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INVESTIGATION OF THE PARAMETRIC VIBRATION OF THE ORTHOTROPIC PLATES SUBJECTED TO PERIODIC IN PLANE FORCES BY MULTI-MODAL APPROXIMATION AND R-FUNCTIONS METHOD

Awrejcewicz J. [1]
Technical University of Lódź, Lodz, Poland
Kurpa L.[2]**,**
Mazur O.
National Technical University, Kharkov, Ukraine

ABSTRACT

The original method of studying parametric vibrations of orthotropic plate with complex shape is proposed. Suggested approach is based on combined application of variational methods and the R-functions theory. Using the proposed method and developed software the regular and chaotic regimes of T-shaped plate are analyzed.

INTRODUCTION

Since the elements modeled by orthotropic plates are used in different branches of industry such as aerospace, ship and transport engineering etc., the investigation of the plates nonlinear vibrations subjected to different types of load is an actual problem. The fundamental theory of studying parametrically excited vibrations plates of the rectangular form had been presented in work [4]. In recent papers [2, 3, 7] and others, new problems of parametric vibrations of the plates including their bifurcation and chaotic dynamics are studied. It should be noted that plates, which are used in applications have the different geometry. Therefore nonlinear dynamics problems of plate with complex planform have received particular interest among scientists. Generally in modern literature in the case of plate with the complex form the universal approaches via FEM (Finite Element Method) or FDM (Finite Difference Method) are used. Application of variational-structural method based on the R-functions method (RFM) [5,6] is relatively new approach for nonlinear problems. Mentioned approach has some preferences; among them first of all it is possibility to construct the system of basic functions in analytical way. The basic functions exactly satisfy the boundary conditions for plates with complex form.

In the given work the new discretization method of the nonlinear system of the differential equations with partial derivatives are proposed. The main idea of the proposed method relies on reduction of the equations governing dynamics of plates of the complex form to a system of nonlinear ordinary differential equations (ODEs) by variational methods joined with the R-functions theory. The proposed method establishes the simple connection between generated coordinates of the unknown functions and allows representation of the coefficients of the obtained ODEs in analytical form in result of solving a series of linear boundary value problems.

1. FORMULATION

Let us consider the nonlinear vibrations of an orthotropic plate with constant thickness *h* loaded by periodic in-plane force. For construction of mathematical model of task Von Karman's non-linear strain-displacement relationships are employed and the equations of motion are developed by applying the principle of virtual work. The movement equations in the mixed form have the following view [8]

$$L_1\Phi = -\frac{C_1}{2}L(w,w), \tag{1}$$

[1] Corresponding author. Email awrejcew@p.lodz.pl
[2] Corresponding author. Email Kurpa@kpi.kharkov.ua

$$L_2 w = L(w,\Phi) - \varepsilon\frac{\partial w}{\partial t} - \frac{\partial^2 w}{\partial t^2}, \tag{2}$$

where w is deflection of the plate, $\Phi(x,y)$ is the stress (Airy's) function being introduced by the stresses σ_x, σ_y and τ through the standard formulas

$$\frac{\partial^2\Phi}{\partial y^2} = \sigma_x,\ \frac{\partial^2\Phi}{\partial x^2} = \sigma_y,\ -\frac{\partial^2\Phi}{\partial x\partial y} = \tau.$$

The linear operators L_1, L_2 in (1), (2) are defined as follows

$$L_1 = C_1\frac{\partial}{\partial x^4} - \left(2\mu_1 - \frac{E_1}{G}\right)\frac{\partial}{\partial x^2\partial y^2} + \frac{\partial}{\partial y^4}, \tag{3}$$

$$L_2 = \frac{1}{12(1-\mu_1\mu_2)}\left(C_1\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + 2(C_3 + C_2)\frac{\partial^4}{\partial x^2\partial y^2}\right), \tag{4}$$

and the expressions for nonlinear operators $L(w,w)$, $L(w,\Phi)$ are

$$L(w,w) = 2\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2\right),\ L(w,\Phi) = \frac{\partial^2 w}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2\Phi}{\partial x^2} - 2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}.$$

Note that equations (1), (2) are already presented in a non-dimensional form, and relations between dimensional and non-dimensional values are defined as

$$\bar{x} = \frac{x}{a},\ \bar{y} = \frac{y}{a},\ \bar{w} = \frac{w}{h},\ \bar{p} = \frac{a^2 p}{h^3 E_2},\ \bar{t} = \frac{h}{a^2}\sqrt{\frac{E_2}{\rho}}t,\ \bar{\varepsilon} = \frac{a^2}{h}\sqrt{\frac{\rho}{E_2}}\varepsilon, \bar{\Phi} = \frac{\Phi}{h^2 E_2}.$$

Later, bars over non-dimensional values will be omitted. The coefficients C_1, C_2 and C_3 appearing in (1)-(4) are defined by the following relations

$$C_1 = \frac{E_1}{E_2},\ C_2 = \frac{G(1-\mu_1\mu_2)}{E_2},\ C_3 = \frac{G(1-\mu_1\mu_2)}{E_2} + \mu_1. \tag{5}$$

In (1)-(5) E_1, E_2 are elasticity (Young) modules, μ_1, μ_2 are Poisson's ratios, G is shear modulus, ρ is the plate density, and ε is a damping coefficient.

