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RATCHETING SIMULATION OF STRUCTURAL STEEL UNDER LOW-CYCLE ASYMMETRIC LOADING

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ABSTRACT

Dependence of the cyclic creep rate in stainless steels 1020 and 1026 on the amplitude and mean value of a loading cycle in the space of stresses is investigated. To simulate the process, constitutive equations of the endochronic theory of plasticity with the improved hereditary kernel were employed. It is shown that the model proposed allows describing with a sufficient accuracy the kinetics of the stress-strain state of specimens under low-cycle asymmetric loading based on the smallest number of basic experiments.

INTRODUCTION

It is known that the operation of actual thin-walled structures, such as pipes and pressure vessels, under cyclic loading with high nominal stresses can be accompanied by the phenomenon of accumulation of oriented deformations whose intensity and nature are responsible for the rate of attaining the limiting state and service life of structure. This effect is called “cyclic creep” or “ratcheting”. It is experimentally observed under stress-controlled loading higher the yield stress of cyclically-anisotropic materials or unsymmetrical loading of cyclically-isotropic materials. The main peculiarity of this effect is that the hysteresis loops induced are never closed and, as a result, the recorded strain gradually creeps in the direction of the mean stress. In the region of low-cycle fatigue this factor influences appreciably the lifetime of the structural materials. The phenomenon of cyclic creep of the material is not necessary caused by time effects as is case of classical creep. To the large extent, it is determined by the anisotropy of the material, both initial and acquired in the process of loading. The intensity of the processes of cyclic creep depends on the properties of the material (isotropic, anisotropic, hardening, softening), the loading mode (stress ratio, nonproportionality of the path of loading cycles, loading frequency), the plasticity margin, temperature, etc.

At present, a significant amount of research [1-4] is devoted to problems of investigations and simulation of cyclic loading. This is explained by both practical needs and by the necessity of having constitution relations capable of describing the inelastic behavior of materials. In the last decades, considerable progress is this field has been attained due to the appearance of numerous experimental and theoretical works [5-7] devoted to the improvement of the applicability of various version of the theory of plasticity to the case of cyclic asymmetrical loading.

The aim of this work is to develop a constitutive model of cyclic plasticity for the prediction of complex processes of loading, both strain- and stress-controlled, for uniaxial and biaxial low-cycle loading.

1. BASIC EQUATIONS OF THE MODEL

We shall restrict our consideration to the mechanical behavior of incompressible plastic materials in the case of low strains. Assuming that the material is initially isotropic we shall use constitutive equations of the endochronic theory of plasticity [9] which are the modification of

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Valanis's endochronic theory [8]. Then in the deviatoric Il'yushin's vector space, the basic equations of this theory are

$$\mathbf{s} = s_y \frac{d\mathbf{e}^p}{dz} + \int_0^z J\left(z - z'\right) \frac{d\mathbf{e}^p}{dz'} dz' , \tag{1}$$

$$d\xi = F\left(z, \Delta\varepsilon^p, l^p, \Phi\right), \tag{2}$$

$$d\xi^2 = q\left(d\mathbf{e}^p \cdot d\mathbf{e}^p\right), \tag{3}$$

where s_y is the yield stress, F is the hardening function and $J(z)$ is the kernel of the integral equation (heredity function). The total strain vector is presented as the sum of elastic and plastic components

$$d\mathbf{e} = d\mathbf{e}^p + \frac{d\mathbf{s}}{2\mu} . \tag{4}$$

In the case of the simple tension-compression loading Eq. (1) can be written as follows:

$$\sigma = \sigma_y \frac{d\varepsilon^p}{dz} + \int_0^z E\left(z - z'\right) \frac{d\varepsilon^p}{dz'} dz' \tag{5}$$

where $\sigma_y = \frac{2}{3} s_y$, $E(z) = 3J(z)$, $q = \frac{2}{3}$, $d\xi = d\varepsilon^p$.

For the cyclically stabilizing materials it is most convenient to use the hardening function

$$F\left(z\right) = C - \left(C - 1\right) e^{-\beta z} \tag{6}$$

and regular heredity function

$$E\left(z\right) = E_1 e^{-\alpha z} + E_2 \tag{7}$$

where E_1, E_2, α are material characteristics.

