

**INFLUENCE OF GAS TURBINE PARAMETERS CHANGES  
TO NONLINEAR VIBRATIONS OF ROTOR-BEARINGS SYSTEM**

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ABSTRACT

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This paper presents results of investigation of the gas turbine rotor dynamics using beam rotor model, which includes the nonlinear stiffness and damping characteristics of oil film in journal bearings. Multidisciplinary rotor dynamic model involving shaft model, rotating disks models, support with fluid film bearing model, is developed for realization of present research. Numerical calculations are carried out for modeling the rotor amplitudes (orbits) in bearings at different operating conditions and analyzing of rotor-bearings system sensitivity on modification of its different parameters such as rotor unbalances values, rotor structural damping and oil viscosity in bearings.

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**INTRODUCTION**

Investigation of gas turbine dynamic behavior should be carried out at different stages of its lifetime (design, assembly, operating and maintenance). At the stage of gas turbine design the engineer is interested in appropriateness of chosen technical solutions. On the other hand during gas turbine operating stage one may interested in analyzing data received from rotor dynamics monitoring system and in this data interpretation. This can be achieved by creating gas turbine rotor dynamics model taking into account gas turbine casing stiffness and damping characteristics, fluid film bearings nonlinear stiffness characteristics and rotor disks inertia characteristics. Gas turbine dynamic parameters determination concerned with rotor dynamics investigation on stationary and transient regimes, eigenvalues and eigenmodes calculation and the presence of sub- and superharmonic vibrations in the system with considerable nonlinearity, determined by fluid film. The solution of this problem is possible with the usage of modern mathematical simulation methods, models and algorithms.

In present paper within the limits of rotor dynamic model creation the gas turbine rotor beam model are created in standard FEM software and used for calculation of natural frequency value of rotor without supports. At second step nonlinear rotor model are created and tested on conformity with standard FEM software model. In further this model are used for direct integration equation of rotor-bearing system motion. The hypothesis of consecutive insertion of elastic and damping elements in unified calculation model is used for developing support model. These elements simulate elastic and damping properties of oil layer, bearing and rotor support case in gas turbine power unit. Oil flow between journal bearing surfaces is described by Reynolds equation with assumptions for parameters of oil flow in bearing taken into account. The system of equations of motion for rotor with bearing is solved by using the Newmark integration scheme, the iterative refinement of stiffness and damping matrices coefficients is performing at each time step.

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## 1. FLUID FILM BEARING ROTOR SUPPORT MODEL

In the general case pressure distribution in a fluid film bearing defined from well-known Reynolds equation [1, 2]. Taking into account boundary conditions for the shaft rotating with angular velocity  $\omega$ , the Reynolds' equation presented in a following form:

$$\frac{\partial}{\partial s} \left( h^3 \frac{\partial p}{\partial s} \right) + \frac{\partial}{\partial z} \left( h^3 \frac{\partial p}{\partial z} \right) = 6\eta \left( \omega R \frac{dh}{ds} - 2e\omega \sin(s/R) \right), \quad (1)$$

where  $p(s, z)$  is pressure in fluid film,  $s$  is coordinate axis located on one of the sliding surfaces in the direction of relative motion;  $z$  is located on the sliding surface perpendicular to relative motion,  $h(s, z)$  is the oil film thickness;  $\eta$  is a viscosity,  $R$  is the shaft journal radius. Lateral rotor force  $Q$  acting on the bearing support is in equilibrium with bearing carrying force defined from fluid film pressure.

If the shaft axis is always parallel to the bearing axis the fluid film thickness has the form:

$$h = \delta - e \cdot \cos(s/R) + \delta_b, \quad (2)$$

where  $\delta = R_b - R$  ( $R_b$  is the bearing radius) is a radial gap in the bearing;  $e$  is the shaft journal eccentricity in the bearing,  $\delta_b(s, z)$  is gap form alteration caused by deformations of bearing and shaft journal working surfaces.

