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NONLINEAR DYNAMICS OF PLATES AND SHALLOW SHELLS INTERACTING WITH MOVING FLUID

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ABSTRACT

Self-sustained vibrations of plates at two-sided interaction with moving fluid are considered. Fluid-structure interaction is described by a hyper singular integral equation, which is solved by Galerkin method. The plate performs geometrical nonlinear vibrations, which is described by finite-degree-of-freedom nonlinear dynamical system. Nonlinear modes are developed to analyze the self-sustained vibrations.

INTRODUCTION

Interaction of thin-walled structures with moving fluid or gas takes place in marine engineering, energetic and aerospace engineering. For example, dynamic stability of ship hydrofoil and dynamics of propeller are encountered in engineering. Many efforts were made to analyze interaction of thin-walled structures with fluid and gas flow. Aero elasticity of plates, shallow and cylindrical shells is treated in the book [1]. Dowell [2] considered the dynamics of one-dimensional structure in the flow, which is described by linear piston theory. Galerkin method is used to derive finite-degree-of-freedom model. Bolotin, Grishko et. al. [3] is considered the elastic plate in the flow with supersonic speed. Many-valued steady states in the finite-degree-of-freedom model are analyzed by the direct numerical integration. Bolotin, Petrovsky et. al. [4] are studied the motions of panel in the region of divergence and flutter instabilities. It is shown [10] that six eigenmodes are enough for adequate simulation of the plate under the action of constant load in a flow. However, for some values of the system parameters, the number of modes for flutter description is equal to 30. Tang, Dowell [20] are analyzed the plate in subsonic flow. It is assumed that the flow is potential. Vortex lattice method is used to describe a fluid-structure interaction. Aero elastic instability of plate in subsonic flow is analyzed in the paper [21]. 2D, incompressible flow is considered; the pressure acting on the plate is described by linear hyper singular integral equation. The vibrations of aerodynamic surface are described by two-degree-of-freedom system in the paper [22]. The action of incompressible flow on vibrating surface is described by the lifting force and moments. Dynamics of the system is described by two nonlinear integro- differential equations.

In this paper moving fluid interacting with a plate is considered; self-sustained vibrations of the plate with geometrical nonlinearity are analyzed. The interaction of a fluid with a plate is described by the hyper singular integral equation, which is solved by Galerkin method. Self-sustained vibrations of a plate are described by finite-degree-of-freedom nonlinear dynamical system. Variant of Shaw-Pierre nonlinear modes is suggested to analyze self-sustained vibrations. Using this approach, the flutter of plate is analyzed.

1. PROBLEM FORMULATION

Dynamics of simply supported plate in the flow of incompressible potential fluid is considered. The flow at a distance from the plate has constant velocity V (Fig.1). The fluid dynamics is described by velocity potential $\varphi(x,y,z,t)$. Lateral displacements of the plate are denoted by $w(x,y,t)$. As normal component of the plate velocities is equal to the normal component of fluid velocities, the following relations are true:

$$\left.\frac{\partial\varphi}{\partial z}\right|_{z=w+0}=V\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t};\left.\frac{\partial\varphi}{\partial z}\right|_{z=w-0}=V\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}. \quad (1)$$

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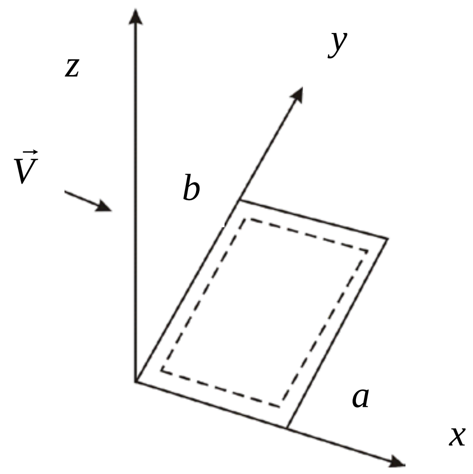

