Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

RESONANT JUMPS IN MULTI-FREQUENCY REGIMES OF MYLTI-CIRCULAR PLATE SYSTEM NON-LINEAR DYNAMICS

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ABSTRACT

By use a double circular plate system dynamics, the multi-frequency analysis of forced non-linear dynamics is pointed out. Series of the amplitude-frequency and phase-frequency graphs as well as eigen-forced time functions–frequency graphs are obtained for stationary resonant states and analyzed according present singularities and triggers of coupled singularities, as well as resonant jumps.
For analyze of stationary forced resonant regimes of forced non-linear oscillations for presented model, we use the graphical presentation of the numerical experiment results over the first asymptotic aproximation of the two amplitudes and two phases of the two-frequency resonant stationary regimes.
For the system of two circular plates connected with non-linear visco-elastic layer with hard or soft non-linear properties on the basis of obtained numerical and graphical results, we can conclude that non-linearity in the interconecting distributed layer intoroduce in the system non-linear part of the potential energy as a energy interactions between circular plates as subsystems (deformable bodies) coupled in the hybrid system with complex component eigen forced non-linear modes, as well as mutual influence and transfer energy through all the system components of the mods. Resonant jumps, as well as „resonant forced oscillatory jumps“, trigger of coupled singularities, as well as coupled triggers of coupled singularities are reason for appearence of new questions for reasearch this non-linear forced dynamics.

INTRODUCTION

Composing the proper mathematical model of mechanical system presents one of the most important steps in the treatment of the system. On the other way said, mathematical modeling regard on the usage of mathematical language for presents the behavior of practical systems. It plays the role of better understanding of systems features. In the more realistic description of the systems non-linearity appears both as an object's natural characteristic and the non-linearity of the systems of differential equations describing the system dynamics, which is a consequence of the choice of the coordinates of the system's description. Since, the problem is to explore and in some way control non-linearity. Theory is useful for presenting the general conclusions to the simple models while the computers are useful for obtaining the special conclusions for more complicated system dynamics.

In this paper, we will present one mechanical system, a double circular plate system with non-linear interconnecting layer, and its mathematical non-linear descriptions then treat that non-linearity in a sense of making the qualitative analysis of the system behavior.

In many engineering systems with non-linearity, single as well multi- frequency excitations are the sources of multi frequency resonant regimes appearance high as well as low frequency modes. That is visible from many experimental research results and also theoretical results (see Refs. [16] and [17]). The interaction between amplitudes and phases of the different modes in the non-linear systems with many degrees of the freedom as in the deformable body with infinite numbers frequency vibration free and forced regimes is observed theoretically in the Refs. [20] and [22] by Stevanović

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K. , (1972) and (1975) by use averaging asymptotic methods Krilov-Bogoliyubov-Mitropolyskiy (see Refs. [10-15] by Mitropolyskiy, Yu. A. (1955), (1965), (1995)). This knowledge has great practical importance. In the monograph [16] by Nayfeh (2004), a coherent and unified treatment of analytical, computational, and experimental methods and concepts of modal nonlinear interactions is presented.

By using averaging and asymptotic methods Krilov-Bogolyubov- Mitropolyskiy for obtaining system of ordinary differential equations of amplitudes and phases in first approximations and expressions for energy of the excited modes depending on amplitudes, phases and frequencies of different non-linear modes are obtained by Hedrih K. in [2, 3] and by Hedrih K. and Simonović J. in [8]. By means of these asymptotic approximations, the energy analysis of mode interaction in the multi frequency free and forced vibration regimes of non-linear elastic systems (beams, plates, and shells) excited by initial conditions for free oscillation regimes was made and a series of resonant jumps as well as energy transfer features for forced regimes were identified.

Recent technological innovations have caused a considerable interest in the study of component and hybrid dynamical processes of coupled rigid and deformable bodies (plates, beams and belts) (see Refs. [2-4] and [6-8]) denoted as hybrid systems, characterized by the interaction between sub-system dynamics, governed by coupled partial differential equations with boundary and initial conditions.