In the general case, function  $h$  depends on two coordinates,  $s$  and  $z$ , and pressure distribution. Its calculation is conducted taking into account the bearing and shaft journal reciprocal displacements counting in their motion as a rigid bodies and deformations of working surfaces. After applying the weighted residual method, considering the standard boundary conditions, applying the Finite Element Method technology and approximating the pressure and the weighting function by triangular finite elements from (1) follows the nonlinear system of FEM equations:

$$[K_f]\{p\} = \{Q_f\}, \quad (3)$$

where  $[K_f(h)]$  is a matrix of system which coefficients for bearing with compliant working surfaces are depending from pressure distribution;  $\{Q_f\}$  is the right side vector which components are depending from pressure distribution too. They both are defined by the bearing geometrical characteristics and thickness of lubrication layer.

Obtained system of equations allows find the pressure distribution in the bearing with an arbitrary law of variation of the fluid film thickness. In general  $\delta_b(s, z)$  in (2) may include displacements caused by angular shaft deformations. This enables to evaluate stiffness characteristics of the fluid film, hydrodynamic forces and moments in the bearing versus the current shaft and bearing linear and angular displacements.

The hypothesis of consecutive insertion of elastic elements in unified calculation model is used for developing support model. These elements simulate elastic properties of oil layer, bearing and rotor support case in gas turbine power unit. Thus, support reaction vector,  $\{R_{sup}^u\}$ , acting on rotor, is related with vector of shaft journal displacements  $\{U_j\}$  as follows

$$[K_{sup}]\{U_j\} = \{R_{sup}^u\}, \quad (4)$$

where  $[K_{sup}] = ([K_b]^{-1} + [K_c]^{-1})^{-1}$  is a special finite element stiffness matrix of support included in matrix  $[K_s]$ ;  $[K_b]$  is a stiffness matrix of oil layer;  $[K_c]$  is a support case stiffness matrix.

Coefficients of matrix  $[K_c]$  are determined in calculation of support case structure. Calculation of support case stiffness may be performed using finite element method. Thus the level of detailed elaboration when creating finite-element model depends on structure features. For calculation of support stiffnesses it is necessary to simulate the whole case for gas turbine power units, which cases may be represented as thin-walled structures. In order to define matrix  $[K_b]$  coefficients the problem of oil flow in gap should be solved.

## 2. ROTOR DYNAMIC MODEL

Multidisciplinary rotor dynamic model involving shaft model, rotating disks models, support model is developed for realization of present research. The beam model with distributed masses and with inertia of cross-section rotational displacement in the case of rotor bending deformation taken into consideration is used as shaft model. Disks are considered as mass points that are attached to prescribed shaft cross-sections. Their inertia characteristics include masses and inertia moments. Shaft cross-section centers of inertia offsets and disks center deviation from shaft geometrical axis are described in each shaft cross-section by the vector of initial disbalances  $\{\varepsilon_0\} = \{\varepsilon_{0x}, \varepsilon_{0y}\}$  which components define rotor inertia center coordinates in fixed coordinate system  $xyz$ . Support model is represented by special finite element which coefficients are determined in calculation process by rotor current position and rotative speed. Rotor model in fixed coordinate system constrained to unit supports is described by FEM equations in the following form

$$[M]\{\ddot{U}\} + [C(\omega)]\{\dot{U}\} + ([K_R] + [K_S])\{U\} = \{F_0\} + \omega^2[M_1]\{E_0\}, \quad (5)$$

where  $[M]$  is mass matrix of shaft and parts, attached to it;  $[C]$  is a matrix, that considers an influence of gyroscopic moments and damping in supports and seals;  $[M_1]$  is a part of matrix  $[M]$  connected with nodal linear displacements;  $\{U\}$ ,  $\{\dot{U}\}$  and  $\{\ddot{U}\}$  are the vectors of rotor nodal displacements, velocities and accelerations correspondingly;  $[K_R]$  is rotor stiffness matrix;  $[K_S]$  is the stiffness matrix that considers supports and seals influence;  $\{F_0\}$  is a vector of external forces acting on rotor;  $\{E_0\}$  is initial shaft cross-section disbalances vector, defined by vector  $\{\varepsilon_0\}$ .