To construct a simple model for describing the anisotropy of materials under unsymmetrical loading we assume that the hardening function $F(z)$ is isotropic. Then the anisotropic behavior arises during nonelastic loading only, and it is characterized by the difference of the kinematic hardening for tension and compression semicycles. Since the strain hardening under unsymmetrical cyclic loading depends on the mode of loading, mean and amplitude stresses of the cycle, one can suggest the dependence of this hardening on both the measure of the deformation process and the stress level attained in the previous semicycle. To take into consideration the latter factor, it is convenient to use the parameter δ proposed by Dafalias and Popov [10]. This parameter is the "distance" in the stress space between the maximum stress state for the given semicycle and the bounding surface . The bounding surface is centered at the origin, grows isotropically each time its stress level is exceeded and represents the highest level of the stress state attained in the loading history. In the uniaxial case the bounding surface is represented by the two lines σ_B , as shown in Fig. 1. Based on the aforesaid, one can write the heredity function as follows:

$$E\left(z, \delta\right) = E_1 e^{-\alpha z} + E_2\left(\delta\right), \tag{8}$$

which implies the different values E_2 for tension and compression semicycles under unsymmetrical loading.

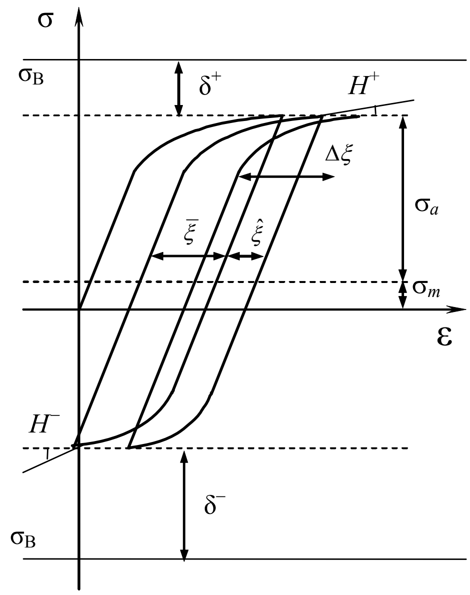

Fig. 1 Block loading scheme and designations

The relationship between the stresses and the internal time (5) under arbitrary uniaxial cyclic loading for *k*-th semicycle can be written in the form

$$\sigma = (-1)^k \left[\sigma_y F(z) + \int_{z_k}^{z} E(z-z')F(z')dz' \right] + \sum_{i=0}^{z_i} (-1)^i \int_{z_{i-1}}^{z_i} E(z-z')F(z')dz' . \tag{9}$$

We define the current modulus of plasticity as

$$H = \frac{d\sigma}{d\varepsilon^p} = \frac{d\sigma}{dz} \cdot \frac{dz}{d\varepsilon^p} = \left| \frac{d\sigma}{dz} \cdot \frac{1}{F(z)} \right| = \frac{1}{F(z)} \left[\sigma_y \dot{F}(z) + \int_0^z \dot{E}(z-z')F(z')dz' \right] + E(0) \tag{10}$$

where the overdot denotes the operation of differentiation with respect to z.

2. MODELING OF UNIAXIAL RATCHETING

Let us use the above equations to describe the deformation of specimens fabricated from the AISI 1020 cyclic softening carbon steel and AISI 1026 cyclic stable carbon steel under block loading [3]. The first block was the strain symmetric cycling in the range of 2%. Then the specimens were unloaded to approximately zero stress, after which the control cyclic loading followed with different values of mean and amplitude stresses of the cycle. Numerical modeling of the such loading involved a step-by-step procedure for the control of the deformation process. The baseline experiments and the calculation and experimental techniques for specifying the basic unknown functions and the material constants are described in detail elsewhere [9].

The main peculiarity of the given model is to set the correct functional dependence of parameter E_2 of the heredity function on the δ [11, 12]. of the preceding semicycle. For initially isotropic materials such dependence can be built on the basis of a single basic experiment performed by complex program. First the strain symmetric loading is performed until the steady state is attained. Then follows the stress unsymmetric cycling at lower stresses also until the steady state and finally monotone loading to the stress level of the first stage is effected. During the first block we determine the parameter E_2 for $\delta = 0$. From the second loading block we can determine the E_2 for the tension and compression semicycles.

