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IMPACT VIBRATION ABSORBER OF PENDULUM TYPE

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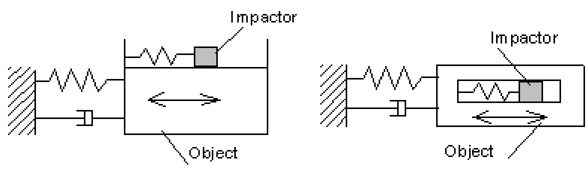
ABSTRACT

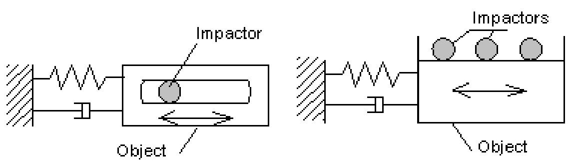
In this work the impact vibration absorber of pendulum type is examined. It consists of pendulum with motion limiting stops attached to the vibrating system. Pendulum vibration absorbers are widely used in practice. The influence of pendulum parameters on the possibility of suppression of vibrations of the basic system under harmonic excitation are discussed in this study.

INTRODUCTION

Vibration is a repetitive, periodic or oscillatory response of mechanical system. Since most of machines and structures undergo some degree of vibrations, engineers have to consider the results of vibrations in the designing process [4], [8]. It is usually required to control the vibrations because it causes fatigue and failure of the vibrating elements and discomfort for the people. One of the most effective passive control methods is adding an impact vibration absorber to the system under excitation [1],[2],[5]. Impact vibration absorbers (IVA) consist of an impact mass which is placed on basic vibrating mass so, that periodically collides with it. The transfer of momentum to the mass from the main mass and dissipation of energy in every impact provides reduction in amplitude response of the main mass. IVA are fulfilled with one, two and more degrees of freedom; noncontrollable and regulated; with unilateral or with bilateral constraints. In accordance with structural type impact vibration absorbers may be spring (Fig.1), floating (Fig.2) and pendular (Fig.3).



a) outer b) inner
Fig.1.a-b. Spring impact absorbers.



a) single –unit b) multi –unit
Fig.2.a-b. Floating impact absorbers.

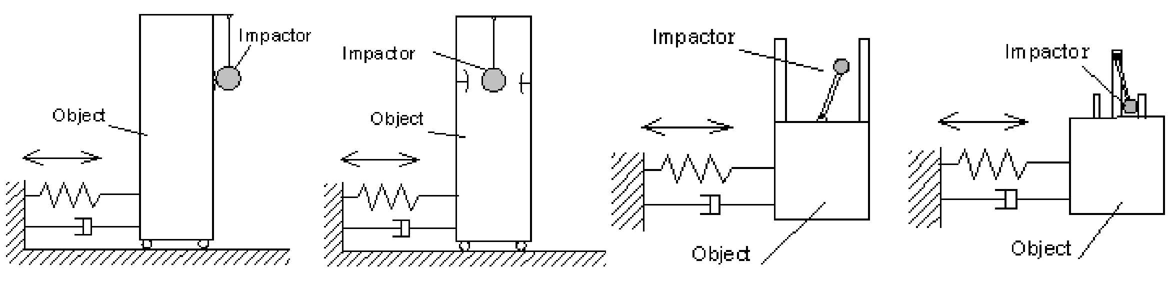


Fig.3. Pendulum impact absorber

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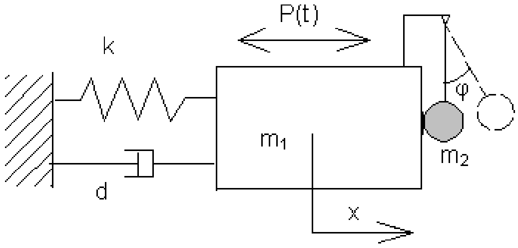
In this work the impact absorber of pendulum type is examined. It consists of pendulum with motion limiting stops attached to the vibrating system. Pendulum vibration absorbers are used in practice for decreasing of vibration level of different engineering structures: flue pipes, television towers, bridges, high-rise buildings, aerial masts, for shaft autobalancing and others [3], [6], [8]. The purpose of this research is to study the influence of parameters of the pendulum on possibility of vibration suppression of the basic system under harmonic excitation, and the effect of the system parameters on system dynamics. This involved determination the effect of mass ratio, excitation amplitude, and clearance between impact stop walls. A pendulum with one and two impacts during the period is considered. Dependence of suppression ability of absorber on pendulum length, coefficient of restitution at impact, mass ratio of the basic system and pendulum, and gap size are found.