The system (1)-(2) are supplemented with initial and corresponding boundary conditions. Initial conditions for plate are taken in the form

$$w|_{t=0} = w_0,\ w'|_{t=0} = 0,$$

The boundary conditions are introduced in various ways and they depend on plate type of support.

2. METHOD OF SOLUTION

Let us present the plate deflection in the following form

$$w(x,y,t) = \sum_{i=1}^{n} f_i(t) w_i(x,y). \tag{6}$$

Here w_i are eigenfunctions of a linear vibrations problem of the corresponded unloaded plate. Thus, the linear problem is reduced to solving the equation

$$L_2 W(x,y) = \omega_L^2 W(x,y), \tag{7}$$

where ω_L is eigenfrequency corresponding to w_i mode of plate vibration. In what follows the problem (7) is further solved by the Ritz method combined with R-function theory.

The stress functions are sought for in the following form

$$\Phi(x,y,t) = p\Phi_0 + \sum_{i,j=1}^{n} f_i(t) f_j(t)\Phi_{ij}(x,y). \tag{8}$$

Here the function $\Phi_0(x,y)$ is solution to the equation

$$L_1\Phi_0 = 0, \tag{9}$$

which satisfies conditions of the form

$$\frac{\partial^2\Phi_0}{\partial\tau^2} = -1, \frac{\partial^2\Phi_0}{\partial n\partial\tau} = 0 \tag{10}$$

on loaded part of border.

The functions $\Phi_{ij}(x,y)$ appearing in (8) are solutions to the following equations

$$L_1\Phi_{ij} = -\frac{C_1}{2}L(w_i, w_j). \tag{11}$$

The boundary conditions for functions Φ_{ij} on unloaded part of counter are follows

$$\frac{\partial^2\Phi_{ij}}{\partial\tau^2} = 0, \frac{\partial^2\Phi_{ij}}{\partial n\partial\tau} = 0 \tag{12}$$

The boundary conditions on unloaded part of the border for functions $\Phi_0(x,y)$and $\Phi_{ij}(x,y)$ depend on support edge. In order to solve the problems (9), (10) and (11), (12) for plate with complex shape the matched method including the Ritz approach as well as the R-function theory is used.

Substituting expressions (6) and (8) for w and Φ into equation (2), and applying the Bubnov-Galerkin method, we can reduce the input system of nonlinear partial differential equations (PDEs) to an appropriate system of ordinary differential equations (ODEs) of the following matrix form

$$\mathbf{f}'' + \varepsilon\mathbf{f}' + (\mathbf{C} - p\mathbf{A})\mathbf{f} + \mathbf{B}(\mathbf{f}) = 0, \tag{13}$$

where

$$\mathbf{A} = \begin{pmatrix} a_1^{(1)} & a_2^{(1)} & \dots & a_n^{(1)} \\ a_1^{(2)} & a_2^{(2)} & \dots & a_n^{(2)} \\ \dots & \dots & \dots & \dots \\ a_1^{(n)} & a_2^{(n)} & \dots & a_n^{(n)} \end{pmatrix}, \ \mathbf{C} = \begin{pmatrix} \omega_{L,1}^2 & 0 & \dots & 0 \\ 0 & \omega_{L,2}^2 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & \omega_{L,n}^2 \end{pmatrix}, \mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ \dots \\ f_n \end{pmatrix}, \ \mathbf{f}' = \begin{pmatrix} f_1' \\ f_2' \\ \dots \\ f_n' \end{pmatrix}, \mathbf{f}'' = \begin{pmatrix} f_1'' \\ f_2'' \\ \dots \\ f_n'' \end{pmatrix}, \ \mathbf{B}(\mathbf{f}) = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix},$$

where b_m is designed as $b_m = \sum_{i,j,k=1}^{n} \beta_{ijk}^{(m)} f_i f_j f_k$.

