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NEW ASPECTS OF CHAOTIC DYNAMICS OF PENDULUM SYSTEMS WITH A LIMITED POWER-SUPPLY

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ABSTRACT

In represented paper some new aspects of chaotic behavior of spherical pendulum with limited (non-ideal) excitation are considered. Chaotic regimes in such system arise due to feedback influence of pendulum oscillations on a mechanism of its excitation. For considered system chaotic attractors and scenarios of its origin were investigated in details. New peculiarities of scenario of transition to chaos through cascade of bifurcations of period doubling were identified. In research map of dynamic regimes, phase portraits, Poincare's sections, distributions of spectral density of regular and chaotic attractors of the system were constructed.

INTRODUCTION

Pendulum systems throughout centuries constantly draw to themselves attention of researchers in various areas of mathematics, mechanics and physics. Recently pendulum models have started to be applied widely at research of dynamic behavior of oscillating systems of the diversified nature in biology, medicine, economy, sociology etc. Problems of global power savings have made especially actual researches of pendulum systems with limited excitations. In such systems it is in essence supposed that the power of source of excitation of oscillations comparable with power consumed by oscillating system. This case is non-ideal for Sommerfeld and Kononenko [1]. It was established later that feedback influence of pendulum oscillations on a mechanism of its excitation leads to chaotic regimes in coupled system [2-4].

In present work we continue previous researches of pendulum system with a limited power-supply [2- 4]. Our main purpose is to investigate new aspects of its chaotic dynamics.

1. DESCRIPTION OF THE SYSTEM AND ITS MATHEMATICAL MODEL

We consider the two-degree-of-freedom pendulum when the point of support is vibrated by an electromotor with a limited power-supply (fig. 1). The mathematical model which takes into account non-ideal of excitation is built in [3, 4]. The mathematical model can be written as

$$
\begin{aligned}
\dot{y}_1 &= Cy_1 - \left[y_3 + \frac{1}{8}\left(y_1^2 + y_2^2 + y_4^2 + y_5^2\right)\right] y_2 - \frac{3}{4}(y_1 y_5 - y_2 y_4) y_4 + 2y_2 \\
\dot{y}_2 &= Cy_2 + \left[y_3 + \frac{1}{8}\left(y_1^2 + y_2^2 + y_4^2 + y_5^2\right)\right] y_1 - \frac{3}{4}(y_1 y_5 - y_2 y_4) y_5 + 2y_1 \\
\dot{y}_3 &= D(y_1 y_2 + y_4 y_5) + Ey_3 + F \\
\dot{y}_4 &= Cy_4 - \left[y_3 + \frac{1}{8}\left(y_1^2 + y_2^2 + y_4^2 + y_5^2\right)\right] y_5 + \frac{3}{4}(y_1 y_5 - y_2 y_4) y_1 + 2y_5 \\
\dot{y}_5 &= Cy_5 + \left[y_3 + \frac{1}{8}\left(y_1^2 + y_2^2 + y_4^2 + y_5^2\right)\right] y_4 + \frac{3}{4}(y_1 y_5 - y_2 y_4) y_2 + 2y_4
\end{aligned}
\tag{1}
$$

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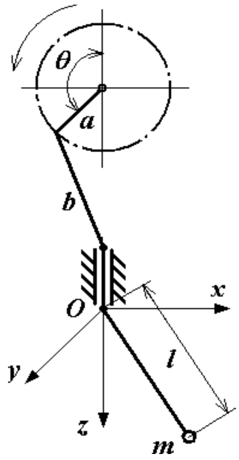


Fig. 1 System "pendulum – electromotor"

It is assumed that conditions of basic parametric resonance are realized, when the speed of the engine shaft is close to double own frequency of the pendulum. There variables y_1, y_2 and y_4, y_5 determine angles of pendulum deviation from coordinate plane *XZ* and *YZ* (fig. 1), variable y_3 determine rotation speed of electromotor shaft.

The system of equations (1) obviously has four control parameters: *C, D, E* and *F* that determined through electrical and mechanical characteristics of the investigated system. Parameter *E* directly determined angle of motor static characteristics, the parameter *C* is proportional to the resistance of environment. *D* and *F* are multi-parameters of dynamical system (1). They depend on the length and mass of pendulum, its own frequency and coefficient of damping, linear dimensions of connecting rod mechanism and the moment of inertia of the rotor and also on the parameters of the static characteristics of an electromotor. Such mathematical model allows specifying the existence of deterministic chaos in investigated system and the main effects of nonlinear interaction between pendulum and electromotor [3, 4].

