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NUMERICAL AND EXPERIMENTAL INVESTIGATIONS OF FRACTURE IN THIN-WALLED STRUCTURES AT IMPACT LOADING

D.Breslavsky[1],
I. Naumov,
A.Onyshchenko
National Technical University "Kharkov Polytechnic Institute",
Kharkov, Ukraine

ABSTRACT

The paper contains the description of combined analytical-numerical method for solution of the problem of impact damage accumulation and fracture in thin free supported plate. The results of experimental investigations of this plate are discussed.

INTRODUCTION

Mechanics of impact interaction of solids is widely developed now due to growing demands of safety in modern industrial applications. The great amount of works had been performed in the direction of impact loading of thin-walled structures [1, 2], which shows the practical necessity of these developments for automobile, rail and aircraft transport, power energetics, nuclear and chemical industry.

However, the important number of failures in thin-walled plates and shells connects with the damage accumulation in the material due to repeating impacts. Such fracture can appear in airplane and space panels, motor and turbine casings etc. Now impact damage problems are poor studied owing to deficit of experimental results in that area.

The presented paper contains the method for calculations of stress-strain state and damage distribution in thin-walled free supported rectangular plate. The method is based on the analytical solutions of boundary problem as well as on the numerical time integration schemes. The experimental results of the low-cycle impact fracture in those plates are given in the second part.

1. PLATE UNDER IMPACT LOADING.

Let us regard thin free supported orthotropic rectangular plate of a constant thickness. The plate is loaded by the impactor (spherical, conical or cylindrical), which at the time of the contact has the velocity v_0 and mass m. Following classical approaches of W.Goldsmith [3] and A.P.Philippov [4] we'll use the combined analytical -numerical method.

The deflection in the place of contact will be considered for elasto-plastic deformation in the following form [5] :

$$\varepsilon = k \cdot P^{2/3} + \chi P^q \, , \; \varepsilon = y - W \tag{1}$$

where P is a contact impact force, ε is a joint deflection of the impactor *y* and plate *W*, χ and *q* are material constants. The coefficient *k* is determined for regarded indentors due to relations presented in [2].

The basic unknown contact force P is determined by the following relations, which is derived from 2nd Newton law:

$$y(t) = v_0 \cdot t - \frac{1}{m} \cdot \int_0^t dt \int_0^t P(t) dt \tag{2}$$

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[1] Corresponding author. Email brdm@kpi.kharkov.ua

Let us use the Timoshenko first order shear deformation theory for the solution of thin plate bending. Linear ε_x, ε_y, ε_z and shear γ_{xy}, γ_{xz}, γ_{yz} strains of the plate are connected with plate middle surface's displacements U_e, V_e, W and shear angles θ_x θ_y by following relations:

$$\begin{cases} \varepsilon_x = \dfrac{\partial U_e}{\partial x} + \dfrac{\partial \theta_x}{\partial x} \cdot z; \varepsilon_y = \dfrac{\partial V_e}{\partial y} + \dfrac{\partial \theta_y}{\partial y} \cdot z; \gamma_{xy} = \dfrac{\partial U_e}{\partial y} + \dfrac{\partial V_e}{\partial x} + \left(\dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \right) \cdot z \\ \gamma_{xz} = \theta_x + \dfrac{\partial W}{\partial x}; \quad \gamma_{yz} = \theta_y + \dfrac{\partial W}{\partial y} \end{cases} \tag{3}$$

The forces and moments in the middle surface $N = \{N_x, N_y, N_{xy}\}^T$, $M = \{M_x, M_y, M_{xy}\}^T$, $Q = \{Q_x, Q_y\}^T$ are connected with the displacements and curvature varyings:

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & \overline{A} \end{bmatrix} \begin{Bmatrix} e^0 \\ k \\ \gamma_z^0 \end{Bmatrix}, \tag{4}$$

where $\{e^0\}=\{ \dfrac{\partial U_e}{\partial x}, \dfrac{\partial V_e}{\partial y}, \dfrac{\partial U_e}{\partial y} + \dfrac{\partial V_e}{\partial x} \}$, $\{k\}=\{ \dfrac{\partial \theta_x}{\partial x}, \dfrac{\partial \theta_y}{\partial y}, \left(\dfrac{\partial \theta_x}{\partial y} + \dfrac{\partial \theta_y}{\partial x} \right) \}$, $\{ \gamma_z^0 \}=\{ \gamma_{xz}, \gamma_{yz} \}$.