In this paper, we will try to present the more realistic model with non-linearity in the connected layer and to investigate the phenomenon of passing through resonant range and appearance of one or several resonant jumps in the amplitude–frequency and phase–frequency curves of different nonlinear modes. In system with non-linearity it is noticeable the energy transfer between coupled sub-systems. For detail see Refs. [5] and [8] which contain analysis of energy transfer in double plate system dynamics.

1. SOLUTION IN THE FIRST ASIMPTOTIC APROXIMATION OF PDEs FOR TRANSVERSAL VIBRATIONS OF A DOUBLE PLATES SYSTEM

If we present a physical model of a double plate system, shown in the Fig. 1.a, then it is clear that the mathematical model of such a system may be expressed by the system of two coupled partial differential equations (1) [3,4] and [6,7,8] which are formulated in terms of two unknowns: the transversal displacement $w_i(r,\varphi,t)$, $i=1,2$ in direction of the axis z, of the upper plate middle surface and of the lower plate middle surface. We present the interconnecting layer as a model of one standard light visco-elastic element [1] with started spring's length l_0 and nonlinearity in the elastic part of the layer as shown in Figure 1b.

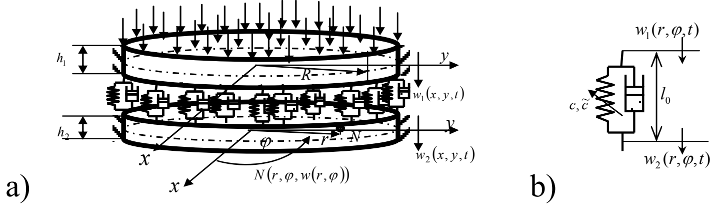


Fig.1. a) A visco-elastically connected double circular plate system; b) model discrete element of visco-elastic non-linear interconnected layer.

The system of partial differential equations (1) is derived using Principle of dynamic equilibrium in the following forms:

$$\frac{\partial^2 w_i(r,\varphi,t)}{\partial t^2}+c_{(i)}^4\Delta\Delta w_i(r,\varphi,t)-2\delta_{(i)}\left[\frac{\partial w_{i+1}(r,\varphi,t)}{\partial t}-\frac{\partial w_i(r,\varphi,t)}{\partial t}\right]- \tag{1}$$

$$-a_{(i)}^2\left[w_{i+1}(r,\varphi,t)-w_i(r,\varphi,t)\right]=\pm\varepsilon\beta_{(i)}\left[w_{i+1}(r,\varphi,t)-w_i(r,\varphi,t)\right]^3+\tilde{q}_{(i)}(r,\varphi,t) \qquad for \qquad i=1,2$$

where are : $\mathrm{D}_i=\frac{E_i h_i^3}{12(1-\mu_i^2)}$, $\varepsilon\beta_{(i)}=\frac{\beta}{\rho_i h_i}$, $a_{(i)}^2=\frac{c}{\rho_i h_i}$, $c_{(i)}^4=\frac{\mathrm{D}_i}{\rho_i h_i}$, $2\delta_i=\frac{b}{\rho_i h_i}$ and the sign $\pm$ on the right hand side corresponds to the feathure of soft (sign +) or hard (sign -) properties of the elastic layer. We suppose that the functions of external excitation at nm-mode of oscillations are the two-frequency process in the form: $\tilde{q}_{(i)nm}(t)=h_{01nm}\cos[\Omega_{1nm}t+\phi_{1nm}]+h_{02nm}\cos[\Omega_{2nm}t+\phi_{2nm}]$, $m,n=1,2,....,\infty$. The solution for system (1) with the visco-linear-elastic connection is taken in the form of the eigen amplitude functions $W_{(i)nm}(r,\varphi)$, $n,m=1,2,3,4,...\infty$, satisfying the same boundary conditions, expansion with time coefficients in the form of unknown time functions $T_{(i)nm}(t)$, that describing their time evolution (see Refs. [4] and [7]):