Since the mathematical model (1) of the system "spherical pendulum–electromotor" is nonlinear with the dimension of phase space equal to five, so complex of numerical methods is used in the research of regular and chaotic regimes. For computer implementation of these methods was developed a specialized package of software modules. In this complex there are such methods as Runge-Kutta methods, algorithm of Benettin, Galgani and others, Henon's methods and so on. General methodology for research of nonlinear dynamics of oscillations systems defined in [4].

2. RESEARCH OF DYNAMICS REGIMES

To observe for the nontrivial evolution of attractors and accordingly regular and chaotic regimes of the system at variation of parameters map of dynamic regimes was constructed (fig. 2). Algorithm of map constructing is based on practical criteria for the existence of deterministic chaos. It consists in diagnostic of regimes of interaction established between the pendulum and electromotor on a set of values of bifurcation parameters [4].

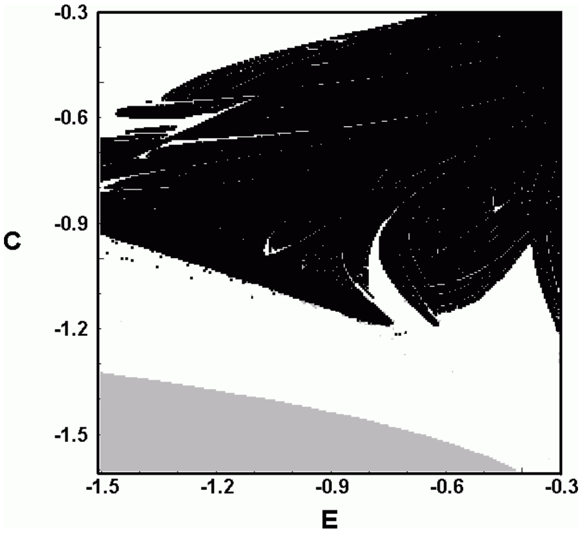


Fig. 2 Map of dynamic regimes

Shown in fig. 2 map of dynamic regimes is obtained due to analysis and data processing of computer experiments. It is built relative to the parameters *C* and *E*. Correspondingly the parameters *D* and *F* are assumed equal to -1 and 0.5. Initial conditions are varied in the neighborhood of origin of coordinates of phase space. There are areas of three different types of dynamic regimes in the fig. 2. In gray marked regions of parameters values equilibrium positions arise in the system. Signature of the spectrum of Lyapunov's characteristic exponents (LCE) in this case will look like $\langle -,-,-,-,- \rangle$. In white marked regions the system "spherical pendulum – electromotor" has limit cycles with signature of spectrum LCE $\langle 0,-,-,-,- \rangle$. In black marked regions of parameters values chaotic attractors arise in the phase space of the system. Signature LCE in this case will look like $\langle +,0,-,-,- \rangle$. As seen from the fig. 2, the black areas of the map have white inclusion, so-called windows of periodicity.

In previous researches of the system "spherical pendulum – electromotor" realization of two main types of scenarios of transition from regular to chaotic regimes were established. These are transition to chaos through intermittency of Pomeau-Manneville [5] and through cascade of bifurcation of period-doubling or Feigenbaum's scenario [6]. For second scenario new specificities of its realization were revealed. Thus, let's analyze the characteristics for last mentioned scenario.

For this purpose let's make a cross-section of the map (fig. 2) at $E = -0.5$ and consider the bifurcations occurring in the system at the change of the parameter C. In fig. 3(a) the dependence on the maximal distinct from zero characteristic exponent of the parameter C are shown. As is known, the main practical criterion for the existence of deterministic chaos in the system is the presence in the spectrum of LCE at least one positive exponent [4]. In fig. 3(a) is shown that there is wide region of chaotic regimes where maximum characteristic exponent have positive value. These regions correspond to black areas of the map (fig. 2).

