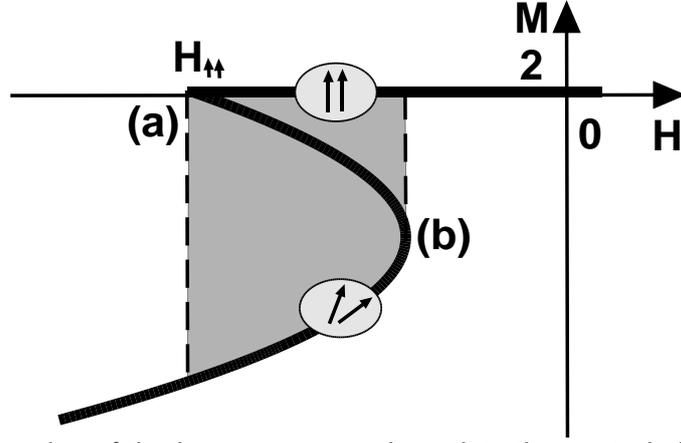


Fig.1. The transformation of the homogeneous phase into the canted phase: (a)- bifurcation point, (b) – the point with  $\frac{dM}{dH} = \infty$ . The hysteresis loop is hatched.

The stability of the collinear structure is determined by the Hessian of the potential energy surface  $E = E(\varphi_1, \varphi_2)$ , i.e.

$$K = \frac{\partial^2 E}{\partial \varphi_1^2} \frac{\partial^2 E}{\partial \varphi_2^2} - \left( \frac{\partial^2 E}{\partial \varphi_1 \partial \varphi_2} \right)^2 \quad (5)$$

A structure is stable for  $K > 0$ , which corresponds to the minimum of the potential energy. In the saddle point of the potential energy surface ( $K = 0$ ) the structure loses stability. For the collinear phase

$$K = (H + \beta)(H + J_0 + \beta) + J(2H + J_0 + 2\beta), \quad (6)$$

and, comparing this with (4,6) we obtain that it loses stability in the bifurcation point.

### 3. THE BOUNDARIES OF THE HYSTERESIS LOOP.

Relation (4) also determines one of the boundaries of the hysteresis loop (or, in general, region of the magnetization reversal) in the  $H$  axis. As it will be shown below, for small enough anisotropy there is no hysteresis and the magnetization switches via the uniform magnetization reversal process through a region of the canted phase. To determine the critical values of the parameters for which the hysteresis appears, we find the slope of the  $M(H)$  curve in the canted phase near the bifurcation point. To do this, we expand the equations (2,3) into the series with respect to the variables  $\varphi_i$  up to the cubic terms:

$$(H + J_0 + J + \beta)\varphi_1 - J\varphi_2 - \frac{1}{6}(H + J_0 + 4\beta)\varphi_1^3 - \frac{J}{6}(\varphi_1 - \varphi_2)^3 = 0, \quad (7)$$

$$(H + J + \beta)\varphi_2 - J\varphi_1 - \frac{1}{6}(H + 4\beta)\varphi_2^3 + \frac{J}{6}(\varphi_1 - \varphi_2)^3 = 0, \quad (8)$$

and look for the solutions in the form of power series with respect to the small deviations of the magnetic field from its bifurcation value  $\varepsilon = \sqrt{H - H_{\uparrow\uparrow}}$ :  $\varphi_i \approx \varphi_i^{(0)}\varepsilon + \varphi_i^{(1)}\varepsilon^3 + \dots$ . In the first order in  $\varepsilon$  we obtain the bifurcation field and the relation between the amplitudes of the angles:

$$\varphi_2 \approx \varphi_1 \left( J_0 + \sqrt{J_0^2 + 4J^2} \right) / 2J, \quad (9)$$