$$w_i(r,\varphi,t)=\mathrm{W}_{(i)nm}(r,\varphi)T_{(i)nm}(t)=\mathrm{W}_{(i)nm}(r,\varphi)\left[K_{inm}^{(1)}e^{-\hat{\delta}_{1nm}t}R_{1nm}(t)\cos\Phi_{1nm}(t)+K_{inm}^{(2)}e^{-\hat{\delta}_{2nm}t}R_{2nm}(t)\cos\Phi_{2nm}(t)\right] \tag{2}$$

where are: K^{s}_{ijnm} cofactors of determinant corresponding to basic homegenous coupled system [7], $-\hat{\delta}_{inm}$ real parts of the corresponding pair of the roots of the characteristic equation [4], and amplitudes $R_{inm}(t)$ and phases $\Phi_{inm}(t)=\Omega_{inm}t+\phi_{inm}(t)$ unknowen time functions which, we are going to obtain using the Krilov-Bogolyubov-Mitropolyskiy asymptotic method (see Refs. [10-15]). It is taken into account that defined task satisfy all necessary conditions for applying asymptotic method Krilov-Bogolyubov-Mitropolskiy concerning small parameter and that external excitation frequencies $\Omega_{1nm}\approx\hat{p}_{1nm}$ and $\Omega_{2nm}\approx\hat{p}_{2nm}$ are in the resonant range intervals of the corresponding eigen frequencies of unperturbed linear system. By applying the asymptotic method, we obtain the system of the first order differential equations according unknovwen amplitude and phases in the first asymptotic approximation [8] as follow:

$$\begin{aligned}
\dot{a}_{1nm}(t)&=-\frac{\left(\delta_{(1)}K^{(2)}_{22nm}+\delta_{(2)}K^{(2)}_{21nm}\right)\left[K^{(1)}_{22nm}-K^{(1)}_{21nm}\right]}{\left(K^{(2)}_{22nm}K^{(1)}_{21nm}-K^{(2)}_{21nm}K^{(1)}_{22nm}\right)}a_{1nm}(t)+\frac{K^{(2)}_{22nm}h_{01nm}}{\left(\Omega_{1nm}+\hat{p}_{1nm}\right)\left(K^{(2)}_{22nm}K^{(1)}_{21nm}-K^{(2)}_{21nm}K^{(1)}_{22nm}\right)}\cos\phi_{1nm}\\
\dot{\phi}_{1nm}(t)&=\left(\hat{p}_{nm1}-\Omega_{1nm}\right)-\\
&\mp\frac{\aleph(W_{nm})\varepsilon\left(\beta_{(1)}K^{(2)}_{22}+\beta_{(2)}K^{(2)}_{21}\right)}{\left(K^{(2)}_{22}K^{(1)}_{21}-K^{(2)}_{21}K^{(1)}_{22}\right)\left(\Omega_{1nm}+\hat{p}_{nm1}\right)}\left[\left(K^{(1)}_{22nm}-K^{(1)}_{21nm}\right)^3\frac{3}{8}a^2_{1nm}(t)+\frac{1}{2}\left(K^{(1)}_{22nm}-K^{(1)}_{21nm}\right)\left(K^{(2)}_{22nm}-K^{(2)}_{21nm}\right)^2a^2_{2nm}(t)\right]-\\
&-\frac{K^{(2)}_{22}h_{01nm}}{\left(\Omega_{1nm}+\hat{p}_{nm1}\right)\left(K^{(2)}_{22nm}K^{(1)}_{21nm}-K^{(2)}_{21nm}K^{(1)}_{22nm}\right)a_{1nm}(t)}\sin\phi_{1nm}
\end{aligned}\tag{3}$$