Mass  $[M]$  and stiffness  $[K_R]$  matrices do not change in process of motion at established rotor rotative speed. In contrast to them, coefficients of matrices  $[K_S]$  and  $[C]$  depend not only on rotor rotative speed but also on position of curved axis of the shaft in supports. Matrix  $[K_S]$  coefficients are defined by parameters of oil flow in bearings and seals, aerodynamic forces in wheels and by the stiffness of gas turbine power unit case. Gyroscopic moments, actuating fluid flow parameters (gas or air) in sealing devices and friction in fluid film bearings have an influence on damping matrix  $[C]$  coefficients. Thus  $[K_S]$  and  $[C]$  are general form matrices, with coefficients, that nonlinearly depend on current position of the shaft axis, rotative speed and oil parameters.

The system of equations of motion (5) is solved by using the Newmark integration scheme, the iterative refinement of stiffness and damping matrices coefficients is performing at each time step.

## 3. ROTOR MOTION CALCULATION RESULTS

Investigated gas turbine rotor consist of two shafts connected by coupling. Each shaft supported in two radial hydrodynamic journal bearings. Bearing and shaft sliding surfaces are applied cylindrically shaped. Nonlinear gas turbine rotor beam model and model in standard FEM software are created. These beam models are used for further investigations of rotor dynamic behavior and are presented on Fig. 1.

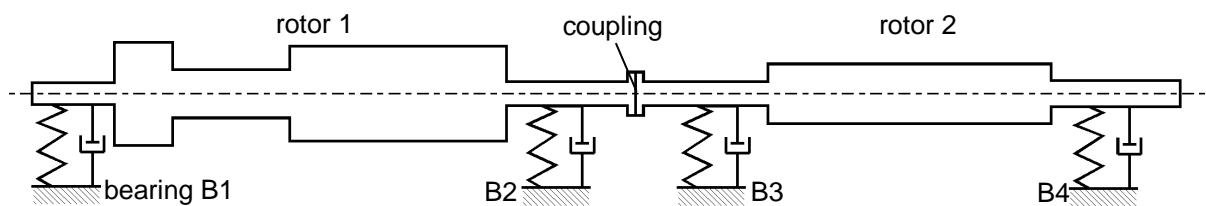


Fig. 1. Gas turbine rotor model

### 3.1 Rotor eigenfrequencies

Rotor model created in standard FEM software testing the nonlinear beam model that used below for direct integration simulation. Nonlinear rotor model verification was carried out for rigid supports and for supports with stiffness 8 MN/mm. Eigenfrequencies verification results presented in Table 1. Frequencies in both models have good conformity between each other. Rotor eigenmodes presented in Fig. 1.

Table 1. Models comparison

№	Eigenfrequencies, Hz			
	Rigid supports		Support stiffness - 8 MN/mm	
	Standard FEM software	Nonlinear beam model	Standard FEM software	Nonlinear beam model
ft1	22,10	22,12	21,68	21,70
ft2	34,35	34,51	32,23	32,35
ft3	56,31	56,77	54,86	55,29
ft4	93,32	94,34	81,81	82,72