In the third order in  $\varepsilon$  we obtain the values of the angle displacements  $\varphi_{1,2}$ :

$$\varphi_{1,2}^2 \approx \frac{\varepsilon^2 \sqrt{J_0^2 + 4J^2} \left( \sqrt{J_0^2 + 4J^2} \mp J_0 \right)}{\beta(J_0^2 + 2J^2) - J^2 \left( \sqrt{J_0^2 + 4J^2} - 2J \right)}. \quad (10)$$

The dependence of the magnetization of the system on the magnetic field near the bifurcation point is given by the formulae

$$M(H) \approx 2 - (H - H_{\uparrow\uparrow}) \frac{(J_0^2 + 4J^2)}{\beta(J_0^2 + 2J^2) - J^2 \left( \sqrt{J_0^2 + 4J^2} - 2J \right)}. \quad (11)$$

For the given values of the parameters  $J$  and  $J_0$  the hysteresis near the homogeneous state ( $\uparrow\uparrow$ ) appears for the critical value of the anisotropy parameter:

$$\beta_c = J^2 \frac{\sqrt{J_0^2 + 4J^2} - 2J}{J_0^2 + 2J^2}. \quad (12)$$

Relation (12) shows that the hysteresis picture is different for different values of  $\beta$ . For  $\beta > \beta_* \approx 0.08J_0$  the relation (12), considered as an equation for  $J$ , has no solution. This means that the hysteresis takes place for any exchange interaction value. For  $\beta < \beta_*$  the equation (12) has two roots  $J_c$  and  $J_{c'}$  which correspond to the points ( $c$ ) and ( $c'$ ) in Fig 2.b. There is no hysteresis in the interval  $J_c < J < J_{c'}$ .

In the domain of existence of the hysteresis the bifurcation point (4) determines the lower boundary of the field dependence of the hysteresis loop. The upper boundary corresponds to the field value for which the derivative  $dM/dH$  becomes infinite (point (b) in Fig.1). Moreover, the derivative  $d\varphi_2/dH$  also becomes infinite. Using this fact in equations (2,3), it is easy to find the dependence of the corresponding field on the exchange constants and the anisotropy:

$$J_2 = -H(H + J_0) \frac{\beta + \sqrt{J_0(2H + J_0)}}{J_0(2H + J_0)}. \quad (13)$$

This dependence is depicted in the Fig. 2 as the curve  $A_2$ . The curve  $A_1$  stands for the dependence (4)  $J = J(H_{\uparrow\uparrow})$  for the bifurcation point of the appearance of the canted phase from the homogeneous state ( $\uparrow\uparrow$ );

$$J_1 = -\frac{(H + \beta)(H + J_0 + \beta)}{(2H + J_0 + 2\beta)}. \quad (14)$$

For  $\beta < \beta_*$  the two curves given by Eqs. (13) and (14) intersect (see Fig.2a). The crossing points correspond to the solutions of equation (12) for  $J$  with the fixed parameter  $\beta$ . For the values of  $J$  between the crossing points there is no hysteresis. For  $\beta > \beta_*$  the curves  $A_1$  and  $A_2$  in Fig.2b do not intersect and the hysteresis takes place for all values of the parameters.

The analysis of the stability of the homogeneous state ( $\downarrow\downarrow$ ) with  $\varphi_{1,2} = \pi$  and the study of the hysteresis of the field dependences near this state can be done in a similar way. The corresponding dependencies are presented in Fig.2 as the curves  $A_3$  and  $A_4$ . It is easy to see that the picture is symmetric with respect to the point ( $M = 0, H = -J_0/2$ ). Notice that this symmetry follows directly from equations (2,3).

Finally, let us consider the antiparallel phase ( $\uparrow\downarrow$ ) which corresponds to the ‘shelf’ (y) domain with  $M = \text{const}$ ) in the field dependence of magnetization with  $\varphi_1 = 0$  and  $\varphi_2 = \pi$ . Linearizing equations (2,3) near this state, we find the bifurcation point which corresponds to the transition from the antiparallel structure of the ferromagnetic subsystem into the canted phase. The corresponding relation between the parameters reads:

$$J_5 = -\frac{(H - \beta)(H + J_0 + \beta)}{(J_0 + 2\beta)}. \quad (15)$$

It is given by the curve  $A_5$  in Fig.2. The curves  $A_i$  in these figure determine the domains of existence of the different structures of the FM system and the hysteresis (marked out).