A, B, D, $\overline{A}$ are the block matrixes which are determined by the elasticity matrix coefficients for orthotropic solid:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{zy} \\ \tau_{zx} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{zy} \\ \gamma_{zx} \\ \gamma_{xy} \end{Bmatrix}, \tag{5}$$

$$A_{ij} = \int_{\overline{z}_0}^{\overline{z}_n} C_{ij} d\overline{z}, \quad B_{ij} = \int_{\overline{z}_0}^{\overline{z}_n} C_{ij} \overline{z} d\overline{z}, \quad D_{ij} = \int_{\overline{z}_0}^{\overline{z}_n} C_{ij} \overline{z}^2 d\overline{z}, \quad i, j = \overline{1,3}$$

$$\overline{A}_{ij} = \int_{\overline{z}_0}^{\overline{z}_n} C_{ij} d\overline{z}, \quad i, j = \overline{4,5}$$

In the case of the coordinate system which is placed in the middle surface of a plate and if it is subjected by symmetric loading, the problem of forced vibrations can be divided into two independent ones. In this case [B] is zero matrix.

$$
\begin{cases}
\dfrac{\partial Q_x}{\partial x}+\dfrac{\partial Q_y}{\partial y}=\rho\cdot h\cdot\dfrac{\partial^2 W}{\partial t^2}-q \\
\dfrac{\partial M_x}{\partial x}+\dfrac{\partial M_{xy}}{\partial y}=Q_x+\rho\cdot\dfrac{\left(\bar{z}_n^3-\bar{z}_0^3\right)}{3}\cdot\dfrac{\partial^2\theta_x}{\partial t^2}+\rho\cdot\dfrac{\partial^2 U_e}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2} \\
\dfrac{\partial M_{yx}}{\partial x}+\dfrac{\partial M_y}{\partial y}=Q_y+\rho\cdot\dfrac{\left(\bar{z}_n^3-\bar{z}_0^3\right)}{3}\cdot\dfrac{\partial^2\theta_y}{\partial t^2}+\rho\cdot\dfrac{\partial^2 V_e}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2} \\
\dfrac{\partial N_x}{\partial x}+\dfrac{\partial N_{xy}}{\partial y}=\rho\cdot h\cdot\dfrac{\partial^2 U_e}{\partial t^2}+\rho\cdot\dfrac{\partial^2\theta_x}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2} \\
\dfrac{\partial N_{xy}}{\partial x}+\dfrac{\partial N_y}{\partial y}=\rho\cdot h\cdot\dfrac{\partial^2 V_e}{\partial t^2}+\rho\cdot\dfrac{\partial^2\theta_y}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2}
\end{cases}
\tag{6}
$$

Here $\bar{z}_n, \bar{z}_0$ are the coordinates of bottom and top plate's surfaces. By substituting (3) into (6), the system of equations of plate's forced oscillations, which is written on displacements, had been obtained:

$$
\begin{cases}
A_{11}\cdot\dfrac{\partial^2 U}{\partial x^2}+A_{12}\cdot\dfrac{\partial^2 V}{\partial x\partial y}+B_{11}\cdot\dfrac{\partial^2\theta_x}{\partial x^2}+B_{12}\cdot\dfrac{\partial^2\theta_y}{\partial x\partial y}+A_{33}\cdot\left(\dfrac{\partial^2 U}{\partial y^2}+\dfrac{\partial^2 V}{\partial x\partial y}\right)+ \\
B_{33}\cdot\left(\dfrac{\partial^2\theta_x}{\partial y^2}+\dfrac{\partial^2\theta_y}{\partial x\partial y}\right)=\rho\cdot h\cdot\dfrac{\partial^2 U}{\partial t^2}+\rho\cdot\dfrac{\partial^2\theta_x}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2} \\
A_{22}\cdot\dfrac{\partial^2 V}{\partial y^2}+A_{12}\cdot\dfrac{\partial^2 U}{\partial x\partial y}+B_{22}\cdot\dfrac{\partial^2\theta_y}{\partial y^2}+B_{12}\cdot\dfrac{\partial^2\theta_x}{\partial x\partial y}+A_{33}\cdot\left(\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 U}{\partial x\partial y}\right)+ \\
B_{33}\cdot\left(\dfrac{\partial^2\theta_y}{\partial x^2}+\dfrac{\partial^2\theta_x}{\partial x\partial y}\right)=\rho\cdot h\cdot\dfrac{\partial^2 V}{\partial t^2}+\rho\cdot\dfrac{\partial^2\theta_y}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2} \\
A_{55}\left(\dfrac{\partial\theta_x}{\partial x}+\dfrac{\partial^2 W}{\partial x^2}\right)+A_{44}\left(\dfrac{\partial\theta_y}{\partial y}+\dfrac{\partial^2 W}{\partial y^2}\right)=\rho\cdot h\cdot\dfrac{\partial^2 W}{\partial t^2}-q \\
B_{11}\cdot\dfrac{\partial^2 U}{\partial x^2}+B_{12}\cdot\dfrac{\partial^2 V}{\partial x\partial y}+D_{11}\cdot\dfrac{\partial^2\theta_x}{\partial x^2}+D_{12}\cdot\dfrac{\partial^2\theta_y}{\partial x\partial y}+B_{33}\cdot\left(\dfrac{\partial^2 U}{\partial y^2}+\dfrac{\partial^2 V}{\partial x\partial y}\right)+ \\
D_{33}\cdot\left(\dfrac{\partial^2\theta_x}{\partial y^2}+\dfrac{\partial^2\theta_y}{\partial x\partial y}\right)=A_{55}\cdot\left(\theta_x+\dfrac{\partial W}{\partial x}\right)+\rho\cdot\dfrac{\left(\bar{z}_n^3-\bar{z}_0^3\right)}{3}\cdot\dfrac{\partial^2\theta_x}{\partial t^2}+\rho\cdot\dfrac{\partial^2 U_e}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2} \\
B_{22}\cdot\dfrac{\partial^2 V}{\partial y^2}+B_{12}\cdot\dfrac{\partial^2 U}{\partial x\partial y}+D_{22}\cdot\dfrac{\partial^2\theta_y}{\partial y^2}+D_{12}\cdot\dfrac{\partial^2\theta_x}{\partial x\partial y}+B_{33}\cdot\left(\dfrac{\partial^2 V}{\partial x^2}+\dfrac{\partial^2 U}{\partial x\partial y}\right) \\
+D_{33}\cdot\left(\dfrac{\partial^2\theta_y}{\partial x^2}+\dfrac{\partial^2\theta_x}{\partial x\partial y}\right)=A_{44}\cdot\left(\theta_y+\dfrac{\partial W}{\partial y}\right)+\rho\cdot\dfrac{\left(\bar{z}_n^3-\bar{z}_0^3\right)}{3}\cdot\dfrac{\partial^2\theta_y}{\partial t^2}+\rho\cdot\dfrac{\partial^2 V_e}{\partial t^2}\cdot\dfrac{\left(\bar{z}_n^2-\bar{z}_0^2\right)}{2}
\end{cases}
\tag{7}
$$

Further the displacements, angles and loading functions are presented as an expansions on trigonometric series. Such representation of solution automatically satisfies the boundary conditions of free supported plate. After substitution of these series into system (6) we use the properties of orthogonality of trigonometric functions. As a result of the system's solution unknown functions $P(t),\ W_{mk}(t),\ F_{mk}(t),\ H_{mk}(t),\ U_{mk}(t),\ V_{mk}(t)$ are obtained. The operational method is used.

