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MATHEMATICAL MODELING OF DEFORMATION PROCESSES OF THE FLEXIBLE VISCOELASTIC PLATES WITH COMPLEX FORM

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ABSTRACT

The investigation method of the vibration of the flexible viscoelastic anisotropic plates is proposed. The nonlinear system of the three integro-differential equations with partial derivatives is mathematical formulation of this problem. The linearization of the problem is carried out by method of successive approximations. The obtained sequence of the linearized problems is solved by quadrature sums. The software for numerical implementation is developed.

Let us consider a mathematical model of vibrations of flexible viscoelastic plates, which is described by system of three nonlinear integro-differential equations in partial derivatives with appropriate boundary and with initial conditions.

In the derivation of the equilibrium equations Kirchhoff-Love hypotheses are used [1]

$$u_1 = u - z\frac{\partial w}{\partial x}, \quad u_2 = v - z\frac{\partial w}{\partial y}, \quad u_3 = w(x,y) \tag{1}$$

On this basis, the Cauchy relations can be written as [1]:

$$\varepsilon_{11} = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \ \varepsilon_{22} = \frac{\partial v}{\partial y} - z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \ \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial y}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \tag{2}$$

Kirchhoff-Love Hooke's law takes the following form [2]:

$$\sigma_{11} = B_{11}^*\varepsilon_{11} + B_{12}^*\varepsilon_{22} + B_{16}^*\varepsilon_{12}; \sigma_{22} = B_{12}^*\varepsilon_{11} + B_{22}^*\varepsilon_{22} + B_{26}^*\varepsilon_{12}; \ \sigma_{12} = B_{16}^*\varepsilon_{11} + B_{26}^*\varepsilon_{26} + B_{66}^*\varepsilon_{12} \tag{3}$$

where ε_{ij} and σ_{ij} ($i = \overline{1,2},\ j = \overline{1,2}$) are respectively, the components of the strain and stress vectors, $B_{ij}^* = B_{ij}(1 - R_{ij}^*)$, $R_{ij}^*[] = \int_0^t R_{ij}(t-\tau)[]d\tau$ is integral operator by Volterra with weak singular kernel of heredity $R_{ij}(t)$, which can be used by Abel core, exponential or Rjanitsina-Koltunova one etc. [2-3].

Using variational principle by Ostrogradsky-Hamilton [1] and taking into account the approach by Boltsman [3, 7], the following equilibrium equations may be obtained

$$\frac{\partial N_{11}^*}{\partial x} + \frac{\partial N_{12}^*}{\partial y} + q_1 = \rho h\frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial N_{12}^*}{\partial x} + \frac{\partial N_{22}^*}{\partial y} + q_2 = \rho h\frac{\partial^2 v}{\partial t^2},$$

$$\frac{\partial^2 M_{11}^*}{\partial x^2} + 2\frac{\partial^2 M_{12}^*}{\partial x \partial y} + \frac{\partial^2 M_{22}^*}{\partial y^2} + N_{11}^*\frac{\partial^2 w}{\partial x^2} + 2N_{12}^*\frac{\partial^2 w}{\partial x \partial y} + N_{22}^*\frac{\partial^2 w}{\partial y^2} + \left(\frac{\partial N_{11}^*}{\partial x} + \frac{\partial N_{12}^*}{\partial y}\right)\frac{\partial w}{\partial x} + \left(\frac{\partial N_{12}^*}{\partial x} + \frac{\partial N_{22}^*}{\partial y}\right)\frac{\partial w}{\partial y} + q_3 = \rho h\frac{\partial^2 w}{\partial t^2} \tag{4}$$

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where w is the transverse displacement of the plate ; u, v are the displacements of the plate in the mid-surface; $q_1,\ q_2,\ q_3$ are the external loads; h is the plate thickness; $M^*_{11},\ M^*_{22},\ M^*_{12}$ are the bending and twisting moments; $N^*_{11},\ N^*_{22},\ N^*_{12}$ – are the normal and tangent forces; x,y are space variables; t is a time.

Let us write the expressions for stress resultants:

$$N^*_{11} = h\left[B^*_{11}(\frac{\partial u}{\partial x}+\frac{1}{2}(\frac{\partial w}{\partial x})^2)+B^*_{12}(\frac{\partial v}{\partial y}+\frac{1}{2}(\frac{\partial w}{\partial y})^2)+B^*_{16}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}) \right],$$

$$N^*_{12} = h\left[B^*_{16}(\frac{\partial u}{\partial x}+\frac{1}{2}(\frac{\partial w}{\partial x})^2)+B^*_{26}(\frac{\partial v}{\partial y}+\frac{1}{2}(\frac{\partial w}{\partial y})^2)+B^*_{66}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}) \right], \quad (5)$$

$$N^*_{22} = h\left[B^*_{12}(\frac{\partial u}{\partial x}+\frac{1}{2}(\frac{\partial w}{\partial x})^2)+B^*_{22}(\frac{\partial v}{\partial y}+\frac{1}{2}(\frac{\partial w}{\partial y})^2)+B^*_{26}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}) \right],$$

$$M^*_{11} = -\left[D^*_{11}\frac{\partial^2 w}{\partial x^2}+D^*_{12}\frac{\partial^2 w}{\partial y^2}+2D^*_{16}\frac{\partial^2 w}{\partial x\partial y} \right],$$

$$M^*_{22} = -\left[D^*_{12}\frac{\partial^2 w}{\partial x^2}+D^*_{22}\frac{\partial^2 w}{\partial y^2}+2D^*_{26}\frac{\partial^2 w}{\partial x\partial y} \right], \quad (6)$$

$$M^*_{12} = -\left[D^*_{16}\frac{\partial^2 w}{\partial x^2}+D^*_{62}\frac{\partial^2 w}{\partial y^2}+2D^*_{66}\frac{\partial^2 w}{\partial x\partial y} \right]$$

$$D^*_{ij} = D_{ij}(1-R^*_{ij}),\ D_{ij} = \frac{h3}{12}B_{ij},\ (i,j=\overline{1,6}).$$

The equations are supplemented by boundary and initial conditions, which depend of type of fixing border of the plate.

The obtained system of nonlinear integro-differential equations are solved on the basis of the algorithm of linearization, which is based on the use of the method of successive approximations [4-5]. The initial values for displacements are put as: $u_0 = 0$ and $v_0 = 0$, and the system of linear integro-differential equations is solved. Then, the obtained solution for deflection (w) is substituted into the first two equations of the system of integro-differential equations. Further, the obtained solutions for displacements u, v are substituted into third equations of the system deflection. This process is fulfilled until satisfactory convergence for the results will be achieved. Separation of the variables is carried out by the Bubnov-Galerkin procedure [6], and R-functions by V.L. Rvachev [4]. The linearized systems of integro-differential equations with the initial condition are solved by quadrature sums method [3].

The software based on the proposed algorithm is developed. In the report, results of computational experiments for the clamped and simply supported viscoelastic flexible anisotropic plates of different shapes are discussed.

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