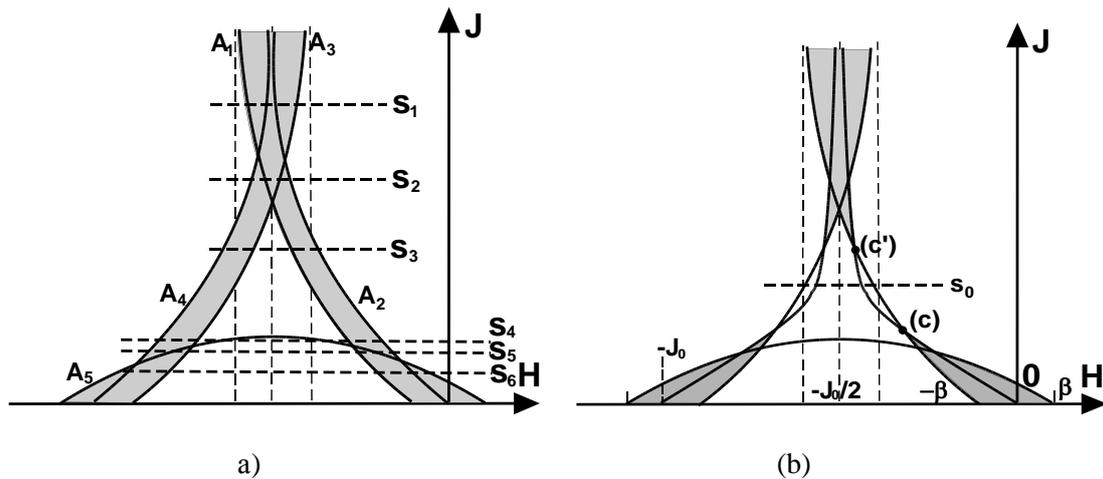


Fig.2. Domains of the hysteresis (hatched) in the plane of the parameters  $(J, H)$  for the fixed values of the anisotropy  $\beta$ :  $\beta > \beta_*$  (a) u  $\beta < \beta_*$  (b).

In Fig.2 the domain of the stability of the parallel phase ( $\uparrow\uparrow$ ) is situated to the right of the curve  $A_1$ , which starts at the point  $H = -\beta$  in the limit  $J \rightarrow 0$  and asymptotically tends to the infinity as  $H \rightarrow -J_0/2 - \beta$ . The domain of the stability of the parallel phase ( $\downarrow\downarrow$ ) is located on the left of the curve  $A_3$ , which starts at the point  $H = -J_0 + \beta$  and asymptotically tends to the infinity as  $H \rightarrow -J_0/2 + \beta$ . The domain under the curve  $A_5$  (with lies between the points  $H = -J_0 - \beta$  and  $H = \beta$ ) corresponds to the antiparallel phase ( $\uparrow\downarrow$ ). Finally, a triangular domain between the curves  $A_1, A_3, A_5$  corresponds to the canted phase. For the fixed anisotropy parameter, the shape of the hysteresis loop changes with the change of parameter  $J$ . The field dependences corresponding to some characteristic values of exchange interaction are depicted in Fig.3 as the lines  $S_i$ . The simplest form of the hysteresis is observed for the large values of exchange interaction (or for the small values of the magnetic anisotropy) for  $J > \tilde{J} \sim 0.1 J_0^2 / \beta$ . This corresponds to the line  $S_1$  in Fig.2. The hysteresis loop is shifted along the field to the value  $-J_0/2$  and has the width  $\Delta = 2\beta - \left(\sqrt{4J^2 + J_0^2} - 2J\right)$  (Fig.3,  $S_1$ ). For lower values of  $J$  (but for  $J > (J_0^2 - 4\beta^2)/8\beta$ ), there appears the domain of the canted phase and the hysteresis loop has the form given in Fig.3 ( $S_2$ ).