Fig. 1 Sketch of mechanical system

Using the Bernoulli's equation, the pressure acting on the plate is obtained in the following form:

$$\frac{p_+ - p_-}{\rho_w} = \frac{\partial(\varphi_- - \varphi_+)}{\partial t} + V\frac{\partial(\varphi_- - \varphi_+)}{\partial x}, \tag{2}$$

where p_+, p_- are fluid pressure acting on upper and lower sides of the plate; $\varphi_+; \varphi_-$ are values of velocities potentials on upper and lower sides of the plate; ρ_w is fluid density. Kutta's hypothesis on the plate edges is used in the following form [18, 32]: $p_+ \to p_-$. The function $\varphi(x, y, z, t)$ is presented as double-layer potential:

$$\varphi(x, y, z, t) = \frac{1}{4\pi}\int_S \gamma(\xi, t)\frac{\partial}{\partial n_\xi}\frac{1}{\sqrt{(x-\xi_1)^2 + (y-\xi_2)^2 + (z-\xi_3)^2}}dS\,, \tag{3}$$

where n_ξ is a unit vector of normal to the plate surface; $\gamma(\xi, t) = \varphi_+ - \varphi_-$ is a circulation of a velocity. The equation (3) is substituted into (1); as a result the following hyper singular integral equation is obtained:

$$V\frac{\partial w}{\partial x} + \frac{\partial w}{\partial t} = \frac{1}{4\pi}\int_S \gamma(\xi, t)\ \frac{\partial^2}{\partial z \partial n_\xi}\left(\frac{1}{\sqrt{(x-\xi_1)^2 + (y-\xi_2)^2 + (z-\xi_3)^2}}\right)dS\,. \tag{4}$$

The review of the results concerning the applications of the singular integral equations to aero elasticity is presented in [36].

If flutter occurs, the plate performs geometrical nonlinear vibrations; this nonlinearity limits the vibrations amplitudes. As thin plates are considered, shear and rotational inertia are not taken into account. Therefore, the plate vibrations are described by von Karman equations:

$$\frac{h^2}{12}\nabla^4 w + \frac{(1-\mu^2)\rho_P}{E}\ddot{w} + \frac{(1-\mu^2)\rho_W}{Eh}(\dot{\gamma} + V\gamma'_X) = \frac{(1-\mu^2)}{Eh}\left(F''_{YY}w''_{XX} - 2F''_{XY}w''_{XY} + F''_{XX}w''_{YY}\right); \tag{5}$$

$$\frac{1}{Eh}\nabla^4 F = (w''_{XY})^2 - w''_{XX}w''_{YY}\,, \tag{6}$$

where F is Airy stress function; h is plate thickness; ρ_p is a density of the plate material; E, μ are Young's modulus and Poisson's ration.

2. FINITE DEGREE-OF-FREEDOM MODEL OF PLATE VIBRATIONS

The circulation of velocities is presented as a series in terms of eigenmodes of simply supported plate:

$$\gamma\left(\xi_1,\xi_2,t\right)=\sum_{l=1}^{N_1}\sum_{m=1}^{N_1}C_{l\,m}(t)\sin\left(\frac{l\pi\,\xi_1}{a}\right)\sin\left(\frac{m\pi\,\xi_2}{b}\right) \tag{7}$$

the lateral displacements of the plate w are the following:

$$w\left(x,y,t\right)=\sum_{r_1=1}^{N_s}\sum_{r_2=1}^{N_s}\theta_{r_1r_2}(t)\sin\left(\frac{r_1\pi\,x}{a}\right)\sin\left(\frac{r_2\pi\,y}{b}\right) \tag{8}$$

The relations (7, 8) are substituted into the singular integral equation (4); the Galerkin method is used. As a result the following system of linear algebraic equations with respect to $C_{l\,m}(t)$ is derived:

$$\sum_{l=1}^{N_1}\sum_{m=1}^{N_1}a_{n_1n_2lm}C_{lm}(t)=b_{n_1n_2}\;;\,n_1=1,...,N_1;\,n_2=1,...,N_1 \tag{9}$$

where

$$a_{n_1n_2lm}=\frac{1}{4\pi}\int_S\sin\left(\frac{n_1\pi\,x}{a}\right)\sin\left(\frac{n_2\pi\,y}{b}\right)dxdy\int_S\frac{\sin\left(\frac{l\pi\xi_1}{a}\right)\sin\left(\frac{m\pi\xi_2}{b}\right)d\,\xi_1d\,\xi_2}{\left[\left(x-\xi_1\right)^2+\left(y-\xi_2\right)^2\right]^{3/2}}$$

