Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

RESEARCH OF THE NONLINEAR FREE VIBRATION OF THE FULLY CLAMPED COMPOSITE LAMINATED PLATES OF AN ARBITRARY PLANFORM

Galina N. Timchenko[1], **Irina O. Morachkovska**, **Nikolay A. Budnikov**
National Technical University "KhPI", Kharkov, Ukraine

ABSTRACT

The method of investigation of the geometrically nonlinear free vibrations of the clamped laminated plates with complex form is proposed. Method is based on using R-functions, variational method by Ritz and projection method by Bubnov-Galerkin. Mathematical formulation is fullfilled on base of the first order shear deformation theory of the plates, which is likes to the theory by Timoshenko.

INTRODUCTION

Nonlinear vibration problems of the laminated plates are very essential for practice because plates are common structural elements in many engineering structures. In spite of the practical importance of these problems, a survey of publications on nonlinear vibrations of plates shows that the theoretical investigations of these problems are insufficient and remains actual up to now. Due to mathematical complexity of the problem the majority of scientists take into consideration only simply supported plates with the rectangular form of the plane. In the given paper we propose the numerical-analytical approach, which can be applied to plates of the complex form.

1. PROBLEM STATEMENT

The mathematical statement of this problem in the framework of the first-order shear deformation theory is based on the hypothesis of a straight line, which was adopted for the whole package. The governing system is nonlinear one of the differential equations with partial derivatives written below [1, 2] in displacements,

$$L_{11}(C_{ij})u + L_{12}(C_{ij})v + Nl_1(C_{ij})w = m_1 \frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$L_{21}(C_{ij})u + L_{22}(C_{ij})v + Nl_2(C_{ij})w = m_1 \frac{\partial^2 v}{\partial t^2} \quad (2)$$

$$L_{33}(C_{ij})w + L_{34}(C_{ij})\psi_x + L_{35}(C_{ij})\psi_y + Nl_3 = m_1 \frac{\partial^2 w}{\partial t^2} \quad (3)$$

$$L_{43}(C_{ij})w + L_{44}(C_{ij}, D_{ij})\psi_x + L_{45}(D_{ij})\psi_y = m_2 \frac{\partial^2 \psi_x}{\partial t^2} \quad (4)$$

$$L_{53}(C_{ij})w + L_{54}(D_{ij})\psi_x + L_{55}(C_{ij}, D_{ij})\psi_y = m_2 \frac{\partial^2 \psi_y}{\partial t^2} \quad (5)$$

The linear differential operators $L_{ij,}\ i, j = 1,2,3$ in the equations (1)-(5) are presented in [5, 6].

Nonlinear operators Nl_1, Nl_2, Nl_3 are defined as follows:

$$Nl_1(C_{ij}) = \frac{\partial}{\partial x}\left\{\frac{1}{2}\left(C_{11}\left(\frac{\partial}{\partial x}\right)^2 + C_{12}\left(\frac{\partial}{\partial y}\right)^2\right) + C_{16}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\right\} + \frac{\partial}{\partial y}\left\{\frac{1}{2}\left(C_{16}\left(\frac{\partial}{\partial x}\right)^2 + \right.\right.$$

[1] Galina N. Timchenko. Email gntimchenko@mail.ru

$$+C_{26}\left(\frac{\partial}{\partial y}\right)^2\Bigg)+C_{66}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Bigg\} \tag{6}$$

$$Nl_2(C_{ij})=\frac{\partial}{\partial x}\left\{\frac{1}{2}\left(C_{16}\left(\frac{\partial}{\partial x}\right)^2+C_{26}\left(\frac{\partial}{\partial y}\right)^2\right)+C_{66}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\right\}+\frac{\partial}{\partial y}\left\{\frac{1}{2}\left(C_{12}\left(\frac{\partial}{\partial x}\right)^2+\right.\right.$$

$$+C_{22}\left(\frac{\partial}{\partial y}\right)^2\Bigg)+C_{26}\frac{\partial}{\partial x}\frac{\partial}{\partial y}\Bigg\} \tag{7}$$

$$Nl_3(C_{ij})=\frac{\partial^2 w}{\partial x^2}\left\{C_{11}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)+C_{12}\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)+C_{16}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right\}+$$

$$+2\frac{\partial^2 w}{\partial x\partial y}\left\{C_{16}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)+C_{26}\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)+C_{66}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right\}+ \tag{8}$$

$$+\frac{\partial^2 w}{\partial y^2}\left\{C_{12}\left(\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)+C_{22}\left(\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right)+C_{26}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right)\right\},$$

where C_{ij}, D_{ij} are rigid characteristics [2], which are defined by the elasticity constants B^i_{jk} for every $i-$th layers.

