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NONLINEAR OSCILLATIONS OF TURBOMACHINERY BLADES

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ABSTRACT

The analysis of the free linear and geometrically nonlinear vibrations of the turbomachinery blades is presented in this paper. The results for the linear vibrations analysis are compared with the experimental ones. The vibrations in the case of multiple internal resonances are investigated. The analysis of nonlinear vibrations is carried out using the combination of nonlinear normal modes and the harmonic balance method.

INTRODUCTION

The turbomachinery blades are one of the most important real applications of the shallow shells theory. Therefore, a lot of studies dial with the blade dynamics. Didkovskii [1] analyzed the parametric vibrations of turbomachinery blades in gas flow. The sufficient conditions of dynamical stability are obtained in this paper. The papers of Ross [2], Hoa [3] are devoted to linear vibration analysis of blades, which are modeled by shells. Venkatsan, Nagaraj [4] studied nonlinear vibrations of rotating blades. They came to the conclusion, that the frequency response can be hard or soft. The data of finite element analysis of turbomachinery blades are compared with experimental results in [5]. The vibrations of turbine blades under the action of longitudinal time periodic force have been considered by Chen, Peng [6]. Using geometrically nonlinear theory and the finite element method, the blade nonlinear model is obtained. Liew, Lim [7] used energetic approach to study linear vibrations of shallow shells with rectangular base and different Gaussian curvature. Mohamed Nabi and Ganesan [8] compared the beam and plate models of turbomachinery blades. They came to the conclusion that the plate models are better. The vibrations of shallow anisotropic blades are treated by Abe, Kobayashi, Yamada [9]. They used the Rayleigh-Ritz method to analyze linear vibrations. The finite-degree-of-freedom model is obtained by Bubnov-Galerkin procedure. Nonlinear vibrations of hydroturbine blades, which are modeled by pre-twisted shell with variable thickness and ring-shaped base, are treated in [10, 11]. The dependence of eigenfrequencies and eigenmodes on pre-twisted angle and thickness are investigated. Choi, Chou [12] analyze the blade vibrations taking into account the shear. The influence of shroud on vibrations is considered.

In this paper the free geometrically nonlinear oscillations of the turbomachinery blades are analyzed. The blades are considered as shallow shells of variable thickness and double curvature. Compressor blade and blade of hydroturbine are studied. The R-function method and the Rayleigh-Ritz approach are used collectively to obtain eigenmodes of linear vibrations. Nonlinear vibrations of shells are approximated by using these eigenmodes. Single-mode and multimode vibrations are studied. The backbone curves are presented and the stability of motions is analyzed.

1. METHOD OF ANALYSIS

The blades are modeled by shallow shells with variable thickness and double curvature. Thin blades are considered, so classical Love theory is applied. The Rayleigh-Ritz method is used to determine the eigenfrequencies and eigenmodes of linear vibrations. The potential energy of the shell can be presented as [13]:

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$$\Pi = \frac{E}{2(1+\mu)}\int_{\Lambda}\left\{\frac{1}{1-\mu}\left[\left(\varepsilon_{11}^2 + 2\mu\varepsilon_{11}\varepsilon_{22} + \varepsilon_{22}^2\right)h(x,y) + \frac{h^3(x,y)}{12}\left(\chi_1^2 + 2\mu\chi_1\chi_2 + \chi_2^2\right)\right] + \right.$$
$$\left. + \frac{1}{2}\left[\varepsilon_{12}^2 h(x,y) + \frac{1}{3}\tau^2 h^3(x,y)\right]\right\} AB dx\, dy,$$

where $\varepsilon_1, \varepsilon_2, \gamma$ are components of membrane strains of shell middle surface; χ_1, χ_2, τ are components of bending deformations of middle surface; A and B are Lamé coefficients; $h(x,y)$ is a variable shell thickness; E, μ are Young's modulus and Poisson's ratio. The kinetic energy of the shell has the following form:

$$T = \frac{\rho}{2}\int_{\Lambda}\left(\dot{w}^2 + \dot{u}^2 + \dot{v}^2\right) h(x,y) AB\, dx\, dy,$$

where $u(x,y,t)$, $v(x,y,t)$, $w(x,y,t)$ are displacements of the middle surface points in the x, y, z directions, respectively; ρ is shell material density.