Inverse Laplace transformation is used for transition to originals. The contact force *P(t)* is obtained by use the convolution theorem:

$$P(\psi)\cdot W_{mk}(\psi) \rightarrow \int_0^t P(\tau)\cdot W_{mk}(t-\tau)d\tau$$

The equations (1) and (2) are rewritten in the following form:

$$k\cdot P(t)^{\frac{2}{3}} + \chi P(t)^q = v_0\cdot t - \frac{1}{m}\cdot\int_0^t dt\int_0^t P(t)dt - W(P(t)) \tag{8}$$

This equation is nonlinear relatively unknown contact force *P(t)*. For its solution let us use the numerical integration [3, 4]. The basic period of oscillation T_1 is divided on *2s* intervals: $\tau = \frac{T_1}{2\cdot s} = \frac{\pi}{\omega_1\cdot s}$. Let us permit, that the force *P(t)* in each time interval is varied by linear law

$$P(t) = P_j - \left(P_j - P_{j-1}\right)\cdot\left(j - \frac{t}{\tau}\right), (j-1)\cdot\tau < t < j\cdot\tau\ . \tag{9}$$

The equation for finding of contact force in the time moment $t=j\tau$ is written as follows:

$$\varepsilon(j\tau) = y(j\tau) - W(j\tau) \tag{10}$$

The force value $P(j\tau)$ is determined consequently, starting from first time interval, for which $P_0=0$, $P(\tau)=P_1$. The calculations are organized in the following procedure. At the instant moment of impact the value of ε (for example, $\varepsilon \approx v_0\tau$) is set, further the P_1 is calculated and the new precised $\varepsilon = k\cdot(P_1)^{2/3} + \chi(P_1)^q$ is determined. The values of P_2, P_3 and other unknowns are calculated in similar way. The calculations perform for the time moment $t=j\tau$, in which $P(j\tau)$ received the zero value, namely for the case of impactor separation from the plate. After finding of contact force another unknowns like displacements, angles θ_x, θ_y, moments, forces and stresses in the plate are determined.

2. IMPACT DAMAGE AND FRACTURE IN RECTANGULAR PLATE

Impact damage accumulation phenomenon occurs in the case of repeated impact action [6]. Its mechanism is close to low-cycle fatigue. Let us use the Rabotnov-Kachanov damage kinetic equation [7]:

$$\dot{\omega} = D\frac{\left(\Delta\sigma^e\right)^m}{\left(1-\omega\right)^m}\ \ \omega(0) = 0\,,\ \omega(t_*) = \omega_*\,, \tag{11}$$

where ω is the damage parameter, $\Delta\sigma^e$ is the equivalent stress calculated by stress amplitudes for each impact, *D* and *m* are the material constants in the damage law.

The calculation order in the case of the analysis of impact damage accumulation in free supported rectangular plate is following. For each case of impact loading the formulated above method is used in order to determine the maximum amplitude stress values $\Delta\sigma_{ij}$ in each plate's point by use of determined value of impact contact force *P*. Further the calculated equivalent amplitude stress $\Delta\sigma^e$ is used in (11) for calculation of the damage parameter value ω in that points. The equivalent stress is determined by three invariants criterion [7]. Calculations are terminated if the damage parameter in one point reaches its critical value ω_* (as a rule $\omega_*=1$).

The impact damage accumulation up to fracture of considered free supported plate was studied by experimental way. The square plates with side 0.18 m and thickness 0.0015m were tested. The material of the plates is steel 12H18N10T.