We use Eq. (10) to get the plastic modulus under unsymmetrical loading in the steady state case. Then for the arbitrary point *B* of the tension semicycle if we use the designation which is accepted in Fig. 1 we can write the equation

$$H^+ = E_1\left(1 - U^+\right) + E_2\left(\delta^-\right), \tag{11}$$

where

$$U^{+}=1-2e^{-\alpha\Delta z}\left(1-\frac{e^{-\alpha\left(2\bar{z}+\hat{z}\right)}-e^{-\alpha\bar{z}}}{e^{-\alpha\left(2\bar{z}+\hat{z}\right)}-1}\right). \tag{12}$$

The intrinsic time intervals $\Delta z,\bar{z},\hat{z}$ correspond the measures $\Delta\xi,\bar{\xi},\hat{\xi}$ which connected by the expression $dz=d\xi/C$. Now we can readily get the $E_2\left(\delta^{-}\right)$ value if the modulus H^{+} is determined from the experiment. Another E_2 value at the δ^{+} can be found from the equality

$$E_2\left(\delta^{+}\right)\bar{\xi}=E_2\left(\delta^{-}\right)\left(\bar{\xi}+\hat{\xi}\right) \tag{13}$$

obtained from the theoretical analysis of the steady hysteresis loops at ratcheting with the constant rate.

The shape of the $E_2\left(\delta\right)$ function can be determined after the approximation of the obtained values by the appropriate function. In our case we use dependence in a following form

$$E_2\left(\delta\right)=E_2\left(0\right)+a\delta^{b}\cdot D^{n} \tag{14}$$

where a , b and n - parameters of model.

The first summand Eq. (14) is the limiting value that corresponds to $E_2\left(\delta\right)$ on the memory surface, and is determined from experimental date. In the second summand expression (14) first factor $a\delta^{b}$ take into account influence on $E_2\left(\delta\right)$ mean stress and second factor - D^{n} take into account influence stress amplitude. Parameter D defined as follows:

$$D=\frac{\sigma_a-\sigma_s}{\sigma_a^{bas}-\sigma_s} \tag{15}$$

where σ_a - amplitude stress, σ_a^{bas} - is the amplitude stress of the basic experiment, σ_s - radius of surface plasticity in the stabilized condition, defined as

$$\sigma_s=\sigma_y F\left(z\to\infty\right)=\sigma_y C \tag{16}$$

3. MODEL VERIFICATION

The equations presented above are now used for description of uniaxial block loading of specimens made of cyclically softening CS 1020 and cyclically stable CS 1026 steels. We use the experimental data presented in [3]. In all cases, the first loading block was realized as straining with symmetric cycles and a range of total strains of 2%. In the second block, we applied stress-controlled asymmetric loading with different values of the mean and amplitude stresses.

For numerical analysis, we created a special computational program. It was used to perform all necessary calculation. Parameters of the model for the two studied materials used in calculation contained in the Table 1.

Table 1 Parameters of the model cyclic plasticity

Steel	*E*, ksi	σ_T, ksi	E_1, ksi	$E_2(0)$, ksi	α	*C*	β_1	β_2
CS1020	25125	40	25298,5	1721	965	0,78	12,4	30
CS1026	26320	20	19600,0	650	1051	0,95	20	20

According to (11) was specified functional dependence $E_2(\delta)$ on value of δ in case of asymmetric cyclic loading studied materials, namely:

$$E_2(\delta) = E_2(0) - 6{,}3 \cdot 10^{-7} \cdot \delta^6 \cdot D^8 \qquad \text{for CS1020}$$
$$E_2(\delta) = E_2(0) - 0{,}51 \cdot \delta^2 \cdot D^4 \qquad \text{for CS1026}$$

The results of numerical calculation are presented in Figs. 2 and 3. In figure 2, we present the results of computation (solid lines), experimental date (doted lines) and results of computation by Hassan and Kyriakides [3] (dashed lines) for CS 1020 steel subjected to asymmetric loading in the form of the dependence of the maximum strain in a cycle on the number of loading cycles for various values of the mean stress (Fig. 2a) and different amplitudes of stress cycles (Fig. 2b). In Fig. 3 results for steel 1026 are accordingly presented.

The comparison of the numerical results with the experimental data shows the efficiency of the proposed model for the description of cyclic creep both in the first loading cycles and in stationary mode. It is worth noting that more precise results were obtained in the case where asymmetric loading is simulated varying the amplitude stress for a constant value of the mean stress. At the same time, all these theoretical predictions are completely covered by spread in the experimental data. For more exact predictions, one may either use other functional dependences for the approximation of the quantities E_2 and δ or perform at least two basic tests.