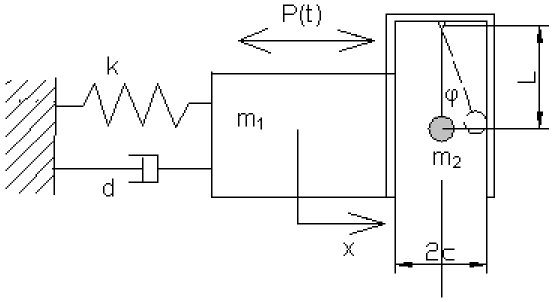
1. ANALITICAL MODEL OF ABSORBER

1.1 Mathematical model

The analytical models of single and double impact pendulum absorbers are presented in Fig.4.



a) single-impact absorber model



b) double impacts absorber model

Fig.4. Model of the pendulum impact absorber.

Parameters of system:

m_1 – mass of the main body; m_2 – mass of the damper; $\mu = m_2/m_1$ - mass ratio;
d – inherent damping coefficient of the main system; k – stiffness coefficient of main system;
λ – natural frequency of the main system; ω – the frequency of pendulum; l – length of pendulum;
c – size of gap; α – max angle for two-impacts absorber, $\tan\alpha = c/l$;
r – coefficient of restitution of the velocity after impact;
The system is considered under harmonic excitation: $P(t) = P_0 \sin \Omega t$**,** where P_0 – amplitude of excitation force; Ω – frequency of excitation.

1.2 The equations of motion of the system

The Lagrange's equations of motion of the examined system are derived:

$$\begin{cases} m_1\ddot{x} + m_2 l\ddot{\varphi}\cos\varphi = -kx - d\dot{x} + m_2 l\dot{\varphi}^2 \sin\varphi + P_0 \sin\Omega t - S\sum_{R=0}^{\infty}\delta(t - RT) \\ m_2 l^2\ddot{\varphi} + m_2\ddot{x}l\cos\varphi = -m_2 gl\sin\varphi + S\sum_{R=0}^{\infty}\delta(t - RT) \end{cases} \tag{1}$$

where S is an impact impulse, $\delta(t - RT)$ is a delta function, T is a period of collisions.
The stereomechanical theory of impact without friction is used for impact impulse definition [7]:

$$S = (1 + r)\frac{m_1 m_2}{m_1 + m_2}(v_{01} - v_{02}), \tag{2}$$

where v_{01} and v_{02} are velocity of main body and velocity of impactor just before impact.
The velocity of impactor consists of translational velocity and relative velocity:

$$v_{01} - v_{02} = v_{01} - (v_{01} + v_{2r}) = -l\dot{\varphi}_{01}, \tag{3}$$

here the angular velocity is pendulum velocity just before impact,

$$\dot{\varphi}_{01} = \dot{\varphi}(T) .$$

Taking into account (3) impact impulse may be represented as:

$$S = (1+r)\frac{m_2}{1+\mu}(-l\dot{\varphi}(T)) \ . \tag{4}$$

Taking into account (4) the equations of system (4) after rearrangement of may be written:

$$\begin{cases} \ddot{x} + b\dot{x} + \lambda^2 x = \mu\dot{\varphi}^2 l\sin\varphi - \mu\ddot{\varphi}\cdot l\cos\varphi + p_0\sin\Omega t + \dfrac{(1+r)\mu}{(1+\mu)} l\dot{\varphi}(T)\sum\limits_{R=0}^{\infty}\delta(t-RT) \\ \ddot{\varphi} + \omega^2\sin\varphi = -\dfrac{\ddot{x}}{l}\cos\varphi - \dfrac{1+r}{1+\mu}\dfrac{\dot{\varphi}(T)}{l}\sum\limits_{R=0}^{\infty}\delta(t-RT) \end{cases} \tag{5}$$

where $\lambda = \sqrt{\dfrac{k}{m_1}}$, $b = \dfrac{d}{m_1}$, $p_0 = \dfrac{P_0}{m_1}$, $\omega = \sqrt{\dfrac{g}{l}}$.

