# NONLINEAR DYNAMICS OF TRAVELING WAVES OF CIRCULAR PLATES WITH CUTOUTS 

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## ABSTRACT

Geometrically nonlinear vibrations of circular plate with two cutouts are simulated by the von Karman equations with respect to displacements. $R$-functions are applied to obtain the vibrations modes of this plate. The nonlinear vibrations of the plate are expanded using these vibrations modes. The nonlinear dynamical system with three degree-of-freedom is derived by the Galerkin method. This system is studied by the multiple scales method.

The circular plate with two cutouts (Fig.1) is considered. It is assumed, that the deformationsdisplacements relations are nonlinear and strains-deformations relations are linear. Vibrations are treated in cylindrical coordinates $(r, \theta, z)$. Then the displacements of the plates material points along $(r, \theta, z)$ are denoted by $u_{r}, u_{\theta}, u_{z}$, respectively.


Fig. 1 Circular plate with two cutouts

The Galerkin method is used to discretize the equations of plate vibrations. Then the nonlinear vibrations of plates with cutouts are expanded using eigenmodes of linear vibrations. The RayleighRitz method is used to obtain eigenmodes of vibrations. In order to satisfy the boundary conditions, the equation of the plate boundary (Fig.1) is obtained analytically. The R-function method is used to construct analytically this boundary.

The eigenmodes of circular plate with two cutouts can be presented in the following form:

$$
\bar{u}_{z}(r, \theta)=\omega^{2}(r, \theta) \sum_{k=0}^{m}\left[Z_{k}^{(c)}(r) \cos (k \theta)+Z_{k}^{(s)}(r) \sin (k \theta)\right] ;
$$

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$$
\begin{aligned}
& \bar{u}_{\theta}(r, \theta)=\omega(r, \theta) \sum_{k=0}^{m}\left[\Theta_{k}^{(c)}(r) \cos (k \theta)+\Theta_{k}^{(s)}(r) \sin (k \theta)\right] ; \\
& \bar{u}_{r}(r, \theta)=\omega(r, \theta) \sum_{k=0}^{m}\left[R_{k}^{(c)}(r) \cos (k \theta)+R_{k}^{(s)}(r) \sin (k \theta)\right] .
\end{aligned}
$$
\]

The nonlinear dynamics of plate is described by three degree-of-freedom nonlinear dynamical system, which can be presented in the following form:

$$
\ddot{q}_{k}+p_{k}^{2} q_{k}=\sum_{i=1}^{3} \sum_{l=1}^{3} \sum_{\mu=1}^{3} G_{i l \mu}^{(k)} q_{i} q_{l} q_{\mu} ; k=\overline{1,3}
$$

The multiple scales method is used to study this system.


Fig. 2 The backbone curves of two conjugate modes
As a result of calculations the backbone curve of traveling waves is shown on Fig.2. The complete analysis of this problem is published in the paper [1].

## REFERENCES

[1] Avramov K.V., Tyshkovets O., Maksymenko-Sheyko K.V. Analysis of nonlinear free vibration of circular plates with cut-outs using R-function method. ASME J. of Vibration and Acoustics, Vol. 132 (in press).


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