$$\begin{aligned}
\dot{a}_{2nm}(t)&=-\frac{\left(\delta_{(1)}K^{(1)}_{22nm}+\delta_{(2)}K^{(1)}_{21nm}\right)\left[K^{(2)}_{22nm}-K^{(2)}_{21nm}\right]}{\left(K^{(1)}_{22nm}K^{(2)}_{21nm}-K^{(2)}_{22nm}K^{(1)}_{21nm}\right)}a_{2nm}(t)+\frac{K^{(1)}_{22nm}h_{02nm}}{\left(\Omega_{2nm}+\hat{p}_{2nm}\right)\left(K^{(1)}_{22nm}K^{(2)}_{21nm}-K^{(2)}_{22nm}K^{(1)}_{21nm}\right)}\cos\phi_{2nm}\\
\dot{\phi}_{2nm}(t)&=\left(\hat{p}_{2nm}-\Omega_{2nm}\right)-\\
&\mp\frac{\aleph(W_{nm})\varepsilon\left(\beta_{(1)}K^{(1)}_{22nm}+\beta_{(2)}K^{(1)}_{21nm}\right)}{\left(\Omega_{2nm}+\hat{p}_{2nm}\right)\left(K^{(1)}_{22nm}K^{(2)}_{21nm}-K^{(2)}_{22nm}K^{(1)}_{21nm}\right)}\cdot\left[\frac{1}{2}\left(K^{(1)}_{22nm}-K^{(1)}_{21nm}\right)^2\left(K^{(2)}_{22nm}-K^{(2)}_{21nm}\right)a^2_{1nm}(t)+\left(K^{(2)}_{22nm}-K^{(2)}_{21nm}\right)^3\frac{3}{8}a^2_{2nm}(t)\right]-\\
&-\frac{K^{(1)}_{22nm}h_{02nm}}{\left(\Omega_{2nm}+\hat{p}_{2nm}\right)\left(K^{(1)}_{22nm}K^{(2)}_{21nm}-K^{(2)}_{22nm}K^{(1)}_{21nm}\right)a_{2nm}(t)}\sin\phi_{2nm}
\end{aligned}\tag{4}$$

where are: $a_{inm}(t)=R_{inm}(t)e^{-\hat{\delta}_{inm}t}$, $m,n=1,2,....,\infty$. and $\aleph(W_{nm})=\dfrac{\int_0^r\int_0^{2\pi}W^4_{(1)nm}(r,\varphi)rdrd\varphi}{\int_0^r\int_0^{2\pi}W^2_{(1)nm}(r,\varphi)rdrd\varphi}$ is coefficient of nonlinearity influence of elastic layer. We observed the case when external distributed two-frequencies force acts at upper surfaces of upper plate with frequencies near eigen circular frequencies of coupled plate systems $\Omega_{inm}\approx\hat{p}_{inm}$, $i=1,2$, $m,n=1,2,....,\infty$. In this case the lower plate is free of load. This means that, we were observed the passing thought main resonant states by discrete values of the forced frequencies. Using the first asymptotic approximation of the amplitudes and phases of multi frequency particular solutions of the non-linear system dynamics (3)-(4), a numerical experiment over the non-linear modes in stationary regimes of non-linear system forced dynamics is realized.

For analyses of the stationary regime of oscillations, we equal the right hand sides of differential equations (3)-(4) for amplitudes $R_{inm}(t)$ and difference of phases $\phi_{inm}(t)$ with null. Eliminating the phases ϕ_{1nm} and ϕ_{2nm}, we obtained system of two algebraic equations by unknown amplitudes a_{1nm} and a_{2nm}, also with elimination of amplitudes a_{1nm} and a_{2nm}, we obtained the forms for phases ϕ_{1nm} and ϕ_{2nm} in the case of two-frequencies forced oscillations in stationary regime of one nm mode of double plate system oscillations. Solving that two systems by numerical *Newton-Kantorovic's method in computer program Mathematica,* we obtained stationary amplitudes and phases curves of two-frequencies regime of one eigen nm-shape amplitude mode oscillations in double plate system coupling with visco-elastic non-linear layer depending on frequencies of external excitation force. If we fixed the value of on external excitation frequency of two possible, we obtained amlitude-frequency curves as well as phase-frequency curves of stationary states of vibration regime in the following forms:

1* for second external excitation frequency with constant discrete value ($\Omega_{2nm}=const$) corresponding amplirude-frequency and phase-frequency curves:

$$a_{1nm}=f_1(\Omega_{1nm}),\ a_{2nm}=f_2(\Omega_{1nm}),\ \phi_{1nm}=f_3(\Omega_{1nm})\ \text{and}\ \phi_{2nm}=f_4(\Omega_{1nm})\ \text{and}\tag{5}$$