1.3 Numerical solution of equations of motion

In this work the numerical solution of system (5) was obtained with help of Euler method using the kinematics conditions – pre-impact and post-impact velocities of moving bodies if coefficient of restitution is known. The velocity of the main body v_1 and velocity of impactor v_2 just after impact are:

$$v_1 = v_{01} + l\dot{\varphi}_{01}\frac{\mu(1+r)}{1+\mu} \ , \qquad v_2 = v_{01} + l\dot{\varphi}_{01}\frac{\mu - r}{1+\mu} \ . \tag{6}$$

Algorithm of Euler's method for the single-impact damper, taking into account (8),(9):

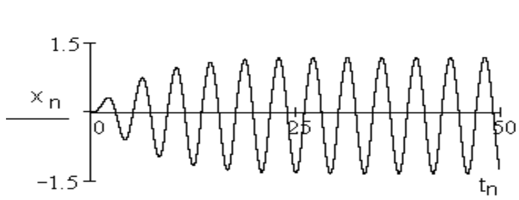
$$\begin{aligned} t_{n+1} &= t_n + \Delta t \\ x_{n+1} &= x_n + \dot{x}_n\Delta t \\ \varphi_{n+1} &= (\varphi_n + \dot{\varphi}_n\Delta t) if\,(\varphi_n \geq 0,0,1) \\ \dot{x}_{n+1} &= \dot{x}_n + \ddot{x}_n\Delta t + l\dot{\varphi}_n\frac{\mu(1+r)}{1+\mu} if\,(\varphi_n \leq 0,1,0) \\ \dot{\varphi}_{n+1} &= \dot{\varphi}_n + \ddot{\varphi}_n\Delta t + \dot{\varphi}_n\frac{\mu - r}{1+\mu} if\,(\varphi_n \leq 0,1,0) \\ \ddot{x}_{n+1} &= -b\dot{x}_n - \lambda^2 x_n + p_0\sin\Omega t_n + \mu\dot{\varphi}_n{}^2 l\sin\varphi_n - \mu\ddot{\varphi}_n l\cos\varphi_n + \frac{(1+r)\mu}{1+\mu} l\dot{\varphi}_n if\,(\varphi_n \leq 0,1,0) \\ \ddot{\varphi}_{n+1} &= -\omega^2\sin\varphi_n - \frac{\ddot{x}_n}{l}\cos\varphi_n - \frac{1+r}{1+\mu}\frac{\dot{\varphi}_n}{l} if\,(\varphi_n \leq 0,1,0) \end{aligned} \tag{7}$$

Here *if* is special logic function in Mathcad program.

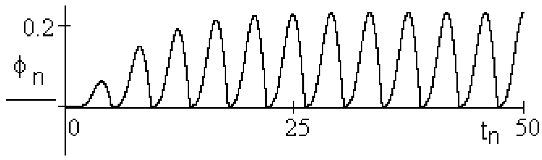
Euler method gives good results if time interval Δt is small. The equations of motion are solved numerically with help of Matcad program. The received results enable to analyze all parameters of motion of the system. Examples of the solution of motion are presented below for single and two-impact absorbers.

2. NUMERICAL EXAMPLE

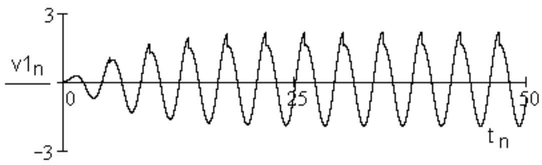
For the numeral solution next value of system parameters are accepted: $\lambda=1.5$, $b = 0.1$, $p_0 = 0.5$. Parameters values are chosen for civil engineering conditions. The structure is modeled as single-degree of freedom system, after adding the pendulum absorber it becomes two freedom degrees, the exiting force is harmonic. Parameters of motion of single-impact absorber are shown in Fig. 5 a-e, two-impact absorbers - in Fig. 6 a-e.



a) x=x(t)

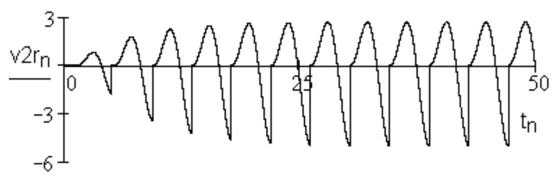


b) φ=φ(t)