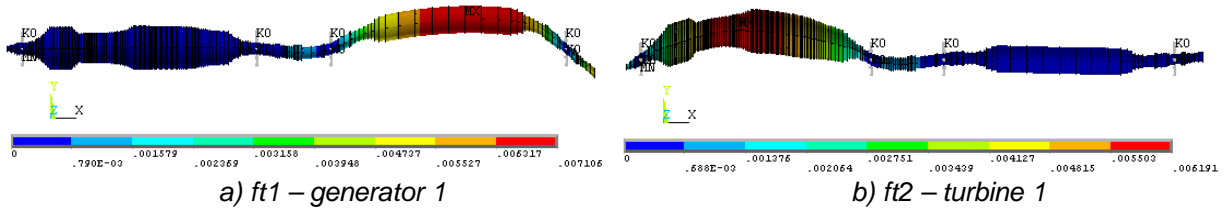


Fig. 2 Rotor eigenmodes

### 3.1 Structural damping sensitivity

The investigation of structural damping influence on rotor dynamic behavior is carried out for a structural damping  $\beta$  of 0.001, 0.002 and 0.02 of structural stiffness matrix. For current parameters calculations were carried out for rotor speeds from 0 and up to 80 Hz with step of 5 Hz. Rotor full spectrums and orbits for structural damping of 0.001, 0.002 and 0.02 presented in Fig. 3.

For  $\beta=0.001$ , in range from 20 Hz to 55 Hz the sufficient rotor vibrations on forward and backward frequencies close to ft2, double ft2 and triple ft2 exist. In contrast with 0.001 case for 0.002 case the sufficient rotor vibrations on forward and backward frequencies close to ft2, double ft2 and triple ft2 exist only in thin range near “rotor 1” first critical speed with amplitudes not exceed maximal allowable limit. For 0.02 case the sufficient rotor vibrations on forward and backward frequencies close to 1X lines exist for speeds in region near “rotor 2” second critical speed (ft3). For all other rotative frequencies not fall within range of sufficient vibrations the rotor vibrates on its rotative speed with very small amplitudes.