$$b_{n_1n_2}=\int_S\left(V\frac{\partial w}{\partial x}+\frac{\partial w}{\partial t}\right)\sin\left(\frac{n_1\pi\,x}{a}\right)\sin\left(\frac{n_2\pi\,y}{b}\right)dx\,dy=0.25bV\sum_{r_1=1}^{N_s}\sum_{r_2=1}^{N_s}\theta_{r_1r_2}(t)r_1\delta_{r_2n_2}\vartheta_{n_1n_2r_1}+$$

$$+0.25ab\sum_{r_1=1}^{N_s}\sum_{r_2=1}^{N_s}\dot{\theta}_{r_1r_2}(t)\delta_{r_1n_1}\delta_{r_2n_2}$$

$\delta_{r_2n_2}$ is Kronecker delta; $\vartheta_{n_1n_2r_1}=\frac{1-\delta_{n_1r_1}}{n_1-r_1}\left[1-(-1)^{n_1-r_1}\right]+\frac{1-(-1)^{n_1+r_1}}{n_1+r_1}$. The solution of the system (9) is presented as:

$$C_{lm}=\sum_{r_1=1}^{N_s}\sum_{r_2=1}^{N_s}C_{l\,m}^{\left(r_1r_2\right)}(t)\,;\left(l,m\right)=1,...,N_1 \tag{10}$$

The equation (10) is substituted into (9); the systems of linear algebraic equations are derived. The solutions of these systems are the following:

$$C_{l,m}^{(r_1r_2)}=0.25Vb\theta_{r_1r_2}(t)\overline{\varphi}_{l,m}^{(r_1r_2)}+0.25ab\dot{\theta}_{r_1r_2}(t)\overline{\overline{\varphi}}_{l,m}^{(r_1r_2)}\,;\left(l,m\right)=1,...,N_1;\left(r_1,r_2\right)=1,...,N_s \tag{11}$$

The parameters $\overline{\varphi}_{l,m}^{\left(r_1r_2\right)}$, $\overline{\overline{\varphi}}_{l,m}^{\left(r_1r_2\right)}$ are solutions of the following systems of linear algebraic equations:

$$\sum_{l=1}^{N_1}\sum_{m=1}^{N_1}a_{n_1n_2lm}\overline{\varphi}_{l,m}^{(r_1r_2)}=r_1\delta_{r_2n_2}\vartheta_{n_1n_2r_1} \tag{12}$$

$$\sum_{l=1}^{N_1}\sum_{m=1}^{N_1}a_{n_1n_2lm}\overline{\overline{\varphi}}_{l,m}^{(r_1r_2)}=\delta_{r_1n_1}\delta_{r_2n_2}\,;\,n_1,n_2=1,...,N_1;\,r_1,r_2=1,...,N_s \tag{13}$$

The finite-degree-of-freedom model of plate geometrical nonlinear vibrations is derived. The equation (8) is substituted into (6); the linear non homogeneous partial differential equation is derived. The solution of this equation can be presented as:

$$F = F_p + F_g \tag{14}$$

where F_p is partial solution of nonhomogeneous equation; F_g is general solution of homogeneous equation. Partial solution of nonhomogeneous equation has the following form:

$$0.5\,F_p = \sum_{r_1,r_2,p_1,p_2=1}^{N_S} \theta_{r_1 r_2}\theta_{p_1 p_2}\left[A^{(1)}_{r_1 r_2 p_1 p_2}\cos\eta(r_2-p_2)\cos\xi(r_1+p_1) + A^{(2)}_{r_1 r_2 p_1 p_2}\cos\eta(r_2+p_2)\cos\xi(r_1-p_1)\right] +$$
$$+\sum_{\substack{r_1,r_2,p_1,p_2=1\\ r_1\neq r_2\ u\ p_1\neq p_2\\ r_1\neq p_1\ u\ r_2\neq p_2}}^{N_S} \theta_{r_1 r_2}\theta_{p_1 p_2}A^{(3)}_{r_1 r_2 p_1 p_2}\cos\eta(r_2-p_2)\cos\xi(p_1-r_1) + \sum_{\substack{r_1,r_2,p_1,p_2=1\\ r_1\neq r_2\ u\ p_1\neq p_2}}^{N_S} \theta_{r_1 r_2}\theta_{p_1 p_2}A^{(4)}_{r_1 r_2 p_1 p_2}\cos\eta(r_2+p_2)\cos\xi(p_1+r_1). \tag{15}$$