Note that the elements of the matrix $\mathbf{A}$ and the vector $\mathbf{B}(\mathbf{f})$ in (13) are defined through double integrals over the investigated domain by the following formulas

$$\alpha_i^{(m)} = \frac{1}{\|w_m\|^2}\iint_\Omega L(w_i, \Phi_0)w_m dxdy, \ \beta_{ijk}^{(m)} = -\frac{1}{\|w_m\|^2}\iint_\Omega L(w_k, \Phi_{ij})w_m dxdy.$$

3. NUMERICAL RESULTS

Let us consider the parametric vibrations of the orthotropic plate of complex shape shown in Figure 1.

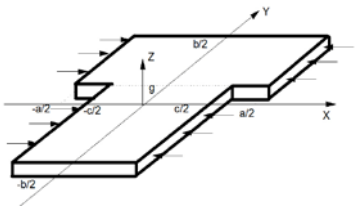


Fig. 1. A T-shaped plate

The plate is subjected to load of the form $p = p_0 \sin\theta t$, which is applied longitudinally along the plate edges being parallel to the axis *OY***.** Let us deflection w satisfy the following boundary conditions

$$w = 0, \ \frac{\partial^2 w}{\partial n^2} = 0.$$

The boundary conditions for stress function Φ have the form

$$\frac{\partial^2\Phi}{\partial\tau^2}=-p\,,\ \frac{\partial^2\Phi}{\partial n\partial\tau}=0\,.$$

Our numerical results are obtained for material (glass-epoxy) with the following relations for the elasticity coefficients: $E_1/E_2=3$, $G/E_2=0.6$, $\mu_1=\mu_2E_1/E_2=0.25$.

It should be noted that the current plate is in a homogenous subcritical state, and the function Φ_0 can be presented in the form $\Phi_0=-\dfrac{y^2}{2}$.

The variational formulations of the tasks (7) and (11) are reduced to finding minimum of following functionals respectively

$$I(W)=\iint_\Omega\left(C_1\left(\frac{\partial^2W}{\partial x^2}\right)^2+2C_1\mu_2\frac{\partial^2W}{\partial x^2}\frac{\partial^2W}{\partial y^2}+\left(\frac{\partial^2W}{\partial y^2}\right)^2+4C_2\left(\frac{\partial^2W}{\partial x\partial y}\right)^2-12(1-\mu_1\mu_2)\omega_L^2W^2\right)d\Omega\,,$$

$$I(\Phi_{ij})=\iint_\Omega\left[C_1\left(\frac{\partial^2\Phi_{ij}}{\partial x^2}\right)^2+\frac{E_1}{G}\frac{\partial^2\Phi_{ij}}{\partial x^2}\frac{\partial^2\Phi_{ij}}{\partial y^2}+\left(\frac{\partial^2\Phi_{ij}}{\partial y^2}\right)^2-2\mu_1\left(\frac{\partial^2\Phi_{ij}}{\partial x\partial y}\right)^2+L(w_i,w_j)\Phi_{ij}\right]d\Omega\,.$$

The series of basic functions, which satisfy boundary conditions, is constructed by the R-function method. At first it is necessary to construct the predicate of the domain

$$\Omega=(\Omega_1\wedge\Omega_2)\wedge(\Omega_3\vee\Omega_4),$$

where

$$\Omega_1=\left(f_1=\frac{1}{a}\left(\left(\frac{a}{2}\right)^2-x^2\right)\ge0\right),\ \Omega_2=\left(f_2=\frac{1}{b}\left(\left(\frac{b}{2}\right)^2-y^2\right)\ge0\right),\ \Omega_3=\left(f_3=\frac{1}{c}\left(\left(\frac{c}{2}\right)^2-x^2\right)\ge0\right)$$

$$\Omega_4=(f_4=y-g\ge0)$$

are sub-domains. According to RFM the equation $\omega(x,y)=0$, where $\omega(x,y)=(f_1\wedge_0 f_2)\wedge_0(f_3\vee_0 f_4)$, is the equation of the boundary domain. The symbols $\vee_0,\wedge_0,-$ (R-disjunction, R-conjunction, R-negation) are defined as follows [5]:

$$x\vee_0 y=x+y+\sqrt{x^2+y^2}\,,\ x\wedge_0 y=x+y-\sqrt{x^2+y^2}\,,\ \bar{x}=-x.$$

For considerable boundary conditions we use the following solution structure

$$w_i=\omega\cdot P_1,\ \Phi_{ij}=\omega^2\cdot P_2\,. \tag{14}$$

The indefinite components P_1,P_2 in (14) are approximated by the following power polynomials

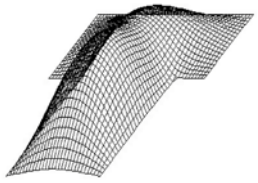
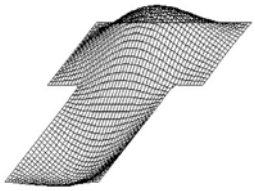
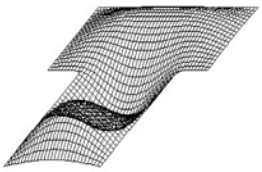
$$P_1,P_2:1,y,x^2,y^2,x^2y,y^3,...$$

The effect of cut-out decrease on natural frequencies has been investigated. Obtained results are presented in Table 1, where $l=g+b/2$, $b/a=1$ (see Figure 1). It should be noted that current plate is symmetric relatively to axis *OY* and frequencies which are corresponded to modes (1,1), (1,2), (1,3) are presented. Degeneration of plate with complex form in square plate leads to decrease of natural frequencies and approaching of their values to corresponding values for square plate.