2* for first external excitation frequency with constant discrete value $\Omega_{1nm}=const$ corresponding amplirude-frequency and phase-frequency curves:

$$a_{1nm}=f_5(\Omega_{2nm}),\ a_{2nm}=f_6(\Omega_{2nm}),\ \phi_{1nm}=f_7(\Omega_{2nm})\ \text{and}\ \phi_{2nm}=f_8(\Omega_{2nm}).\tag{6}$$

In this extend abstract, we will present some of the amplitede-frequencies and phase-frequencies curves of stationary kinetic state in continuously exchange of fixed discrete values of one external excitation frequencies and in that sense regard system in stationary regime, and some characteristic diagrams of that amplitide-frequency and phase-frequency curves are presented on the following Figs. 2 and 3.

Let us to make a quantitative analyses of passing through discrete stationary states alog resonant frequency intervals and apperance of new non-stable branches on amplitude (phase)-frequencies curves like as changes on that characteristics for the frequencies of external force in the range of eigen frequencies of coupling in one nm -eigen amplitude mode of corresponding linearized system oscillations. We take into account that system for the case when the plates are with the same boundary and material characteristics and when the upper plates has the height twice then the lower one, $h_2 = h_1/2$, and obtained the eigen frequencies of visco-elastic linear coupling with values: $\hat{p}_{111} = 135.55\left(s^{-1}\right)$ and $\hat{p}_{112} = 301.14\left(s^{-1}\right)$.

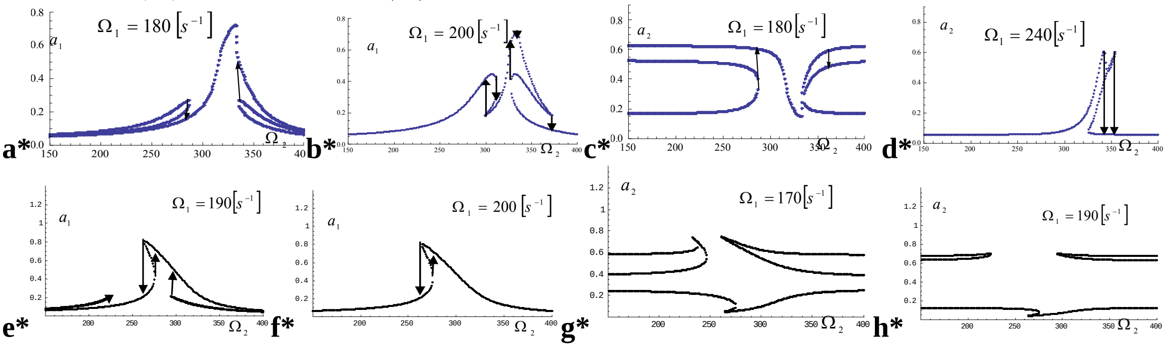


Fig. 2. Amplitude-frequency characteristic curves for the amplitudes of the first $a_{1nm} = f_5(\Omega_{2nm})$ and second $a_{2nm} = f_6(\Omega_{2nm})$ time harmonics for hard (a, b*, c*, d*) and for soft (e*, f*, g*, h*) characteristics of interconnected layer and for the different value of excited frequency Ω_{2nm} for discrete value of excited frequency $\Omega_{1nm} = const$ with noted corresponding one or more resonant jumps. Arrows means directions of the resonant jumps.*

The amplitude-frequency responses for two frequency like stationary vibration regimes, contain amplitudes a_1 and a_2 presented in Fig. 2. These shown diagrams exhibit a hardening, Figs. 2a*,b*,c* and d*, and softening, Figs.2e*,f*,g* and h*, characteristic as a non-linear interactions between time non-linear modes of the two-frequency external excitation in the resonant interval of two external excitation frequencies close to the eigen linearized system frequencies. This is a property of hard and soft non-linearity of a visco-non-linear elastic layer and corresponding non-linear characteristic is in accordance with governing system of partial differential equations (1) for the case of the lower sign for hard non-linear characteristic, and of the upper sign for soft non-linear characteristics. That shapes are results of the modes interaction and of the particular discrete values choice of the external excitation frequencies Ω_{1nm} used in the resonant frequencies intervals belonging to corresponding eigen frequencies $\hat{p}_{1nm}$ of the corresponding nm- th eigen amplitude shape mode of plate linear system taken in the simulations.