Information and measuring system (IMS) had been developed in the Department of Control Systems and Processes of National Technical University 'Kharkiv Polytechnic Institute' was used for experimental analysis [8]. The appointment of system is strain registration in the processes of impact loading of thin plates. It composes from the strain gauges, signal unit sensors, unit of interface and protection, ADC board ADA-1406 and personal computer.

The IMS is the part of laboratory testing system (LTS), which additionally includes the device of plate fixation and the loading system, which works by use of electrical-mechanical pulse converter. The acceleration of the impactor is performed by use of magnetic field coil. The cylindrical impactors with diameter 0.004m were used.

Determination of the constants for damage law (11) was performed on the specimens which had been cut from same steel sheet. 6 specimens and 3 plates were tested.

The test sequence includes four groups of experiments: 1) static plate loading; 2) elastic impact of plate; 3) impact low-cycle uniaxial tests; 4) impact low-cycle plate fracture.

Experiments from first and second group were performed for calibration of the developed LTS. Static loading of the plate was used for strain gauge's calibration. The correlation between measured voltage and strains were established. After that the impact elastic loading of plates with spherical and cylindrical impactors were studied. For each strain gauge the dependencies from time were determined. Fig. 1 contains the signal plots have been obtained by the developed IMS.

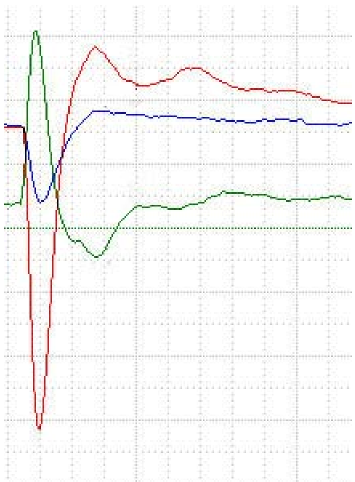

Fig. 1 Strain measuring by IMS

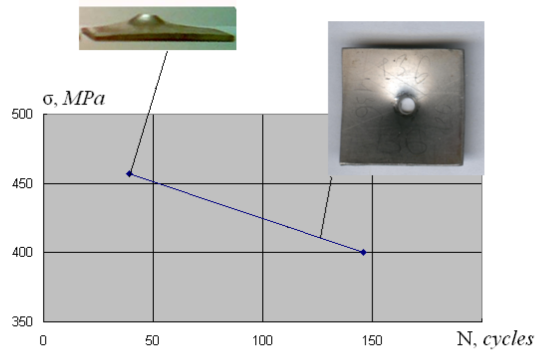

Fig.2 Impact long term data

As a result of uniaxial impact test the averaged values of the numbers of cycles to fracture were determined. For the first group with stress 400 MPa it was 146 cycles, for second, with another stress 457 MPa, it was 39 cycles. Fig. 2 contains impact low cycle long strength data, which have been obtained in these experiments, as well as the illustrations of the one destroyed specimen from each group. The fracture of specimens occurred by punching.

Material constants involve in kinetic damage equation (11) were obtained by use of experimental data. There values are: $D=1\ 10^{-29}$ $(\text{MPa})^{-1}$/cycle, m=9.9.

Let us describe the experimental investigation of impact low-cycle plate fracture. The plates were placed in fixation device realizing the free support and were impacted by repeating loading up to punching. The velocity of impactor was 0.0625 m/s. The average value of the number of cycles to fracture was 79 impacts, variation of the data did not exceed 16%. Let us stress the reasonably local character of plastic deformation of the plates, the area does not exceed of 6% of total plate's area (Fig. 3).

Fig.3 Impactor and plate after punching

CONCLUSIONS

The method for solution of impact problems for thin free supported plates is given. It is based on analytical solution of boundary problem as well as on numerical for initial value one. The use of determined stress fields and kinetic damage equation for simulation of impact damage accumulation allows determine the values of lives to fracture of the plates. As usual in Nonlinear Mechanics the validation of calculational method needs the comparison between calculated and experimental data. This comparison for steel square plates will be possible after another group of experiments connected with determination of constants χ and q in equation (1).

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