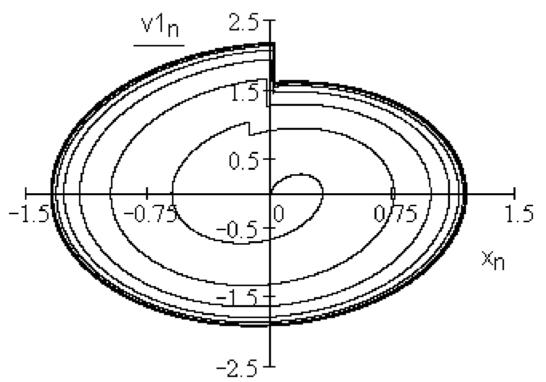


c) v1=v1(t)

c) *v1=v1(t)*

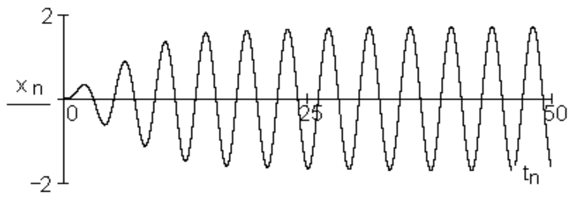


d) $v_2r=v_2r$ (t)

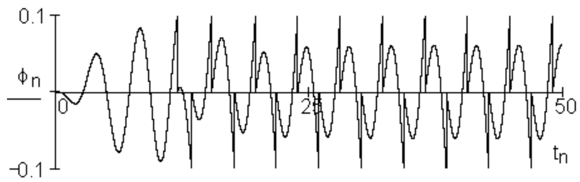


e) v1=v1(x)

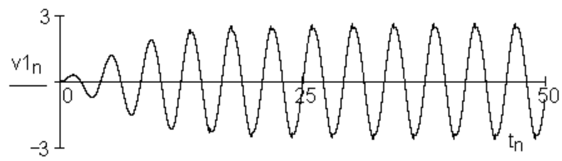
Fig. 5.Plots of dependence of the motion parameters on time and phase map for single-impact absorber in case of: Ω=1.5, λ=1.5, ω=0,75, μ=0.04, r=0.6.

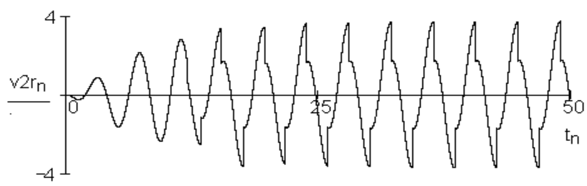


a) x=x(t)

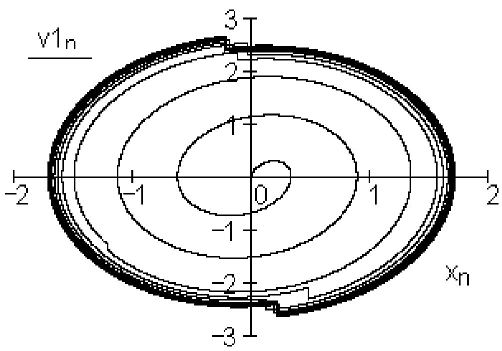


b) φ=φ(t)





d) v_2 =v_2r (t)



e) v1=v1(x)

Fig. 6. Plots of dependence of the motion parameters on time and phase map for double-impact absorber in case of: Ω=1.5, λ=1.5, ω=0.75, α=0.1, μ=0.04, r=0.6.

Plots in Fig.5-6 represent: a) *x=x(t)* - displacement of mass m_1, b) *φ=φ(t)* - rotation angle of pendulum, c) $v_2r=v_2r$ *(t)* - relative velocity of mass m_2 , d) *v=v(t)* - velocity of mass m_1, as functions of time *t*, e) *v=v(x)* - velocity of mass m_1 as function of mass displacement *x*.

Plots of maximal amplitude A_{max} of main body in relation to exiting force frequency Ω for different pendulum frequences ω are presented in Fig.7, plots of A_{max} depending on mass ratio μ are in Fig.8, depending on the coefficient of restitution are in Fig.9, depending on angle α are in Fig.10.

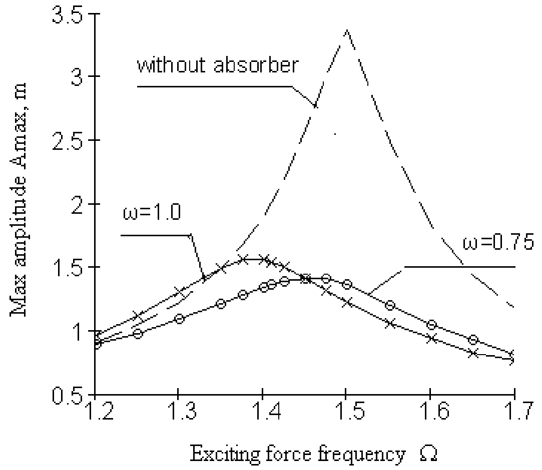


Fig.7. Maximal amplitude Amax in relation to exiting force frequencies Ω for single impact pendulum absorber (μ=0.04, r=0.6).