The general solution of the homogeneous equation is equal to zero $F_g = 0$.

Now the solution (15) is substituted into the equation (5); the Galerkin method is applied. As a result the following dynamical system is obtained:

$$\sum_{l,m=1}^{N_S}\left(M_{n_1 n_2 l m}\ddot{\theta}_{lm} + D_{n_1 n_2 l m}\dot{\theta}_{lm} + K_{n_1 n_2 l m}\theta_{lm}\right) + R_{n_1 n_2}\left(\theta_{1,1},\theta_{1,2},\ldots\right) = 0;\ n_1, n_2 = 1,\ldots,N_S, \tag{16}$$

where

$$R_{n_1 n_2}\left(\theta_{1,1},\theta_{1,2},\ldots\right) = -\int_S \left(F''_{YY}w''_{XX} - 2F''_{XY}w''_{XY} + F''_{XX}w''_{YY}\right)\sin\left(\frac{n_1\pi x}{a}\right)\sin\left(\frac{n_2\pi y}{b}\right)dxdy =$$
$$\sum_{r_1,r_2,p_1,p_2,l,m=1}^{N_S} \alpha^{(n_1,n_2)}_{lmr_1r_2p_1p_2}\theta_{lm}\theta_{r_1r_2}\theta_{p_1p_2}$$

3. APPLICATION OF NONLINEAR MODES FOR SELF-SUSTAINED VIBRATIONS ANALYSIS

The general approach for nonlinear modes of self-sustained vibrations analysis is suggested. In the next section these nonlinear modes are used for analysis of the plate self-sustained vibrations. The nonlinear dynamical system (16) can be presented in the following matrix form:

$$\ddot{\eta} + A\dot{\eta} + B\eta = f(\eta,\dot{\eta}) \quad f_j = \sum_{l,r,p=1}^{N} G^{(j)}_{lrp}\eta_l\eta_r\eta_p;\ j = 1,\ldots,N \tag{17}$$

where $\eta = \{\eta_1,\eta_2,\ldots,\eta_N\}$; $f = \{f_1,\ldots,f_N\}$; $A = \{\alpha_{kj}\}$; $B = \{\beta_{kj}\}$. It is assumed, that the trivial equilibrium $\eta = 0$ undergoes Hopf bifurcation and the self-sustained vibrations appear. These self-sustained vibrations are presented as the Shaw-Pierre nonlinear modes:

$$\eta_j = \overline{R}_j = a_{j1}\eta_k + a_{j2}\dot{\eta}_k + R_j(\eta_k,\dot{\eta}_k);\ \dot{\eta}_j = \overline{F}_j = a_{N+j,1}\eta_k + a_{N+j,2}\dot{\eta}_k + F_j(\eta_k,\dot{\eta}_k)$$
$$j = 1,\ldots,k-1,k+1,\ldots,N \tag{18}$$

where $a_{j1}; a_{j2}; a_{N+j,1}; a_{N+j,2}$ are unknown coefficients. The variables $(\eta_k,\dot{\eta}_k)$ are chosen as master coordinates. The nonlinear functions R_j; F_j are presented in the following form:

$$R_j(\eta_k,\dot{\eta}_k) = \delta^{(j)}_1\eta^3_k + \delta^{(j)}_2\eta^2_k\dot{\eta}_k + \delta^{(j)}_3\eta_k\dot{\eta}^2_k + \delta^{(j)}_4\dot{\eta}^3_k + \ldots$$
$$F_j(\eta_k,\dot{\eta}_k) = \varepsilon^{(j)}_1\eta^3_k + \varepsilon^{(j)}_2\eta^2_k\dot{\eta}_k + \varepsilon^{(j)}_3\eta_k\dot{\eta}^2_k + \varepsilon^{(j)}_4\dot{\eta}^3_k + \ldots;\ j = 1,\ldots,k-1,k+1,\ldots,N \tag{19}$$