Nonlinear vibrations of the blade are approximated by eigenmodes of linear vibrations. Then the shell bending vibrations $w(x,y,t)$ can be presented as:

$$w(x,y,t) = \sum_{i=1}^{N}\eta_i(t)\overline{w}_i(x,y), \qquad (1)$$

where $\overline{w}_i(x,y), i = \overline{1,N}$ are normalized eigenmodes of free linear vibrations. The displacements u and v can be presented as:

$$u(x,y,t) = \sum_{i=1}^{N}\eta_{i+N}(t)\overline{u}_i(x,y), \; v(x,y,t) = \sum_{i=1}^{N}\eta_{i+2N}(t)\overline{v}_i(x,y), \qquad (2)$$

where $\overline{u}_i(x,y), i = \overline{1,N}$ and $\overline{v}_i(x,y), i = \overline{1,N}$ are in-plane eigenmodes of vibrations. Using the kinetic and the potential energy, the Lagrange equations are derived. If the eigenfrequencies of longitudinal vibrations are significantly higher, than the eigenfrequencies of the bending vibrations, it is possible to neglect the in-plane inertial terms. The dependences of $\eta_{N+1},...,\eta_{3N}$ on $\eta_1,...,\eta_N$ can be derived from the last $2N$ Lagrange equations. The solution of the linear algebraic equations is substituted into the system of first N ordinary differential equations. As a result the system of N ordinary differential equations with respect to $\eta_1,...,\eta_N$ is derived. After the transformation to the dimensionless modal variables $\xi_1,...\xi_N$ the system has following form:

$$\ddot{\xi}_k = f_k(\xi_1,...\xi_N) = -\overline{\Omega}_k^2\xi_k - \sum_{i=1}^{N}\sum_{j=1}^{N} l_{kij}\xi_i\,\xi_j - \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{g=1}^{N} l_{kijg}\xi_i\,\xi_j\xi_g, \quad k = \overline{1,N} \qquad (3)$$

where $\overline{\Omega}_k = \dfrac{\Omega_k}{\Omega_1}$ are dimensionless eigenfrequencies.

The Shaw-Pierre nonlinear modes [14, 15] are used to analyze the vibrations of the finite-degree-of-freedom system (30). Lat us suppose, that the general coordinates $\xi_1,...,\xi_M$ is active. Then the invariant manifolds of the system with internal resonances can be presented as [15]:

$$\xi_k = X_k\left(\xi_1,\dot{\xi}_1,..\xi_M,\dot{\xi}_M\right); \; \dot{\xi}_k = Y_k\left(\xi_1,\dot{\xi}_1,..\xi_M,\dot{\xi}_M\right), \; k = M+1,..2N \qquad (4)$$

Nonlinear modes (4) satisfy the following partial differential equations:

$$\begin{aligned}&\sum_{i=1}^{M}\frac{\partial X_k}{\partial \xi_i}\dot{\xi}_i+\sum_{i=1}^{M}\frac{\partial X_k}{\partial \dot{\xi}_i}f_i\left(\xi_1,..\xi_M,X_{M+1},..X_{2N}\right)=Y_k \\ &\sum_{i=1}^{M}\frac{\partial Y_k}{\partial \xi_i}\dot{\xi}_i+\sum_{i=1}^{M}\frac{\partial Y_k}{\partial \dot{\xi}_i}f_i\left(\xi_1,..\xi_M,X_{M+1},..X_{2N}\right)=f_k\left(\xi_1,..\xi_M,X_{M+1},..X_{2N}\right)\end{aligned} \qquad k=M+1,..2N \quad (5)$$

The functions X_k, Y_k can be found as polynomials with respect to $\xi_1,\dot{\xi}_1,..\xi_M,\dot{\xi}_M$. Now the equations (4) are substituted into the first M equations of the system (3). As a result, the system of M ordinary differential equations describing the motions on nonlinear mode is obtained:

$$\ddot{\xi}_k = f_k\left(\xi_1,..\xi_M, X_{M+1}\left(\xi_1,\dot{\xi}_1,..\xi_M,\dot{\xi}_M\right),...X_{2N}\left(\xi_1,\dot{\xi}_1,..\xi_M,\dot{\xi}_M\right)\right),\ k=1,..M$$

These equations can be studied by harmonic balance method, multiple scales or other methods. The stability of motions in this work is analyzed by the Floquet-Lyapunov theory [16].