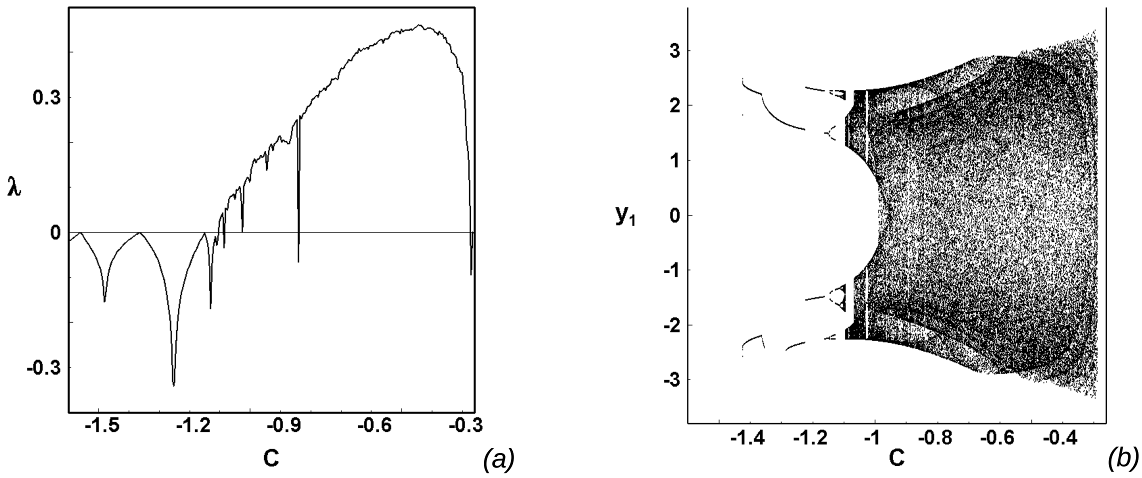


Fig. 3 The dependence on the maximum distinct from zero characteristic exponents of the parameter C (a); Bifurcation tree at E = −0.5 (b)

In fig. 3(b) the bifurcation tree of the system is shown. Close study of fig. 3(b) allows determining areas of existence of regular and chaotic regimes. The light sites of "crown" of this tree correspond to periodic regimes of the steady state oscillations of the system, and densely blacked out – to chaotic. Points of a bifurcation, at which transition from regular periodic regime to the non-regular chaotic one occurs, are precisely visible. So, as can be seen from the fig. 3(b) there is cascade of bifurcation of period-doubling in the interval $C \in (-1.3, -1.1)$. In this interval current tree has specificity structure. After each bifurcation tree branches break off and appear in another area. Let's analyze the system dynamics in this case.

At $C = -1.21$ there is stable limit cycle in the system. Its phase portrait is shown in fig. 4(a). The signature of spectrum LCE of this cycle looks like $\langle 0,-,-,-,- \rangle$. In the system "spherical pendulum – electromotor" regular regime of interaction is fixed in which the pendulum has periodic oscillations.

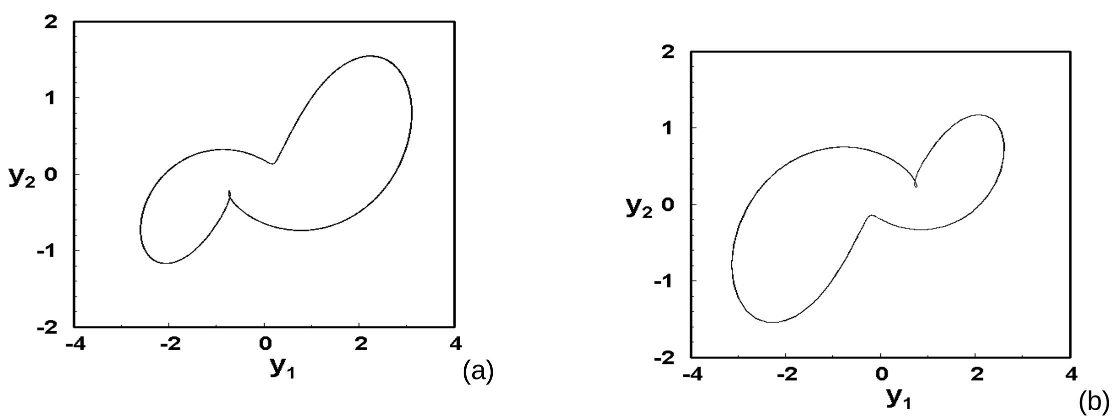


Fig. 4 Projections of phase portrait of limit cycles at C = −1.21 (a), at C = −1.2 (b)

At reducing the absolute value for the parameter C another stable limit cycle appears in the system. Arisen cycle is symmetrical for previous one and has the same period. The projection of phase portrait of such cycle at $C = -1.2$ is built in fig. 4(b).