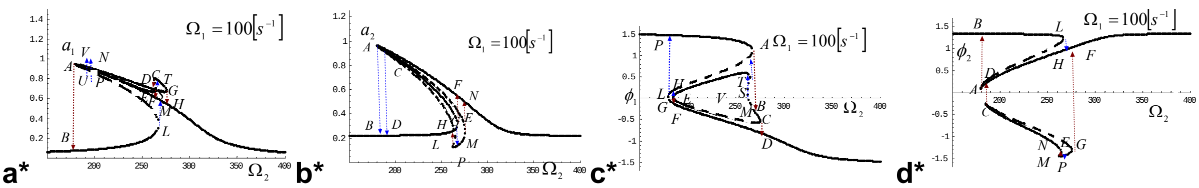


Fig. 3. Characteristic resonant jumps on the amplitude-frequency (a, b*) curves $a_{1nm} = f_5\left(\Omega_{2nm}\right)$ and $a_{2nm} = f_6\left(\Omega_{2nm}\right)$, and phase-frequency (c*, d*) curves $\phi_{1nm} = f_7\left(\Omega_{2nm}\right)$ and $\phi_{2nm} = f_8\left(\Omega_{2nm}\right)$ as a characteristic cases of the large resonant interactions betwee external two frequency excitation and non-linear properties of the double plate system dynamics when both frequencypes Ω_{1nm} and Ω_{2nm} take values from the resonant frequency intervals and casses for the appearing of the large interaction of the coupled stationary resonance regime.*

Characteristic for both series of the amplitude-frequency curves for two frequency like non-linear stationary vibration regimes is that more then one pair of the resonant jumps appear, together with more then one instability branch in the corresponding amplitude-frequency and phase-frequency curves. It is visible that in the listed Figs 3 a*,b*,c* and d*. In the listed figures branch presented in dot line correspond to unstable stationary vibration regimes.

2. THE TIME HARMONICS SHAPES AND THEIR MUTAL INFLUENCE

If we presents the time functions at nm-mode of oscillations of the plate systems in form of sum of two harmonics:

$$T_{(i)nm}(t)=\sum_{j=1}^{2} T_{(i)nm_j}(t)=\sum_{j=1}^{2} K_{inm}^{(j)} a_{jnm}(t)\cos\Phi_{jnm}(t) \quad , \; i=1,2 \tag{7}$$

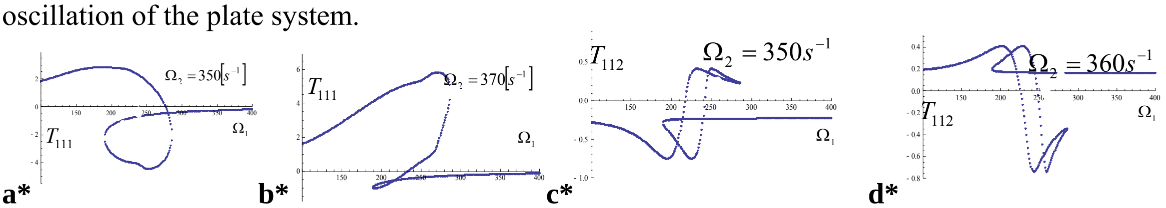
where, we use the change of amplitudes and phases by (3)-(4), and since, we have the numerical results, we are in position to present the shape of harmonics depending of frequencies of external excitations. For the chosen parameters of the system thas two harmonics in the 11-mode of oscillations has the following form:

$$T_{111_1}(\Omega_1,\Omega_2)=5.906\cdot a_1(\Omega_1,\Omega_2)\cos\left[\Omega_{2(1)}\cdot t+\phi_1(\Omega_1,\Omega_2)\right] \tag{8a}$$

$$T_{111_2}(\Omega_1,\Omega_2)=3.32\cdot a_2(\Omega_1,\Omega_2)\cos\left[\Omega_{2(1)}\cdot t+\phi_2(\Omega_1,\Omega_2)\right] \tag{8b}$$