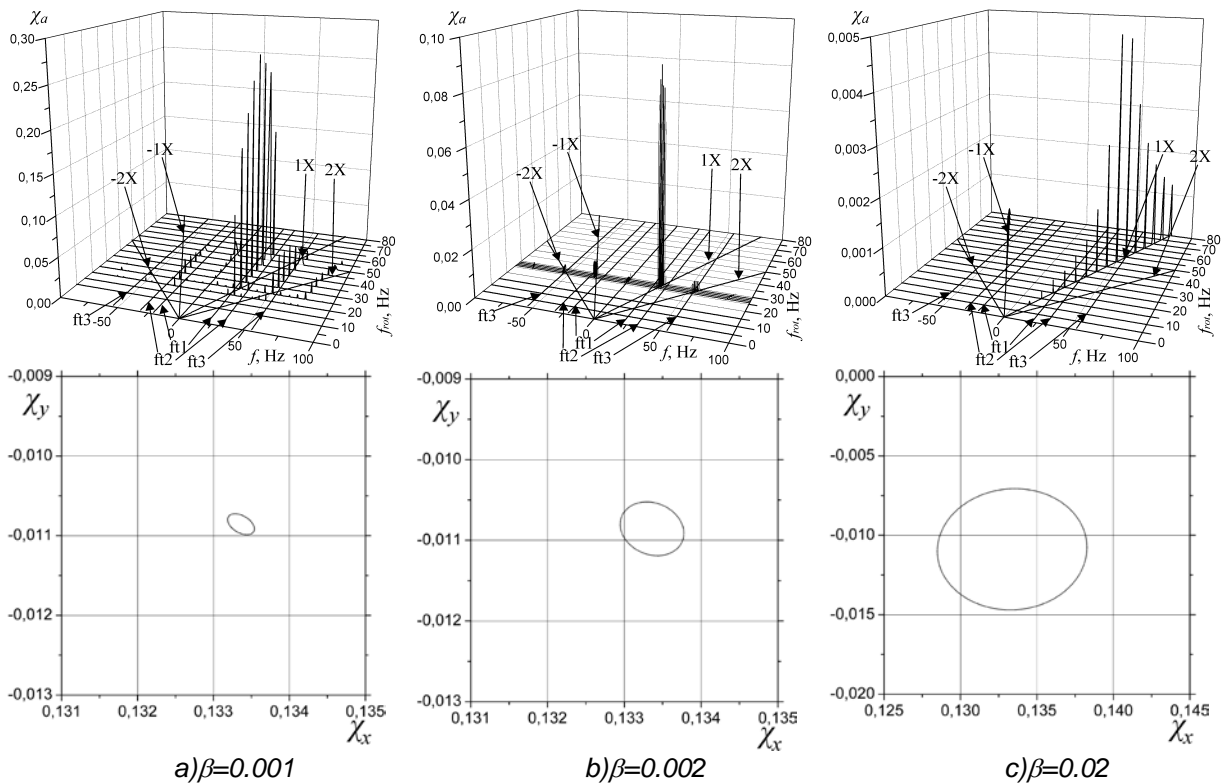


Fig 3. Rotor orbits in bearings(60Hz) and full spectra with different structural damping and lubrication viscosity 0.04 MPa·s

### 3.2 Oil viscosity sensitivity

The investigation of lubrication viscosity influence on rotor dynamic behavior were carried out for lubrication viscosity  $\eta$  of 0.02 MPa·s, 0.04 MPa·s and 0.06 MPa·s. For current parameters calculations were carried out for rotor speeds from 0 and to 80 Hz with step of 5 Hz. Rotor full spectrums and orbits for lubrication viscosity of 0.02 MPa·s, 0.04 MPa·s and 0.06 MPa·s presented in Fig. 4.

As it follows from rotor orbits and its full spectrum analysis for 0.02 MPa·s lubrication viscosity case the sufficient “rotor 1” vibrations on forward and backward frequencies close to  $ft_2$ , double  $ft_2$  and triple  $ft_2$  exist for rotor speeds in region from 30 to 65 Hz. “Rotor 2” have vibrations in this region too but with amplitudes ten times less then similar for turbine. Besides that generator have vibrations on it first critical speed ( $ft_1$ ). As it follows from rotor orbits and its full spectrum analysis for 0.04 MPa·s the sufficient vibrations on forward and backward frequencies close to  $ft_2$  and double  $ft_2$  exist for speeds in region near “rotor 1” first critical speed ( $ft_2$ ). The “rotor 2” vibrations amplitudes ten times less then similar for “rotor 1” for region began from rotor rotation frequency equal to  $ft_2$ . As it follows from rotor orbits and its full spectrum analysis for 0.06 MPa·s lubrication viscosity case its vibrates on forward and backward rotational frequencies ( $\pm 1X$ ) with small amplitudes.