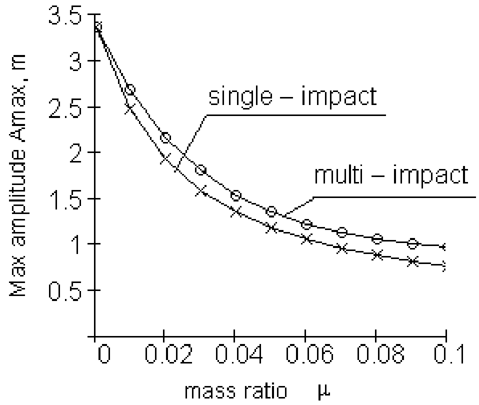


Fig.8. Maximal amplitude depending on μ –ratio for single-impact absorber *(ω=0.75, r=0.6)* and *multi-impact absorber (ω=0.75, r=0.6, α=0.05) and Ω=1.5 for both case.*

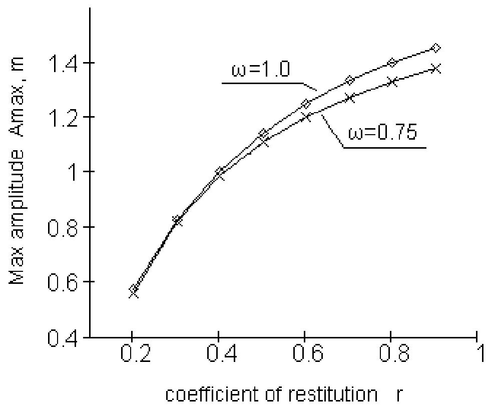


Fig.9. Maximal amplitude Amax in relation to coefficient of restitution r for single impact pendulum absorber (μ=0.04, r=0.6).

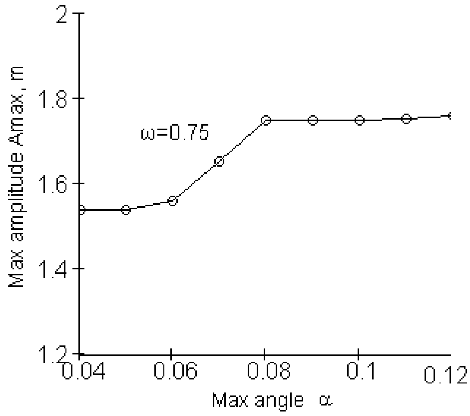


Fig.10. Maximal amplitude Amax depending on pendulum clearance angle α (μ=0.04, r=0.6) for multi-impact absorber shown in Fig.4b.

3. COMPARISON WITH CLASSICAL IMPACT ABSORBER

The analytical models of classical impact absorbers are presented in Fig.11.

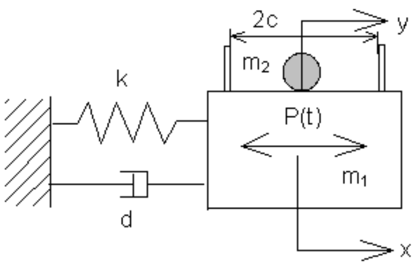


Fig.11. Model of classical impact absorber.

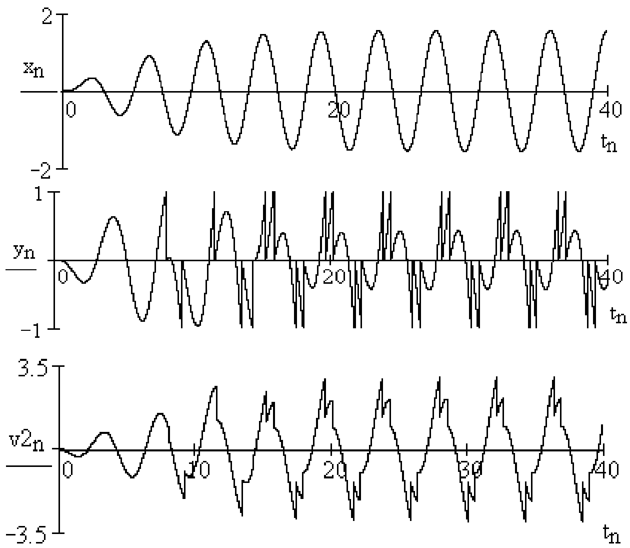
The differential equations of the system motion:

$$\begin{cases} m_1\ddot{x} = -kx - d\dot{x} + P_0 \sin\Omega t - S\sum_{R=0}^{\infty}\delta(t - RT) \\ m_2\ddot{y} = -m_2\ddot{x} + S\sum_{R=0}^{\infty}\delta(t - RT) \end{cases} \tag{8}$$