Table 1 Comparison of frequency parameter $\lambda_i=a^2/h\sqrt{\rho/E_2}\,\omega_{L,i}$ for various values of cutout

Geometrical parameters	λ_1	λ_2	λ_3
$c/a=0.35$, $l/a=0.2$	9.365	19.957	33.050
$c/a=0.4$, $l/a=0.1$	8.131	17.095	31.403
$c/a=0.45$, $l/a=0.05$	7.664	16.123	30.432
$c/a=0.48$, $l/a=0.02$	7.553	15.909	30.189
Square plate[1]	7.536	15.875	30.150

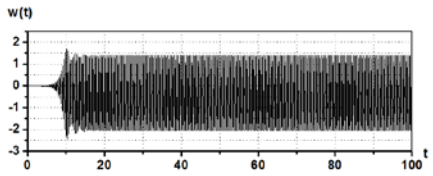
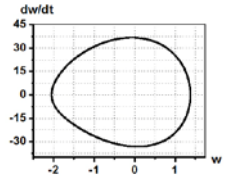
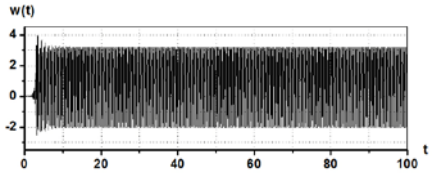
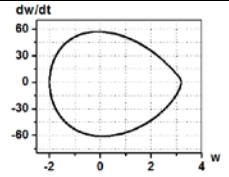
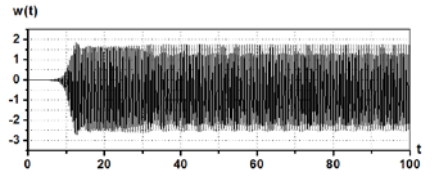
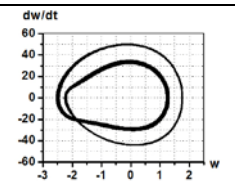
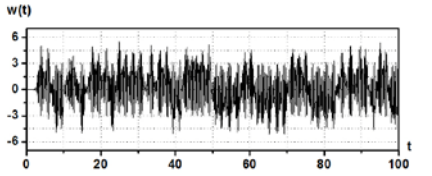
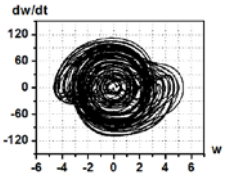
Table 2. Modes of plate vibrations

		
$\lambda_1 = 19.373$	$\lambda_2 = 22.665$	$\lambda_3 = 37.013$

Further results are obtained for $c/a = 0.6$, $g/a = 0.1$, $b/a = 1$, $\varepsilon = 1$, $w_0 = 0.001$. In Table 2 the first three modes (considering symmetry of the plate relatively to axis *OY*) with the corresponding linear frequency parameters $\lambda_i = a^2/h\sqrt{\rho/E_2}\,\omega_{L,i}$ are presented.

In what follows the effect of multimode approximation (three modes) on the investigated plate characteristics is further studied for a fixed value of the excited frequency $\theta = 19$ taken near the eigenfrequency and for various values of the parameter p_0. Dependencies $w(t)$ and $\dot{w}(w)$ are computed in the point of first mode maximum $M(0,-0.141a)$, and the obtained results are reported in Table 3.

Table 3. Dependences $w(t)$, $\dot{w}(w)$ obtained via third order approximation

p_0	One mode approximation	
12.2		
21		
p_0	Three modes approximation	
12.2		
21		

Analyzing of data in Table 3 one may see that results obtained with use one-mode approximation differ from results obtained applying three-mode approximation and draw a conclusion that for chaotic behavior of plate investigation one-mode approximation cannot used.

CONCLUSIONS

The effective numerically-analytical method of parametric regular and chaotic vibrations investigation of orthotropic plates with complex shapes and different types of boundary conditions has been proposed**.** The approach is based on applying a variational methods and R-function theory. The numerical results are obtained for plate with complex shape using first and third order of approximations. In addition to the general theoretical results our numerical analysis has shown that