OSCILLATION OF SYSTEMS WHICH HAVE FORCE-DISPLACEMENT CHARACTERISTICS WITH RECTANGULAR LOOPS OF HYSTERESIS

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ABSTRACT

The article is devoted to nonlinear oscillation of passive systems which have force-displacement characteristics with rectangular loops of hysteresis resulting from the dry friction force. Under certain conditions when force-displacement characteristic has segment of zero stiffness the resonance frequencies are shifted in area of higher frequencies defined by the size of this segment.

INTRODUCTION

The possibility of creation systems with quasi-zero-stiffness for protection of dynamic objects is now well known nowadays [1]. In the article [2] shows the possibility to creation systems with force-characteristics with rectangular hysteresis's loops resulting from the dry friction force (Fig.1). Height of hysteresis's loops is determined by the dry friction force R (where $R = q \cdot F_*$, $F_* - const$, q is the coefficient, which determines the height of loops, F_* is the restoring force without the friction force). If the coefficient q is more than 1 then the restoring force are absent, so $0 \le q \le 1$. Variants when $q > 1$ are not checked.

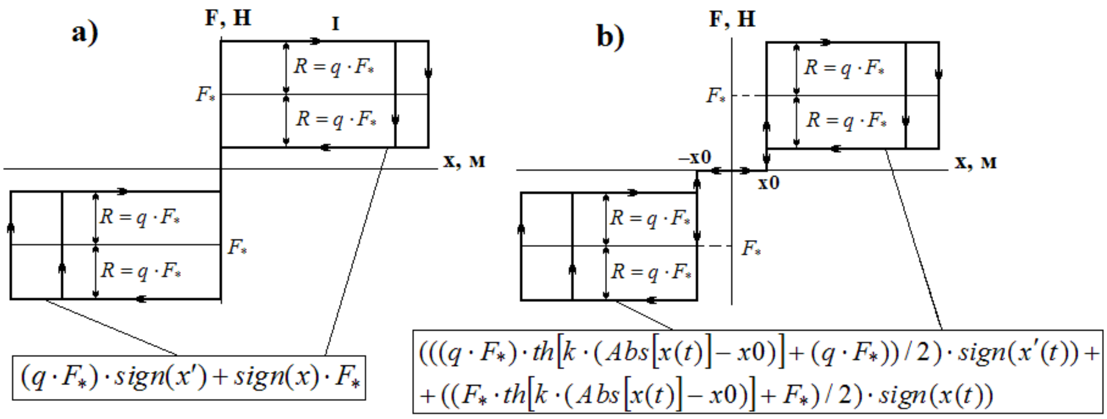

Fig. 1 Force characteristics with loops of hysteresis
a – without segment of zero stiffness; b – with segment of zero stiffness ($-x0, \ x0$)

Oscillation under the harmonic excitation $F_0 \cdot \cos(p \cdot t + \varphi)$ (where F_0 is the amplitude of the correction force; p is the frequency; φ is the initial phase) are determined both analytically and numerically. Loops of hysteresis are defined analytically by functions, shown in Fig. 1; (where k is the coefficient, which determines the inclination of loop's sides; for diagrams in the Fig. 1, k = 10000; x0 is the value, which determines the size of segment of zero stiffness).

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1. OSCILLATION OF SYSTEMS WHICH HAVE FORCE-CHARACTERISTICS WITH RECTANGULAR LOOPS OF HYSTERESIS WITHOUT SEGMENT OF ZERO STIFFNESS

Six transcendental equations are solved in order to determine analytically the oscillation of systems with force-characteristics described in Fig.1, a. Oscillations under the subject to harmonic excitation ($F_0 \cdot \cos(p \cdot t + \varphi)$) are determined numerically as well. Loops of hysteresis are described by functions shown in Fig. 1, a. The differential equation of the moving object with mass m is:

$$m \cdot \ddot{x} = F_0 \cdot \cos(p \cdot t + \varphi) - \{(q \cdot F_*) \cdot sign(x') + sign(x) \cdot F_*\}, \tag{1}$$