In this case we should consider the system oscillations by the temporary realization, for example y_2. In fig. 5 temporary realizations of the system at $C = -1.21$ and $C = -1.2$ combined by phase are shown. These realizations are built after transition process of the system is passed. Limit cycle at $C = -1.21$ (fig. 4(a)) corresponds to black dot line in fig. 5. Another limit cycle of the system at $C = -1.2$ corresponds to gray line in fig. 5. As can be seen from fig. 5 the inverse of oscillations occurs in the system at such changes of dynamic regimes. Thus, arisen at $C = -1.2$ limit cycle we will call inverted cycle relative to previous limit cycle. And such property of rotation of a phase portrait of a cycle we will name inversion.

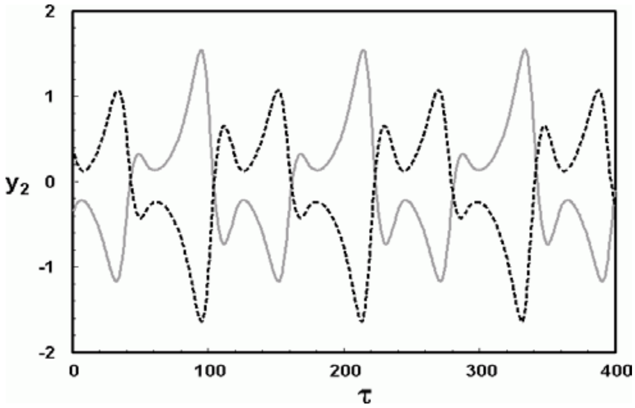


Fig. 5. Temporary realizations of the system at C = −1.21 and C = −1.2

As a result of the period doubling bifurcation of inverted cycle a new limit cycle arises in the system at $C = -1.125$. Projection of its phase portrait is built in fig. 6(a). At reducing the absolute value for the parameter C the inversion of limit cycles is taken place again. The inverted limit cycle of the same period arises in the system (fig. 6(b)). At the further increase of value C the next bifurcation of period doubling is taken place. After this bifurcation the limit cycle presented in fig. 6(c) arises in the system. Then at the further increase of value C after second bifurcation new inversion of limit cycle is taken place. The inverted limit cycle (fig. 6(d)) arises.

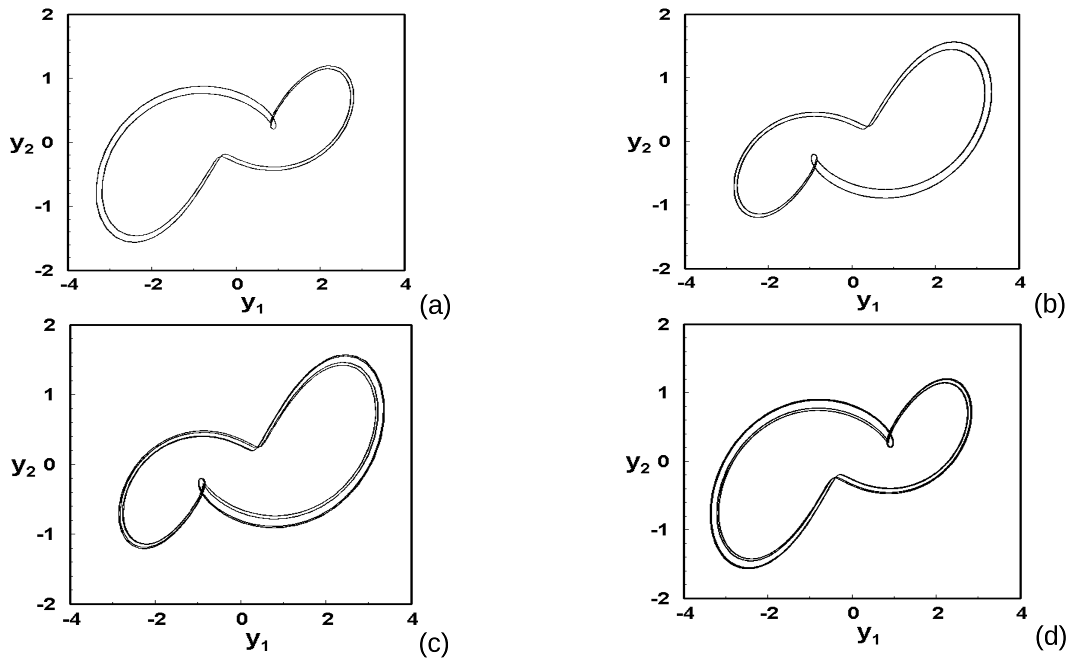


Fig. 6 Projections of phase portrait of limit cycles at C = −1.125 (a), at C = −1.115 (b), at C = −1.110 (c), at C = −1.108 (d)