The coefficients $a_{j,1}; a_{j,2}; a_{N+j,1}; a_{N+j,2}$ of linear part of the nonlinear mode (18) are determined. The linear part of the system (18) is considered, which can be presented as

$$\dot{z} = \Gamma z = \begin{bmatrix} 0 & E \\ -B & -A \end{bmatrix} z , \tag{20}$$

where $z = [z_1,..., z_{2N}] = [\eta, \dot{\eta}]$. The solution of the system (20) is the following:

$$z = \sum_{j=1}^{N} \left[\Theta_{2j} W_{2j} \exp(\lambda_{2j} t) + \Theta_{2j-1} W_{2j-1} \exp(\lambda_{2j-1} t) \right], \tag{21}$$

where λ_i, W_i are eigenvalues and eigenvectors of the matrix Γ; $\lambda_{2j} = \overline{\lambda}_{2j-1}; W_{2j} = \overline{W}_{2j-1}; \Theta_{2j} = \overline{\Theta}_{2j-1}$ are constants of integration. If a pair of eigenvalues of the matrix Γ takes a form: $\lambda_{1,2} = \pm i \chi_1$, the self- sustained vibrations appear. The solution of the system (20) on the central manifold is presented in the following form:

$$z = \Theta_2 W_2 \exp(\lambda_2 t) + \Theta_1 W_1 \exp(\lambda_1 t) \tag{22}$$

where $W_1 = \gamma_1 - i\,\delta_1$; $\gamma_1 = \left[\gamma_1^{(1)};...;\gamma_1^{(2N)}\right]$; $\delta_1 = \left[\delta_1^{(1)};...;\delta_1^{(2N)}\right]$; $\Theta_1 = K_1^{(1)} - i\,K_1^{(2)}$; $\lambda_1 = \alpha_1 - i\,\psi_1$. Two elements $\eta_k, \dot{\eta}_k$ of the vector z are presented in the following form:

$$\eta_k = \gamma_1^{(k)} \vartheta_1(t) + \delta_1^{(k)} \vartheta_2(t)\,;\ \dot{\eta}_k = \gamma_1^{(N+k)} \vartheta_1(t) + \delta_1^{(N+k)} \vartheta_2(t) \tag{23}$$

where $\vartheta_1(t) = 2\exp(\alpha_1 t)\left[K_1^{(1)} \cos\psi_1 t - K_1^{(2)} \sin\psi_1 t\right]$; $\vartheta_2(t) = -2\exp(\alpha_1 t)\left[K_1^{(1)} \sin\psi_1 t + K_1^{(2)} \cos\psi_1 t\right]$.
The rest elements of the solutions (23) are the following:

$$\eta_i = \gamma_1^{(i)} \vartheta_1(t) + \delta_1^{(i)} \vartheta_2(t)\,;\ \dot{\eta}_i = \gamma_1^{(N+i)} \vartheta_1(t) + \delta_1^{(N+i)} \vartheta_2(t)\,;\ i = 1,..., k-1, k+1,..., N \tag{24}$$

Solving jointly the equations (24, 23), the coefficients of linear part of the nonlinear normal mode (18) is obtained in the form:

$$a_{i1} = \frac{\gamma_1^{(i)} \delta_1^{(N+k)} - \delta_1^{(i)} \gamma_1^{(N+k)}}{\gamma_1^{(k)} \delta_1^{(N+k)} - \delta_1^{(k)} \gamma_1^{(N+k)}}\,;\ a_{i2} = \frac{\gamma_1^{(i)} \delta_1^{(k)} - \delta_1^{(i)} \gamma_1^{(k)}}{\gamma_1^{(N+k)} \delta_1^{(k)} - \delta_1^{(N+k)} \gamma_1^{(k)}}$$
$$i = 1,..., k-1, k+1,..., N, N+1,..., N+k-1, N+k+1,..., 2N \tag{25}$$

In future analysis, the ordinary procedure for nonlinear normal mode calculations [38] is used.