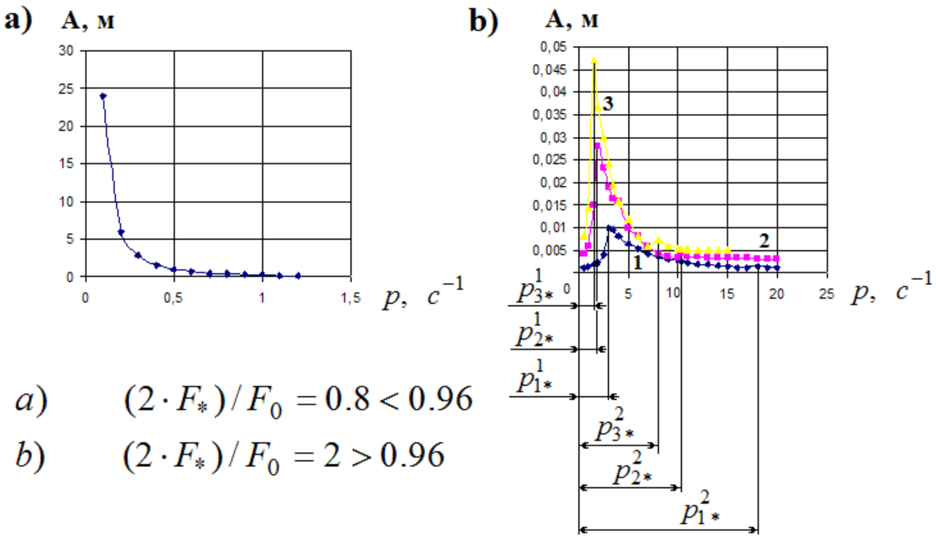
For numerical solution it is possible to determine the oscillations for different coefficients q (Fig. 1, a). For $q = 1$, under the next relation $(2 \cdot F_*)/F_0 \geq 0.96$, the oscillations are vanished. The amplitude-frequency characteristics are derived for the next relation: $2 \cdot F_* / F_0 \leq 0.96$. The amplitude-frequency characteristic for $2 \cdot F_* / F_0 = 0.8$ and $F_0 = 100 \quad H$ is described in Fig. 2, a. The results of analytical and numerical solutions are considered coincident.

2. OSCILLATION OF SYSTEMS WHICH HAVE FORCE-CHARACTERISTICS WITH RECTANGULAR LOOPS OF HYSTERESIS WITH SEGMENT OF ZERO STIFFNESS

In order to determine analytically the oscillations for force characteristics shown in Fig. 1 ,b with segments of zero stiffness ($x0 \neq 0$) and disturbing force $F_0 \cdot \cos(p \cdot t + \varphi)$ it is necessary to solve nine transcendental equations [2]. The oscillations are determined numerically by solving the next differential equation:

$$\begin{aligned} m \cdot x'' = F_0 \cdot \cos(p \cdot t + \varphi) - (((q \cdot F_*) \cdot th[k \cdot (Abs[x] - x0)] - \\ -(q \cdot F_*))/2) \cdot sign(x') - ((F_* \cdot th[k \cdot (Abs[x] - x0)] + F_*)/2) \cdot sign(x) \end{aligned} \tag{2}$$

Ti obtain the analytical solution, the nine transcendental equations are reduced to the one which is solved by the dichotomy method. Author can solve it in a specific frequency range which less than p^1 (Fig. 2, b).



$m = 500 \quad kg$; $\quad F_0 = 100 \quad H$;

b) 1 – $|x0| = 0.001 \quad m$; $\quad$ 2 – $|x0| = 0.003 \quad m$; $\quad$ 3 – $|x0| = 0.005 \quad m$;

p^1_{1*}; p^1_{2*}; p^1_{3*} are first resonance's frequencies; p^2_{1*}; p^2_{2*}; p^2_{3*} are second resonance's frequencies;