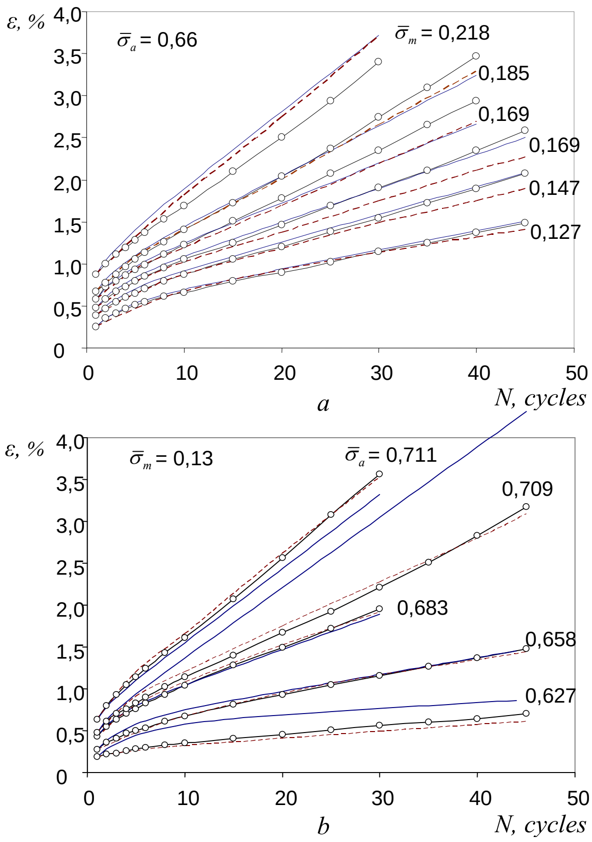

Fig. 2 Dependence of the maximum strain in a cycle on the number of loading cycles for CS 1020 steel

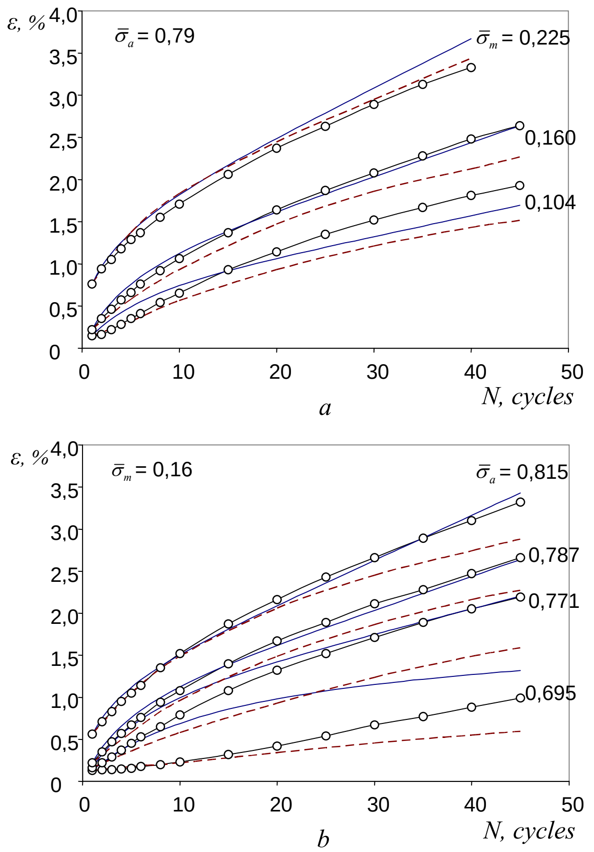

Fig. 3 Dependence of the maximum strain in a cycle on the number of loading cycles for CS 1026 steel

CONCLUSIONS

Constitutive equations of the endochronic theory of plasticity for describing of the unsymmetrical stress-controlled loading are presented. New rule of the kinematic hardening is introduced for characterizing an induced anisotropy under such loading. A discrete scale of the intrinsic time and the evolutionary equation of the hardening function suggested in the work make it possible to obtain simple constitutive equation for modeling the complex histories of cyclic loading.

Analysis of the modeling results of uniaxial ratcheting testifies we have obtained a satisfactory description of the stress-strain kinetics under unsymmetrical stress cycling.

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