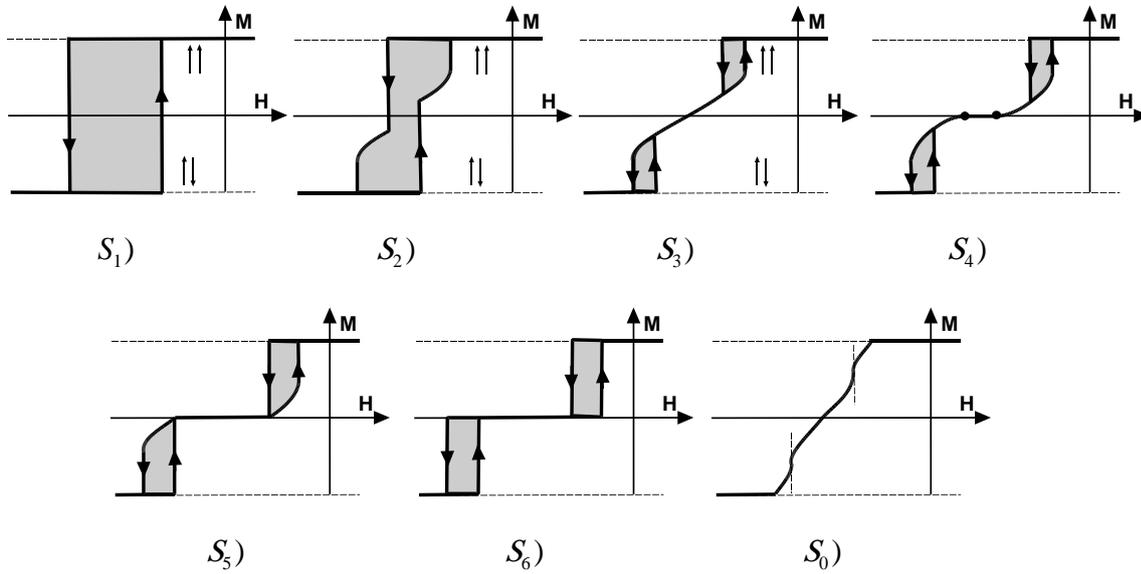


Fig.3. Different shapes of the  $M(H)$  hysteresis loop for different values of the magnetic anisotropy  $\beta$  and the FM exchange parameter  $J$ .

For the domain  $S_3$  (see Fig.2) the hysteresis loop splits into two loops (Fig. 3,  $S_3$ ). For the line  $S_4$  (with  $J < J_0 + 2\beta$ ) we observe the “shelf” of the antiparallel phase ( $\uparrow\downarrow$ ) in the  $M(H)$  dependence (Fig. 3,  $S_4$ ). Upon further decreasing of the exchange interaction, the shelf occupies all the domain of the fields between the hysteresis loops ( $S_5$ ), but the canted phase still remains in the two hysteresis loops. Finally, for the smaller values of  $J$  ( $S_6$ ), the hysteresis loops corresponds to the transitions between parallel and antiparallel phases, and the canted phase disappears. If the magnetic anisotropy is small enough (Fig. 2b), there exists a domain of parameter  $J$  for which there is no hysteresis (in contrast to the FM-systems without exchange bias).

## CONCLUSION

In the present paper, we analytically studied the exchange bias phenomenon in the framework of a simple model of ferromagnetic subsystem with two layers in contact with a hard antiferromagnet. The different shapes of  $M(H)$  hysteresis loops were founded for different values of the parameters of the system (anisotropy and exchange interaction) of the ferromagnetic layers. The results can be used to explain the experimentally observed features of the exchange bias phenomenon.

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