2. VIBRATIONS OF THE COMPRESSOR BLADE

The blade is modeled by pre-twisted shallow shell with trapezoidal base and variable thickness (Fig 1.).

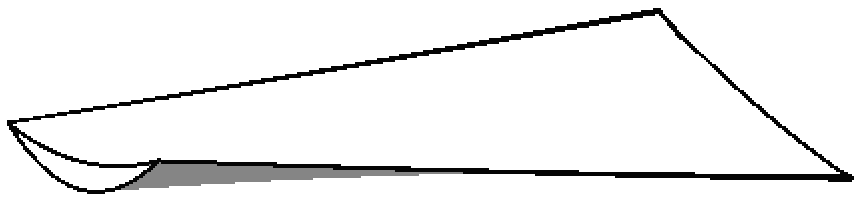

Fig. 1 Sketch of blade

The boundary conditions on the clamped edge can be presented as:

$$w\big|_{x=0}=\frac{\partial w}{\partial n}\bigg|_{x=0}=0,\ u\big|_{x=0}=0,\ v\big|_{x=0}=0$$

The node lines of the first eigenmodes are shown on the Figure 2. The results of the eigenmodes calculations (Fig. 2) are close to the data from [17, 18]. As follows from the calculations, the following internal resonances exist in the system: $\overline{\Omega}_4 \approx \overline{\Omega}_5 ; 2\overline{\Omega}_3 \approx \overline{\Omega}_4 ; 2\overline{\Omega}_3 \approx \overline{\Omega}_5$.

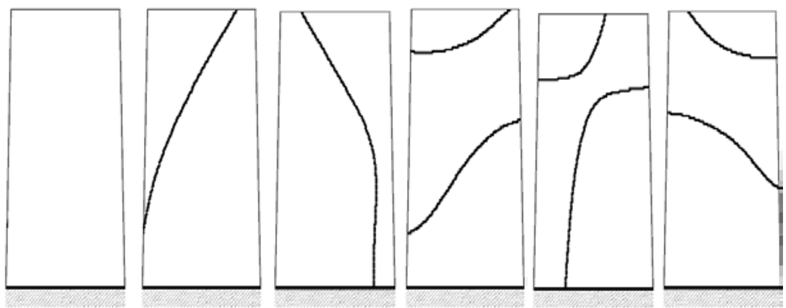

Fig. 2 Nodal lines vibrations eigenmodes

The first five eigenmodes are used in the expansions (1, 2) to obtain the finite degree-of-freedom model of the blade nonlinear vibrations. The nonlinear dynamics of this system is analyzed by nonlinear modes with ξ_3,ξ_4,ξ_5 as independent variables. Figures 3-4 show the backbone curves on this invariant manifold. Unstable motions are shown by dotted lines. The multimode (Fig. 3) and single-mode motions (Fig. 4) are observed. The multimode motions can be stable (Fig. 4).