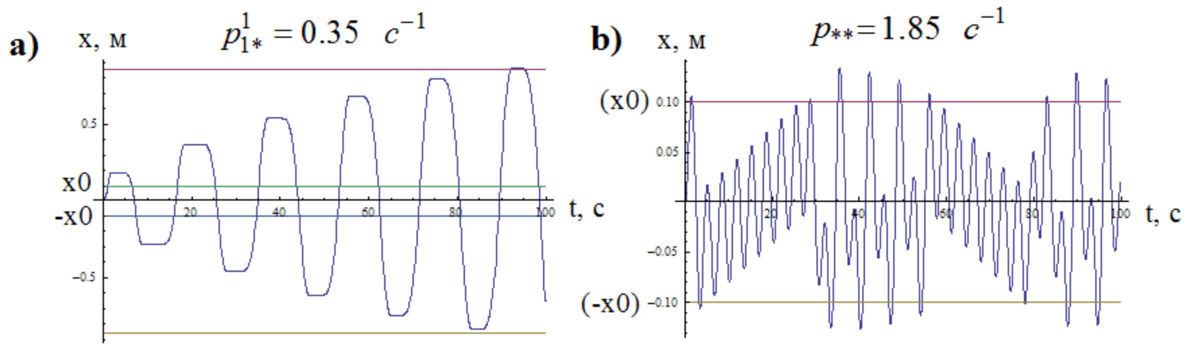
Fig. 2 Amplitude-frequency characteristics (q = 1)

The results of the analytical solution were coincided with results of the numerical one, but numerical solution of differential equation (2) was obtained for any frequencies of harmonic excitation p.

For $x0 = 0$ the amplitude-frequency characteristic obtained by solution of differential equation (2) is in coincident with amplitude-frequency characteristic obtained by solution of differential equation (1) (Fig. 2, a).

For $q = 1$ and $(2 \cdot F_*)/F_0 \geq 0.96$ for the case when $x0 \neq 0$ (Fig. 1, b) oscillation do not vanish as for the case when $x0 = 0$ (Fig. 1, a), but resonance's frequencies are shifted in area of higher frequencies. The shift depends on the area of section with zero stiffness, $x0$: the more smaller is the area, the more bigger is the shift (Fig. 2, b). For the numerical solution of differential equation (2) for set-up parameters two resonance's frequencies (p_*^1, p_*^2 - Fig. 2, b) are determined. Resonance oscillation (first resonance's frequencies) are shown in Fig. 3, a. "Oscillation stop" takes place starting from the specific frequency p_{**} (Fig. 3, b).

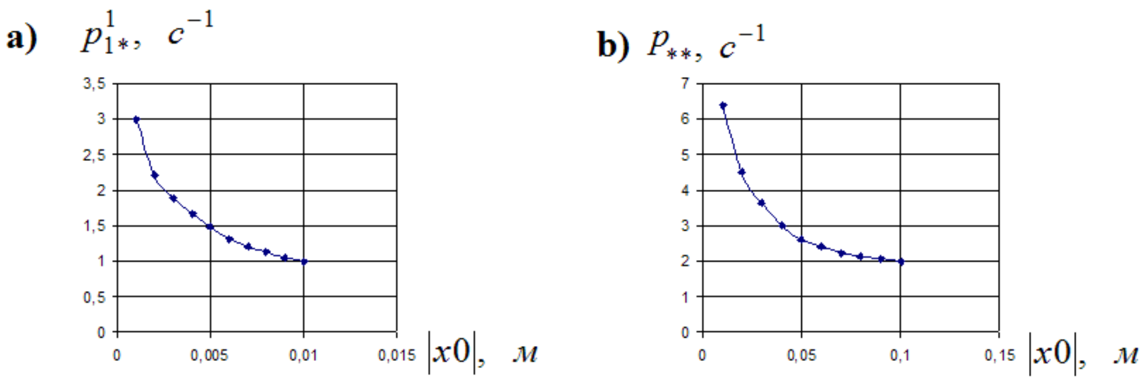
Relations between the first resonance's frequencies, "oscillation stop" frequencies and the size of segment of zero stiffness are shown in the Fig. 4.



$m = 500 \quad kg; \qquad F_0 = 100 \quad H; \qquad 2 \cdot F_* / F_0 = 2; \qquad |x0| = 0.1 \quad m$

a) - first resonance's frequency (p_{1*}^1); b) "oscillation stop" (p_{**})

Fig. 3 Oscillations



$m = 500 \quad kg; \qquad F_0 = 100 \quad H; \qquad 2 \cdot F_* / F_0 = 2$

a) first resonance's frequencies б) frequencies of "oscillation stop"