The cascade of bifurcations of period doubling with inversion repeats infinite number of times. An end result of such process is origin in system of a chaotic attractor at $C \approx -1.1$. The projection of phase portrait of arisen chaotic attractor is built in fig. 7(a). In this case signature of spectrum LCE will look like $\langle +,0,-,-,- \rangle$. An important feature of this cascade is preservation of attractor inversions in chaotic region. Really, arisen at $C = -1.08$ chaotic attractor (fig. 7(b)) is inverted relative to chaotic attractor presented in fig. 7(a).

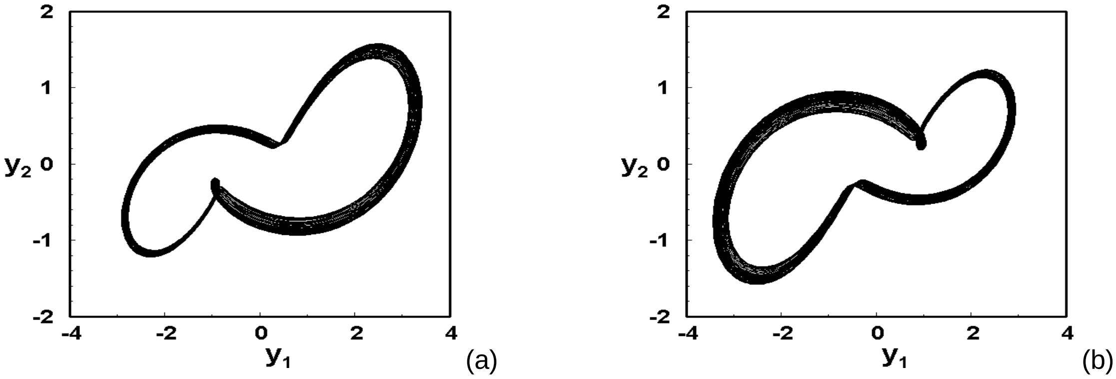


Fig. 7 Projections of phase portraits of chaotic attractors at C = −1.09 (a) and at C = −1.08 (b)

After each period-doubling bifurcation in the Fourier-spectrum appears sub-harmonic component exactly midway between the main harmonics of previous period (fig. 8). In the critical point a reverse process of destruction of sub-harmonic components is beginning and intervals of continuous spectrum are occurring.