In a case of the clamped edge the differential equations (1) – (5) are supplemented by the following boundary conditions:

$$u=0,\ v=0,\ \psi_x=0,\ \psi_y=0,\ w=0. \tag{9}$$

The initial conditions are taken as follows:

$$\mathrm{w}=\mathrm{w}_{\max},\ \frac{\partial \mathrm{w}}{\partial \mathrm{t}}=0\,. \tag{10}$$

2. METHOD OF SOLUTION

The first step of the proposed method is a solving of the corresponding linear problem of the laminated plates free vibration. In detail the solving algorithm of this problem has been described in [7].The distinctive feature of the proposed method is application of the R-functions theory and variational methods. Namely, such approach allows find natural frequencies and functions in analytical form for any domain and kind of the boundary conditions, which are very important to solve the nonlinear problem. The natural modes corresponding to linear vibrations of the plates have been chosen as basic functions for the representation of unknown functions.

On the next step the unknown functions w,ψ_x,ψ_y are presented in the form

$$w(x,y,t)=y_1(t)\cdot w_1(x,y),\ \psi_x(x,y,t)=y_1(t)\cdot\psi_{x1}(x,y),\ \psi_y(x,y,t)=y_1(t)\cdot\psi_{y1}(x,y) \tag{12}$$

here $w_1(x,y),\ \psi_{x1}(x,y),\ \psi_{y1}(x,y)$ are the components of the eigenfunctions vector. It's obviously that equations (4) and (5) will be satisfied. In order to satisfy the equation (1) and (2) let us present the unknown functions u and v in the following form:

$$u(x,y,t)=y_1^2(t)\cdot u_2(x,y)$$
$$v(x,y,t)=y_1^2(t)\cdot v_2(x,y), \tag{13}$$

where (u_2,v_2) is solution of the following system of the equations

$$L_{11}(C_{ij})u_2+L_{12}(C_{ij})v_2=-Nl_1(C_{ij})w_1$$
$$L_{21}(C_{ij})u_2+L_{22}(C_{ij})v_2=-Nl_2(C_{ij})w_1 \tag{14}$$

The last system coincides with the similar system for 2-dimensional elasticity problems for which the right parts play the role of mass forces. The boundary conditions are the same with (9). The RFM is applied to find these functions [3, 4]. We will ignore the inertia terms in equation (1) and (2) then it's easy to check that they be satisfied after substitution of the expressions (13).

Substituting the expressions (12), (13) for $u(x,y,t), v(x,y,t), w(x,y,t), \psi_x(x,y,t), \psi_y(x,y,t)$ in movement equation (3) and applying the method by Bubnov-Galerkin one obtains the nonlinear ordinary differential equation in unknown functions $y_1(t)$:

$$y_1''(t) + \omega_L^2 y_1(t) + \beta \cdot y_1^3(t) = 0 \tag{15}$$

where coefficient β is defined as

$$\beta = -\frac{\int\limits_{\Omega} \left(Nl_{32}\left(C_{ij}, u_2, v_2, w_1\right)\right) \cdot w_1 d\Omega}{\rho h \|w_1\|^2},$$

$$Nl_{32}(C_{ij}) = \frac{\partial^2 w_1}{\partial x^2}\left\{ C_{11}\frac{\partial u_2}{\partial x} + C_{12}\frac{\partial v_2}{\partial y} + \frac{1}{2}\left(C_{11}\left(\frac{\partial w_1}{\partial x}\right)^2 + C_{12}\left(\frac{\partial w_1}{\partial y}\right)^2 \right) + \right.$$