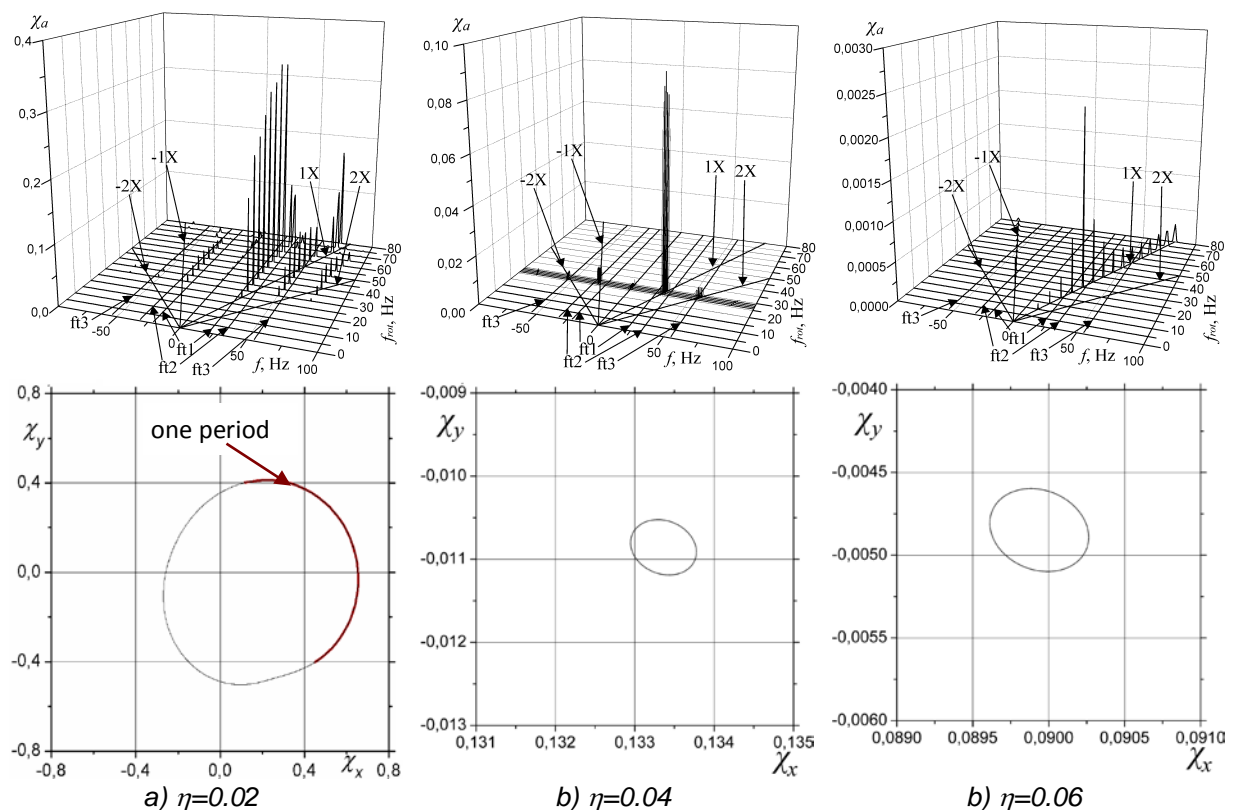


Fig 4. Rotor orbits in bearings (60Hz) and full spectra with different oil viscosity and structural damping 0.002

### 3.3 High unbalance sensitivity

The investigation of rotor orbits in bearings sensitivity were carried out for unbalance values 5 kg, 10 kg and 15 kg on radius of 1 m. For current parameters calculations were carried out for rotor speeds from 0 and up to 80 Hz with step of 5 Hz. Rotor full spectrums and orbits for unbalance values 5 kg, 10 kg and 15 kg on radius of 1 m presented in Fig. 5.

For 5 kg unbalance case the sufficient rotor vibrations on forward and backward frequencies on  $\pm 1X$  and  $\pm 2X$  lines exist in whole considered range with amplitudes of “rotor 1” vibrations amount to approximately 120  $\mu\text{m}$  and “rotor 2” vibrations amount to approximately 30  $\mu\text{m}$ . “Rotor 2” receives just frequency excitation from “rotor 1” without sufficient amplitudes growth. For 10 kg unbalance case the sufficient rotor vibrations on forward and backward frequencies on  $\pm 1X$ ,  $\pm 2X$  and  $\pm 3X$  lines exist in whole considered range with amplitudes of “rotor 1” vibrations amount to approximately 150  $\mu\text{m}$  and “rotor 2” amplitudes less then 30  $\mu\text{m}$ . For 15 kg unbalance case the sufficient rotor vibrations on forward and backward frequencies on  $\pm 1X$ ,  $\pm 2X$  and  $\pm 3X$  lines exist in whole considered range.

Besides that there are sufficient vibrations with frequency 9 Hz. In full spectrum one can see a lot of small amplitudes on different frequencies in investigated range.