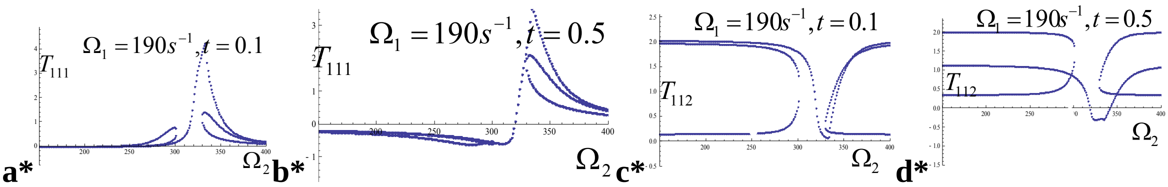
where $\Omega_{2(1)}$ means that, we use the discrete values of the $\Omega_2=const$ or of the $\Omega_1=const$ of the external frequencies, and $a_i(\Omega_1,\Omega_2)$ or $\phi_i(\Omega_1,\Omega_2)$ means that, we use amplitude or phase stationary response $a_i(\Omega_1,\Omega_2=const)$ and $\phi_i(\Omega_1,\Omega_2=const)$ in one case and $a_i(\Omega_1=const,\Omega_2)$ and $\phi_i(\Omega_1=const,\Omega_2)$ in the other case.

In regard that, we considered the stationary regimes of vibrations in nm-mode of oscillations we have to use the particular moment in time so we use that the $t=1\left[s^{-1}\right]$. In the Figs. 4 and 5 we present the shapes of the some time harmonics like as the shape of the time function in the 11-mod of oscillation of the plate system.



a* b* c* d*

Fig. 4. The shapes of the first (a, b*) and second (c*, d*) eigen forced time non-linear harmonic* $T_{111_1}(\Omega_1,\Omega_2)=5.906\cdot a_1(\Omega_1,\Omega_2=const)\cos[\Omega_1\cdot t+\phi_1(\Omega_1,\Omega_2=const)]$ *(a*, b*) and* $T_{111_2}(\Omega_1,\Omega_2)=3.32\cdot a_2(\Omega_1,\Omega_2=const)\cos[\Omega_2\cdot t+\phi_2(\Omega_1,\Omega_2=const)]$ *(c*, d*), for the different value of excited frequency* Ω_{1nm} *and for discrete value of excited frequency* $\Omega_{2nm}=const$ *.*



a* b* c* d*

Fig. 5. The evolution of the shape of the first (a, b*) and second (c*, d*) eigen forced time non-linear harmonic* $T_{111_1}(\Omega_1,\Omega_2)=5.906\cdot a_1(\Omega_1=190,\Omega_2)\cos[190\cdot t+\phi_1(\Omega_1=190,\Omega_2)]$ $T_{111_2}(\Omega_1,\Omega_2)=3.32\cdot a_2(\Omega_1=190,\Omega_2)\cos[190\cdot t+\phi_2(\Omega_1=190,\Omega_2)]$ *for the different value of excited frequency* Ω_{2nm} *and for discrete value of excited frequency* $\Omega_{1nm}=190s^{-1}$*, with* $t=0.1\left[s^{-1}\right]$ *and* $t=0.5\left[s^{-1}\right]$

CONCLUSIONS

For analyze of stationary regimes of non-linear oscillations for presented model, we solved system of PDE`s (1) semi analyticaly in asymptotic first approximation. One part of solutions, were obtained numerically and presents amplitudes-frequencies and phase-frequency characteristics with identification, in the first asymptotic approximations, interaction of the non-linear component mods and non-linear resonant interactions, in the displacement of the plate middle surface points. For the case of the external excitation by two frequency force and resonant range of the frequencies, we conclude complexity in the system non-linear response, depending of initial conditions and also of other system kinetic parameters and corresponding relation between these sets of the kinetic parameters.

Acknowledgment. *Parts of this research were supported by the Ministry of Sciences and Environmental Protection of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON144002 and Faculty of Mechanical Engineering University of Nis.*

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