Impact impulse S and post-impact velocities of the main body v_1 and velocity of impactor v_2 are:

$$S = (1+r)\frac{m_2}{1+\mu}(-\dot{y}(T)), \quad v_1 = \dot{x}(T) + \dot{y}_0(T)\frac{\mu(1+r)}{1+\mu}, \quad v_2 = \dot{x}(T) + \dot{y}(T)\frac{\mu - r}{1+\mu}. \tag{9}$$

Further the system with the same characteristics as for pendulum absorber is considered: $\lambda=1.5$, $b = 0.1$, $p_0 = 0.5$, $\Omega=1.5$. In Fig.12 the parameters of motion in dependence on time are presented. The absorber with parameters *c=1 m c=1, μ=0.04, r=0.6* admits multi-impacts; it shows four impacts during a period.



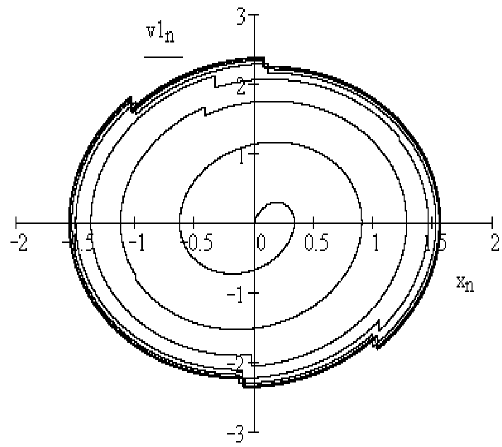


Fig.12. Plots of dependence of motion parameters on time: $x=x(t)$, $y=y(t)$, $v_2r=v_2r(t)$ and phase map for classical impact absorber in case of: $\Omega=1.5$, $\lambda=1.5$, $c=1$, $\mu=0.04$, $r=0.6$.

CONCLUSIONS

The differential equations of motion of the vibrating system are derived on the basis of Lagrange's equation of the second type. The impacts in the system are described as impacts of perfectly rigid bodies taking into account the coefficient of restitution. The equations of motion are solved numerically with help of Matcad program, using Euler's method. Numerical solution allows calculating not only the parameters of motion in the steady-state mode, but also in a transitional process. All parameters of transient motion and steady-state motion were defined, results were analyzed. Dependences of amplitude of vibrations are shown graphically on correlation of the masses, maximal of pendulum amplitude in the graphs is shown maximal, instead of amplitude of the set motion. For a one-impact absorber, adjusted on resonance frequency, attenuation ability is greater, but velocity of collisions is great, that can result in the damage of material. In the future it is necessary to take into account resilient properties of impact contacts using the dynamics conditions – to add the contact forces in impact contact point. Taking into account that in real structures the velocity of impact or maximum amplitude of the main system may be limited due to the danger of damage, choice of the damper parameters is made accordingly.

REFERENCES

[1] Cheng Jianlian, Hui Xu. Inner mass impact damper for attenuating structure vibration. *International Journal of Solids and Structures* .Vol. 43, Issue 17, pp. 5355-5369, 2006.

[2] Cheng C.C., Wang J.Y. Free vibration analysis of a resilient impact damper. *International Journal of Mechanical Sciences*, Vol. 45, pp. 589– 604, 2003.

[3] Chua Sy G., Pacheco B.M., Fujuino Y. and Ito M. Classical impact damper and pendulum impact damper for potential civil engineering application. *Structural Eng./Earthquake Eng.*,Vol.**7**, No1, pp. 110-112, 1990.

[4] Clarence W. de Silva. *Vibration. Fundamental and Practice.* CRC Press LLC, New-York, 2000.

[5] Ema S., Marui E. A fundamental study on impact dampers, *International Journal of Machine Tools and Manufacturers*, Vol. 36, pp. 93–306, 1996.

[6] Korenev B.G., Reznikov L.M. *Dynamic vibration absorber*. Nauka, Moscow, 1988. (in Russian)

[7] Viba J. *Optimisation and syntesis of vibro-impact mashines*. Zinatne, Riga 1988. (in Russian)

[8] *Vibrations in engineering: V.6. Protection from vibrations and impacts.* Frolov, K.F., editor, Mashinostroenie, Moscow, 1995. (in Russian)