$$\left. + C_{16}\left(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x} + \frac{\partial w_1}{\partial x}\frac{\partial w_1}{\partial y} \right) \right\} + 2\frac{\partial^2 w_1}{\partial x \partial y}\left\{ C_{66}\left(\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_1}{\partial x}\frac{\partial w_1}{\partial y} \right) + \right.$$

$$\left. + C_{16}\left(\frac{\partial u_2}{\partial x} + \frac{1}{2}\left(\frac{\partial w_1}{\partial x}\right)^2 \right) + C_{26}\left(\frac{\partial v_2}{\partial x} + \frac{1}{2}\left(\frac{\partial w_1}{\partial y}\right)^2 \right) \right\} + \frac{\partial^2 w_1}{\partial y^2}\left\{ C_{12}\left(\frac{\partial u_2}{\partial x} + \frac{1}{2}\left(\frac{\partial w_1}{\partial x}\right)^2 \right) + \right.$$

$$\left. + C_{22}\left(\frac{\partial v_2}{\partial x} + \frac{1}{2}\left(\frac{\partial w_1}{\partial y}\right)^2 \right) + C_{26}\left(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x} + \frac{\partial w_1}{\partial x}\frac{\partial w_1}{\partial y} \right) \right\}$$

Let us present the unknown function in the form: $y(t) = A\cos\omega_N t$. Then after application of the method by Bubnov-Galerkin to the equation (15) it is possible to get the explicit dependence $\nu = \frac{\omega_N}{\omega_L}$, of the ratio of the nonlinear frequency to linear one on the vibration amplitude A [1]

$$\frac{\omega_N}{\omega_L} = \sqrt{1 + \frac{3}{4}\beta \cdot A^2} \tag{16}$$

As we noted the finding of functions (u_2, v_2) is connected with solving the system (14). Obviously the system is supplemented by corresponding boundary conditions. Taking into account that we consider fully clamped plate, it is possible to prove, that this problem may by reduced to the finding point of stationary of the following functional

$$I(\vec{U}_2) = \int\limits_{\Omega}\left\{ C_{11}\left(\frac{\partial u_2}{\partial x}\right)^2 + C_{22}\left(\frac{\partial v_2}{\partial y}\right)^2 + 2C_{12}\frac{\partial u_2}{\partial x}\frac{\partial v_2}{\partial y} + C_{66}\left(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x}\right)^2 + 2\left(C_{16}\frac{\partial u_2}{\partial x} + \right.\right.$$

$$\left.\left. + C_{26}\frac{\partial v_2}{\partial y} \right)\left(\frac{\partial v_2}{\partial x} + \frac{\partial u_2}{\partial y} \right) + \left(Nl_1\left(C_{ij}\right) w_1 \right) u_2 + \left(Nl_2\left(C_{ij}\right) w_1 \right) v_2 \right\} d\Omega \tag{17}$$

where $U_{2n} = u_2 l + v_2 m, V_{2n} = -u_2 m + v_2 l$.

The discretization of functional (11) and (17) is fulfilled by RFM and Ritz method.

The proposed method is numerically realized in the framework of software “POLE-RL” and widely tested on many nonlinear vibration problems for plates at large amplitudes. The proposed method can be applied not only for composite plates with identical elasticity constants of layers but also for the plates of "sandwich" type.

3. NUMERICAL RESULTS

Let us consider the geometrically nonlinear free vibration of the laminated plates (Fig. 1) with geometric sizes: $l = 0.75м, a = 0.4м$, $h_i = 8 \cdot 10^{-5} м$.

.

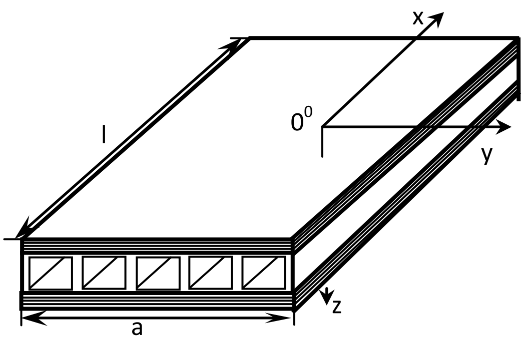

Fig. 1. The laminated plates

The shell is carried from the material with the physical characteristics:

Face layers

$E_1 = 1.35 \cdot 10^5 МПа, E_2 = 8 \cdot 10^3 МПа, G_{12} = 6 \cdot 10^3 МПа,$

$\nu_{12} = 0.286,$

Core

$E_1 = 0.7264 \cdot 10^5 МПа, E_2 = 1.4528 \cdot 10^5 МПа,$

$G_{12} = G_{23} = 1.1622 \cdot 10^5 МПа, G_{13} = 0.5811 \cdot 10^4 МПа,$

$\nu_{12} = 0{,}2552.$

The obtained backbone curves for different lamina scheme of the layers are presented in Fig.2. Here the backbone curve L_1 corresponds to lamina scheme of the layers $\lfloor 0^0/90^0 \rfloor_2$ *core* $\lfloor 90^0/0^0 \rfloor_2$. Curve L_2 is backbone for $\lfloor \pm 80^0 \rfloor_2$ *core* $\lfloor \mp 80^0 \rfloor_2$, curve L_3 is backbone fore $\lfloor \pm 60^0 \rfloor_2$ *core* $\lfloor \mp 60^0 \rfloor_2$, curve L_4 is backbone for $\lfloor \pm 30^0 \rfloor_2$ *core* $\lfloor \mp 30^0 \rfloor_2$. From the presented results it follows that the curve L_1 is more rigid. The obtained results can be used at designing of similar elements under the transverse load.

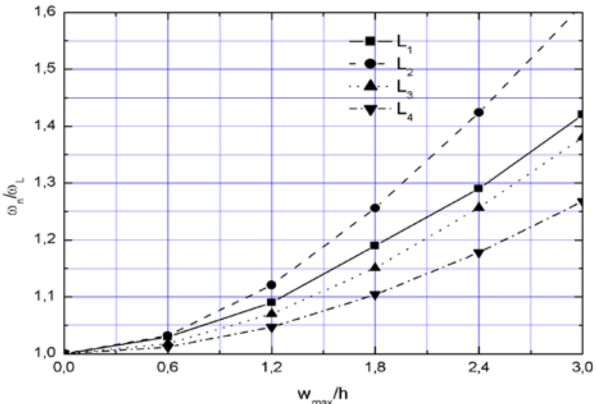

Fig. 2. Backbone curves of plate for different values of packing layers

CONCLUSION

In this work, the method of investigation of nonlinear free vibrations of the symmetrically laminated clamped plates with an arbitrary plane form is proposed. This approach is based on *R*-functions theory, variational methods, and the Runge-Kutta method. The software "POLE-RL" is applied to obtain the numerical results. The investigation is carried out for the "sandwich" plate. The investigation is carried out for the "sandwich" plates. The ratio of the nonlinear frequency to linear one depending on amplitudes of vibration (w_{max}/h) of laminated plates pack with various face layers are received.

REFERENCES

[1] Vol'mir A.S. *Nonlinear Dynamics of Plates and Shells* ,Nauka, Moskow, 1972 (in Russian).

[2] Ambartsumyan S.A. *General Theory of Anisotropic Shells*, Nauka, Moskow, 1974 (in Russian).

[3] Rvachev V.L. *The R-Functions theory and its Some Application*, Naukova Dumka, Kiev, 1982 (in Russian).

[4] Rvachev V.L., Kurpa L.V. *The R-Functions in Problems of Plates Theory*, Naukova Dumka, Kiev, 1987 (in Russian).

[5] L.V. Kurpa, G.N Timchenko, Investigation into nonlinear vibrations of composite plates using the R-function theory, *Strength of Materials* 39 (5), pp. 529-538, 2007.

[6] Kurpa L.V., Timchenko G.N. Investigation geometrically nonlinear vibrations composite plates with complex form, *Dynamics and Strength of Machines* 21, pp.113 – 118, 2006 (in Russian).

[7] Kurpa L.V., Timchenko G.N. Investigations of vibrations of laminated plates of an arbitrary form, In: *Proc. of the 8 th SSTA Conference, Jurata, Poland*, pp. 371-375, 2005.