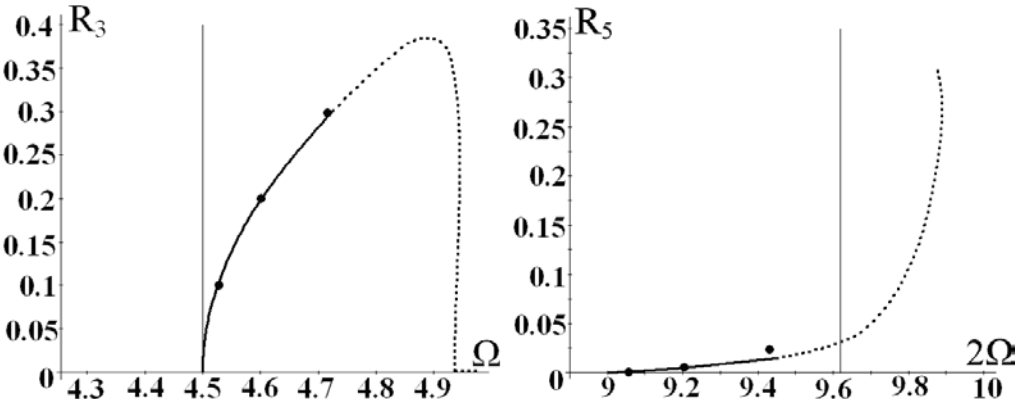

Fig. 3 The backbone curves of the vibrations with dominant general coordinate ξ_3, which excite autoparametrically the motions ξ_5

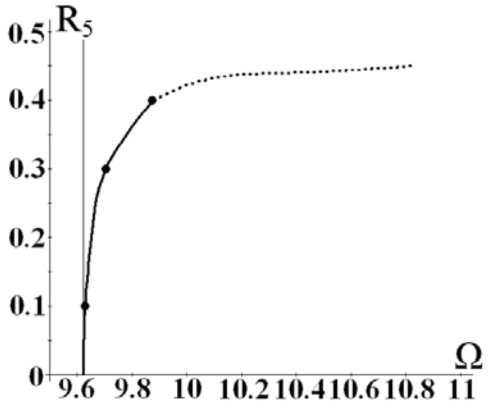

Fig. 4 The vibrations of the shell with dominant general coordinate ξ_5

3. THE VIBRATION OF HYDROTURBINE BLADE

The blade of the axial flow turbine is described by the double-curved shallow shell with variable thickness. The shell base has a shape of ring sector, which has one rounded angle (Fig. 5). The node lines of the first eigenmodes are shown on the Figure 6, which is close to the known experimental and calculated results [19].