4. NUMERICAL ANALYSIS OF VIBRATIONS

The dynamics of the plate in water flow is investigated for the following parameters:

$$E = 2 \cdot 10^{11}\ Pa\,;\ \rho_p = 7.8 \cdot 10^3\ \frac{kg}{m^3};\ \rho_W = 1 \cdot 10^3\ \frac{kg}{m^3};\ \nu = 0.3\,;\ h = 0.02\,m;\ a = b = 0.5\,m$$

The self-sustained vibrations are studied on the basis of finite degrees-of-freedom model (23). These vibrations start-up due to Hopf bifurcation and they analyzed by the nonlinear modes. At first, nonlinear mode is determined by solution of the system of linear algebraic equations; the motions on the mode are analyzed. The calculations are performed for different Mach numbers. The results of the calculations are presented on the bifurcation diagram (Fig. 2). Stable and unstable trivial equilibrium are shown by solid and dotted lines, respectively. Limit cycle start-up at Hopf bifurcation. Behavior of such self-sustained vibrations, when the Mach number is increased, is shown by solid lines on Fig. 2.

The direct numerical integration of the system (17) is performed to check the obtained self-sustained vibrations. Points on nonlinear mode are used as initial conditions. The results of the calculations are shown by rhombs on Fig.2. Thus, the results of the direct numerical integration are close to the data obtained by nonlinear modes.

CONCLUSIONS

Interaction of the vibrating plate with a fluid flow is analyzed in this paper. It is assumed that a fluid is incompressible, frictionless and irrotational; the model of fluid motions is linear. Fluid-plate interaction is described by the linear hyper singular integral equation. Galerkin method is used for approximate solution of this integral equation.

For analysis of self-sustained vibrations, geometrical nonlinearity includes in the model of plate vibrations. It limits the vibrations amplitudes in the region of trivial equilibria instability.

The generalization of the Shaw-Pierre nonlinear modes for self-sustained finite degree-of-freedom system vibrations is suggested in this paper. As nonlinear modes are determined in power series, the suggested approach is valid only for moderate vibrations amplitudes.

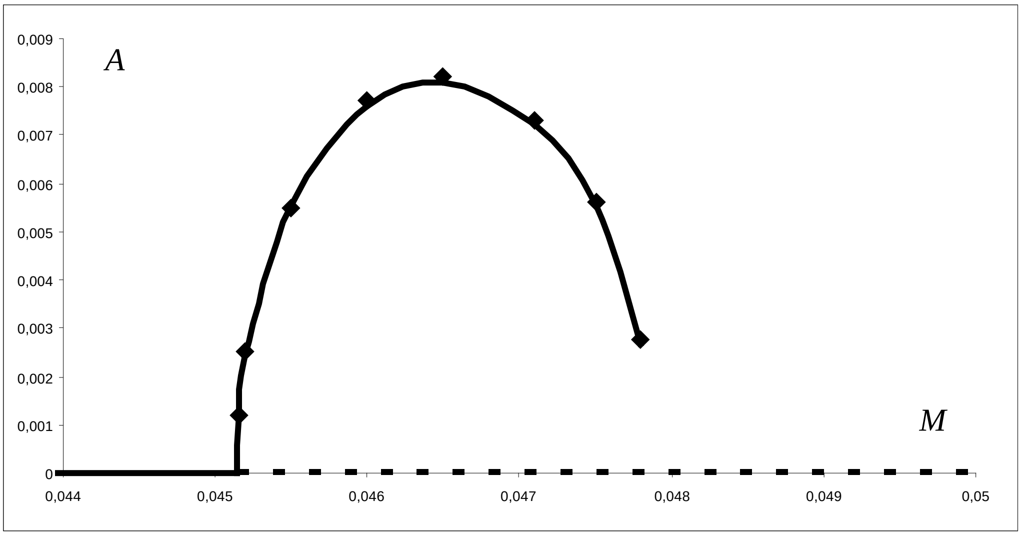

Fig.2 Bifurcation diagram of the system

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