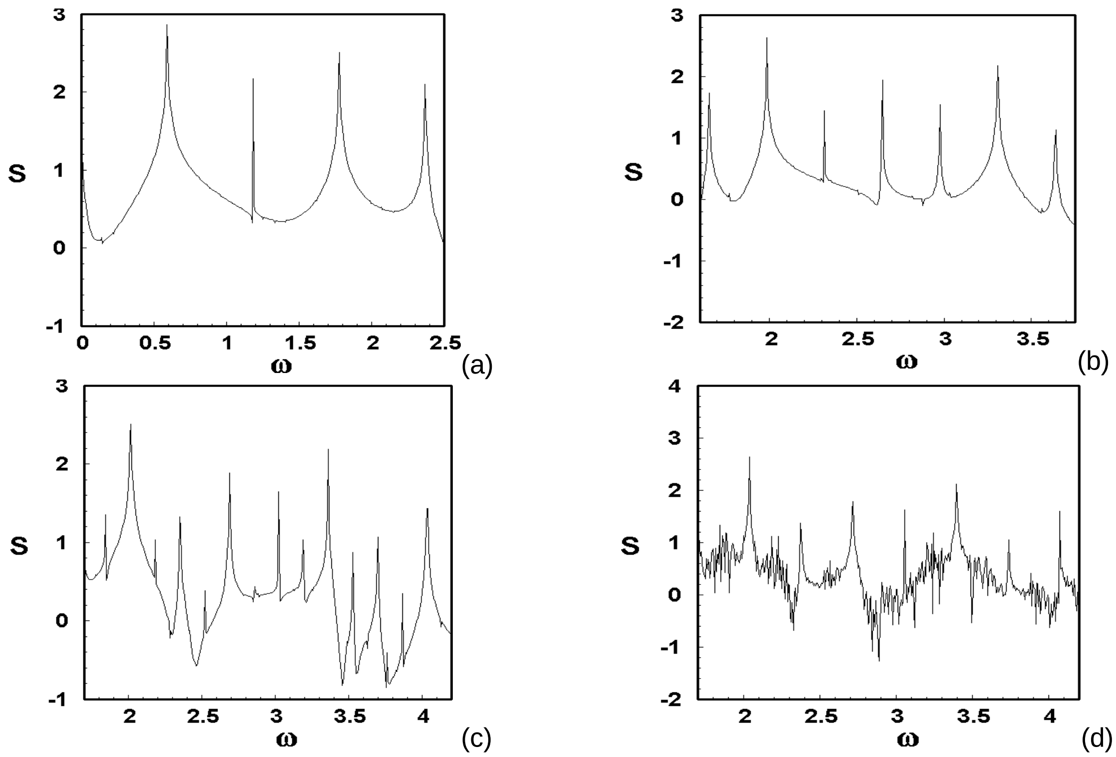
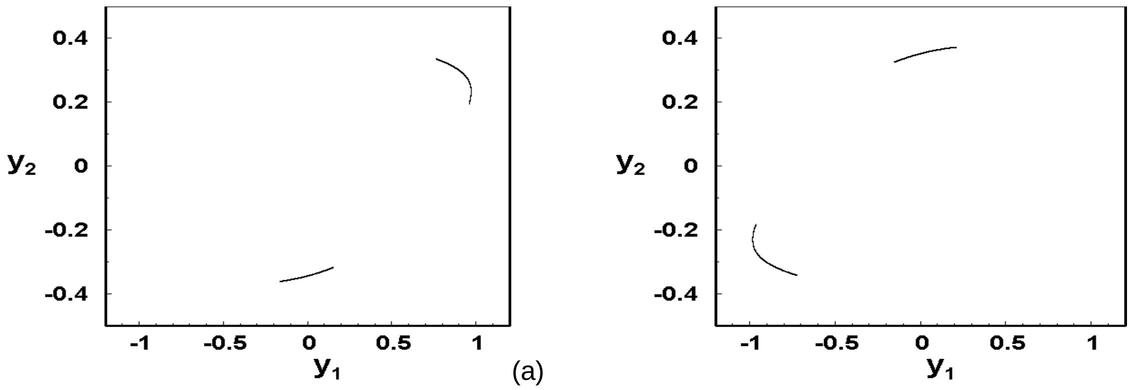


Fig. 8 Distributions of spectral densities of limit cycles at C = −1.125 (a), at C = −1.115 (b), at C = −1.110 (c), at C = −1.108 (d)

Let's consider Poincare sections of arisen chaotic attractors. Projection of Poincare section of chaotic attractor at $C=-1.09$ by the plane $y_3 = -2$ is built in fig. 9(a). This section represents some chaotic set which number of points increases with increasing time of numeric integration of system. As arisen chaotic attractor at $C = -1.08$ rotates with another inverted chaotic attractor, the same its Poincare section rotates with inverted section at $C=-1.08$ (fig. 9(b)).



(a) (b)

Fig. 9 Projections of Poincare section of chaotic attractors at C = −1.09 (a) and at C = −1.08 (b)

CONCLUSIONS

In current paper we obtain new aspects of chaotic dynamics of the system "spherical pendulum – electromotor". Constructed map of dynamic regimes shows existence of regular and chaotic attractors in the system. Herewith, chaotic regimes are not unusual. They occupy largest area of system parameters in such map.

New peculiarity of scenario of transition to chaos through cascade of bifurcations of period doubling was identified. This peculiarity consists in the rotation of limit cycles with inverted to them after each bifurcation of period doubling. Also such process is preserved after origin of chaotic regimes in the system when arisen chaotic attractor rotates with inverted one. This peculiarity is traced in bifurcation tree, phase portraits, temporary realizations and Poincare section of regular and chaotic attractors.

Thus, received in work results extend previous researches and in aggregate with results of works [3, 4] expose a great variety of chaotic behavior of the system "spherical pendulum – electromotor".

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