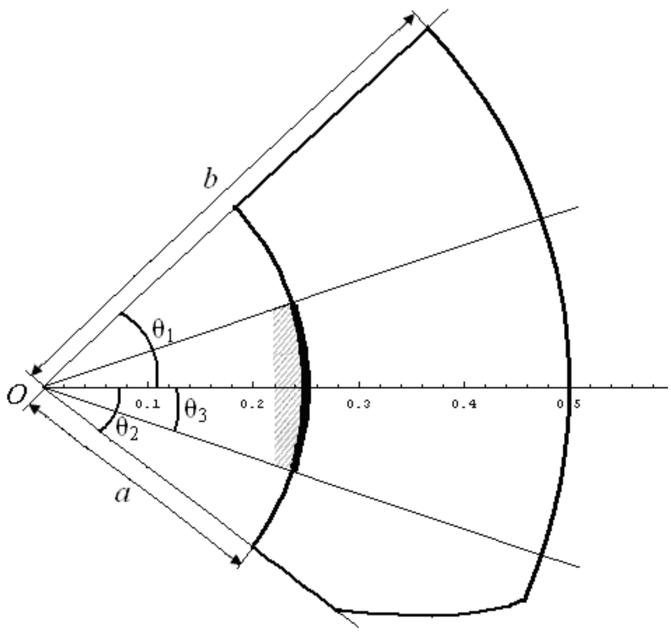

Fig. 5 Base of the shallow shell

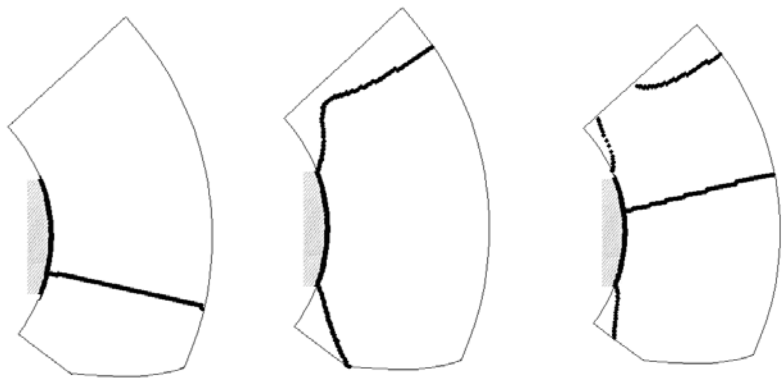

Fig. 6 Nodal lines of eigenmodes

The free nonlinear vibrations close to the first two eigenfrequencies are analyzed. In this case, three terms $N = 3$ are enough in the expansion (1-2). Two invariant manifolds with independent variables ξ_1 and ξ_2 are considered. Figures 7-8 show the backbone curves.

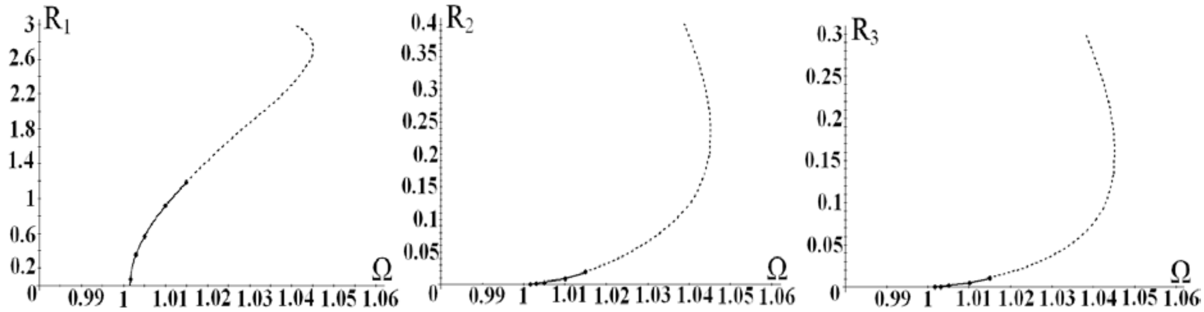

Fig. 7 The backbone curves of vibrations near the first eigenfrequency

CONCLUSIONS

The analysis of free vibrations of turbomachinery blades are presented in this paper. The different modes of blades' nonlinear vibrations are described by hard backbone curves. It is mentioned that the amplitudes of stable vibrations of blade edges are in excess of blade thickness in this region. Therefore, nonlinear theory is required for description of blade dynamics. It is shown, that due to the presence of the internal resonances stable multimode vibrations exist in the system.

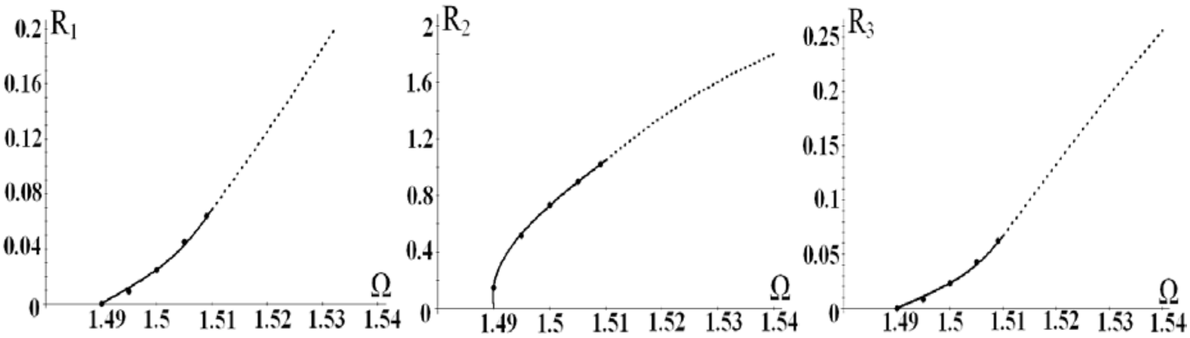

Fig. 8 The backbone curves of vibrations near the second eigenfrequency

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