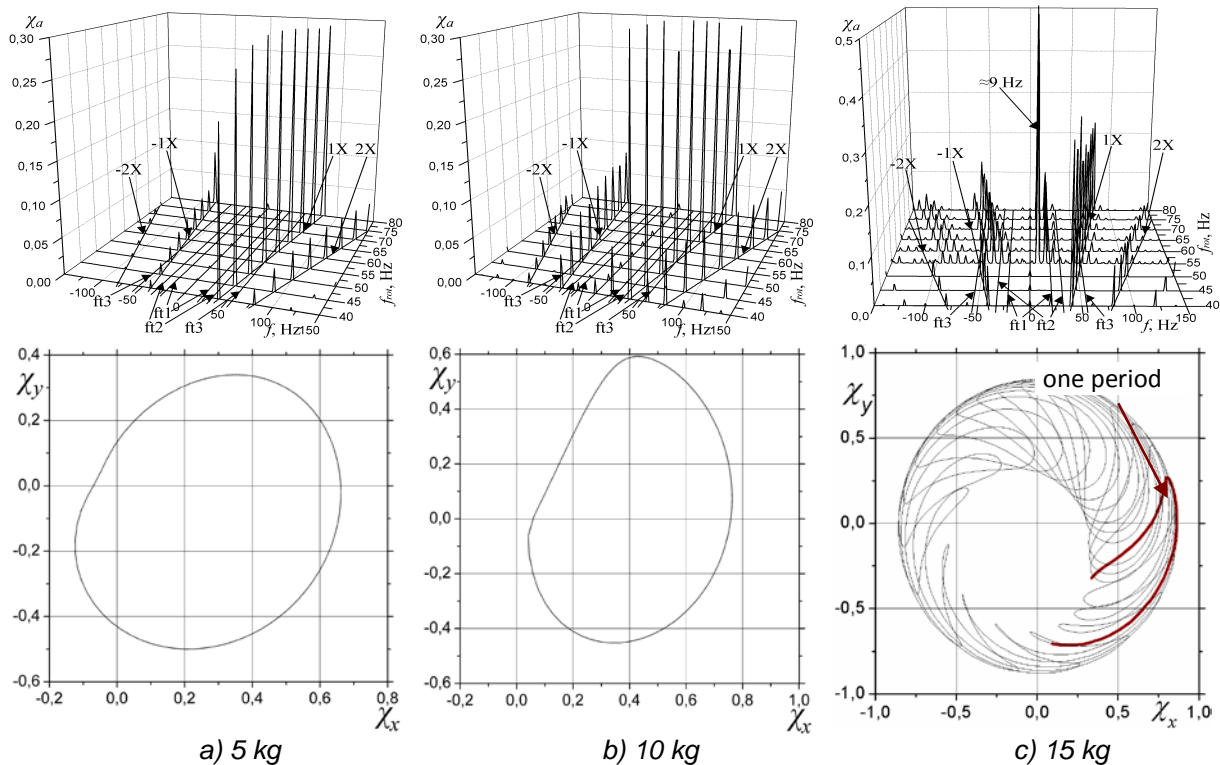


Fig 5. Rotor orbits in bearings(60Hz) and full spectra with different unbalance values

## CONCLUSIONS

The nonlinear rotor dynamics model was developed and verified using corresponding standard FEM software model. Developed model consist of beam shaft model with attached inertia elements and support with fluid film bearing model and allows calculate eigenvalues and eigenmodes, stable and unstable rotor orbits for varying model parameters. On the base of rotor dynamic model the rotor sensitivity to structural damping, oil in bearings viscosity and unbalance value varying were investigated. Influence of these system parameters on rotor orbits and vibration frequencies are shown. Borders between stable and unstable rotor operating regimes depending upon system parameters values are determined.

Presented methodology allows to change rotor system parameters such as damping, oil viscosity and unbalance values at any stage of gas turbine lifetime for decreasing rotor vibration amplitudes and improving rotor stability.

## REFERENCES

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