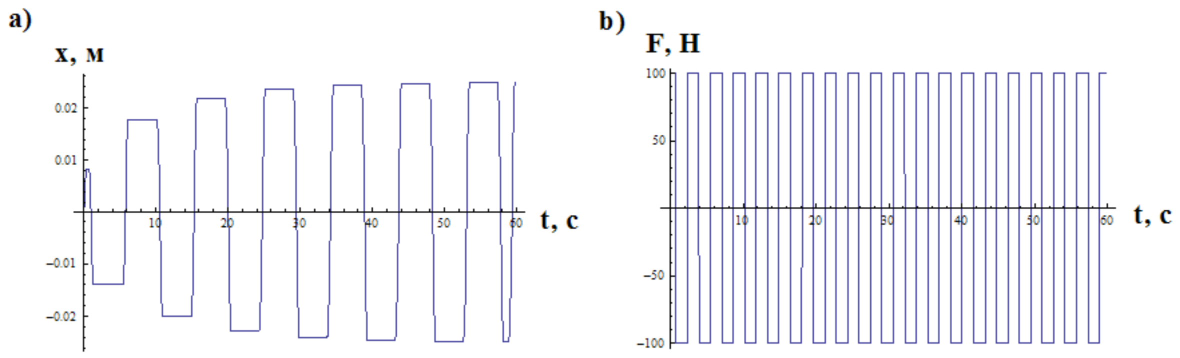
Fig. 4 Dependences of critical frequencies from size of segment of zero stiffness

Oscillation of systems, which force characteristics shown in Fig. 1, b, were determined also for excitation $F_0 \cdot sign[\cos[p \cdot t + \varphi]]$. The next differential equation is solved:

$$m \cdot x'' + (F_* \cdot th[k \cdot (Abs[x] + x_*)] + F_*)/2 \cdot sign[x'] +$$
$$(F_* \cdot th[k \cdot (Abs[x] + x_*)] + F_*)/2 \cdot sign[x] = F_0 \cdot sign[\cos[p \cdot t + \varphi]] \qquad (3)$$

Within specific parameters the oscillation frequency becomes in several times less than the frequency of excitation (Fig. 5). The frequency range of that effect exists is sufficiently narrow. (for

$F_0 = 100 \quad H; \; F_* = 125 \quad H; \; |x0| = 0.005 \quad m; \; \varphi = 0; \; q = 1 \quad p \subset [1.9 - 2.4]$). For these parameters the relation of the oscillation frequency to the excitation frequency is little bit more than three (Fig. 5). This effect is not observed for the excitation $F_0 \cdot \cos(p \cdot t + \varphi)$.



$p = 2 \quad c^{-1}; \qquad F_0 = 100 \quad H; \qquad F_* = 125 \quad H; \qquad |x0| = 0.005 \quad m; \qquad \varphi = 0$

a) coordinate of oscillating body; b) excitation

Fig. 5 Oscillation contraction for the excitation $F_0 \cdot sign[\cos[p \cdot t + \varphi]]$

It should be noted that for the described cases the oscillations with stopping can be observed (dependences of the coordinates from time are rectangular, as shown in the Figs. 3, 5), that is, specific for the systems with dry friction.

CONCLUSIONS

The considered systems of passive type with force characteristics, shown in Fig. 1, can be widely applied in scientific and technical areas, such as seismic protection, suspension brackets, impact protection and so on. The studies showed that numerical approach of the oscillations determination for described systems is more preferable than the analytical one. Some interesting effects were revealed for the numerical approach.

For the system with force characteristics shown in Fig. 1, a, the resonance frequency converges to zero. For definite ratio of the correction force to the amplitude of excitation force the oscillation vanish. If for that ratio the segment of zero stiffness appears on the characteristics (Fig. 1, b) then oscillations do not vanish but the resonance frequency is shifted into the area of high frequencies (Fig 2, b). In this paper the frequencies of "oscillation stop" were defined.

For the excitation force $F_0 \cdot sign[\cos[p \cdot t + \varphi]]$ the multiple decreasing of oscillation frequencies was revealed in comparison to the frequency of excitation force in the small diapason of the frequencies with force characteristics shown in Fig. 1, b.

For system with force characteristics, shown in Fig.1 the frequencies with "oscillation stops" were determined, both for excitation force $F_0 \cdot \cos(p \cdot t + \varphi)$ and $F_0 \cdot sign[\cos[p \cdot t + \varphi]]$.

REFERENCES

[1] Alabuzhev P., Gritchin A., Kim L, Migirenko G., Chon V., Stepanov P. *Vibration Protecting and Measuring Systems with Quasi-Zero Stiffness.* Hemisphere Publishing, New York, 1989.

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