Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

STABILITY FOR A CLASS OF SYSTEMS WITH UNCERTAIN STRUCTURE

Ievgen Ivanov[1]
Taras Shevchenko National University of Kyiv
Kyiv, Ukraine

ABSTRACT

In the article a class of continuous-discrete dynamical systems with switching at uncertain moments of time is constructed on the basis of possibility theory. A notion of stability with given necessity level is introduced and stability properties of systems of constructed class are investigated.

INTRODUCTION

Recent years have witnessed growing interest in hybrid systems, e.g. dynamical systems which combine continuous dynamics and switching between several different discrete states. Well known formal models of such systems include switched systems [1], hybrid automata [2], etc. In many practical applications exact circumstances or moments of switching in these systems are unknown and therefore at any moment of time, active set of laws which govern continuous dynamics ("structure" of the system) can not be determined. In such cases it is reasonable to model switching as a random or likewise "uncertain" process. One promising approach of this kind is the use of systems with random structure [3], in which switching is modeled as (continuous time) jump Markov process. However stochastic models are difficult to apply in cases when statistical information about switching process is absent, because the distribution of a random process can not be estimated. In this situation models based on non-probabilistic uncertainty theories can be more adequate.

One promising theory of such kind is the possibility theory [4, 5, 6, 7], which allows one to estimate a level of credibility of some event with respect to other events. In this theory each event is characterized by two numeric values – levels of possibility and necessity. Furthermore only relative comparison of levels (more, less, equal) is meaningful. For chosen basic events these levels are usually determined on the basis of expert opinions (instead of statistics). For non-basic events levels can be determined with a help of possibility and necessity composition rules, or more generally, possibility and necessity measure extension theorems [5, 8, 9].

In this paper we present a formal model – a system with fuzzy structure (note that the term *fuzzy* is used in this paper in sense of possibility theory rather then L. Zadeh fuzzy set theory), analogous to systems with random structure, but constructed on the basis of possibility theory. Despite its formal similarity to stochastic models, its properties and associated methods of investigation are quite different. But there is a reason to believe that it can be useful in the case of absence of statistical information. Systems with fuzzy structure require introduction of special notions of stability. We will define a notion of stability with given necessity level and provide sufficient conditions for stability of trivial equilibrium point of systems with fuzzy structure.

The paper is organized as follows: in section 1 we give necessary preliminaries on fuzzy Markov processes and possibility theory, in section 2 we introduce systems with fuzzy structure, in section 3 we introduce the notion of stability with given necessity level and investigate stability of trivial equilibrium points of systems with fuzzy structure with a help of a variant of Lyapunov comparison principle [10].

1. FUZZY MARKOV PROCESSES

Much like in probability theory, in possibility theory uncertain quantities (possibilistic variables) are formalized as (measurable) functions defined on a space of (atomic) events. But instead of probability measure, possibility and necessity measures are used. Fuzzy processes can be viewed as

[1] Corresponding author. Email ivanov.eugen@gmail.com

time-varying possibilistic variables. A notion of independence of possibilistic variables [4, 6] allows to define class fuzzy Markov processes [11] analogously to the class of stochastic Markov processes.

Let X be a space of atomic events. Let $P : 2^X \to [0,1]$ be a possibility measure, i.e. $P(\bigcup_k A_k) = \sup_k P(A_k)$ for any family $\{A_k\}_k$ of subsets of X (non-atomic events), and $N : 2^X \to [0,1]$ be a necessity measure, i.e. $N(\bigcap_k A_k) = \inf_k N(A_k)$ for any family $\{A_k\}_k$. Note that assumption of totality of measures is not restrictive because possibility measure extension theorem [5, 9] guarantees that possibility (and necessity) measure on algebra of sets can be extended to power set. We assume that measures are normed and coherent, i.e. equalities $N(A) = 1 - P(\neg A)$ for all $A \subseteq X$ (here $\neg A$ denotes complement of a set), $P(X) = 1$ and $N(X) = 1$ hold.

Now we briefly recall some notions of possibility theory. Denote by X_0 the set of atomic events of non-zero possibility, T – the timeline $[0,+\infty)$, Y – a set. For total of measures, possibilistic variable is an arbitrary partial function $X \to Y$, defined on a superset of X_0. Similarly, a (continuous-time) fuzzy process in possibility theory is a partial function $T \times X \to Y$, defined on a superset of $T \times X_0$.

A (possibility) distribution of possibilistic variable $\xi : X \to Y$ is a mapping $y \mapsto P\{x \in X_0 \mid \xi(x) = y\}$. A distribution of a fuzzy process $p : T \times X \to Y$ is a functional $F_p : 2^{T \to Y} \to L$, defined by equality $F_p(q) = P\{x \in X \mid \forall t\, p(t,x) = q(t)\}$, where $q : T \to Y$, i.e. F_p gives a possibility of q to be a trajectory of p. An α-trajectory of p (where $\alpha \in [0,1)$) is a mapping $q : T \to Y$, such that $F_p(q) > \alpha$, i.e. q is a trajectory of p with possibility greater then α.

Like in theory of stochastic processes, an important role in fuzzy processes play fuzzy Markov processes. Let q_1, q_2 be trajectories of a fuzzy process p, which intersect at some moment $t^* \in T$, i.e. $q_1(t^*) = q_2(t^*)$. Then cross trajectories of q_1 and q_2 at t^* are defined as functions $\overline{q}_1$, $\overline{q}_2$, such that $\overline{q}_1(t) = q_1(t)$ if $t \le t^*$, $\overline{q}_1(t) = q_2(t)$ if $t \ge t^*$ and $\overline{q}_2(t) = q_2(t)$ if $t \le t^*$, $\overline{q}_2(t) = q_1(t)$ if $t \ge t^*$. Informally, $\overline{q}_1$ and $\overline{q}_2$ are obtained by gluing together parts of q_1 and q_2 before and after t^*.

Definition 1.1 [11]. A fuzzy process p has Markov property if for every α-trajectories q_1, q_2 of p, such that $q_1(t^*) = q_2(t^*)$ for some t^*, cross trajectories of q_1 and q_2 at t^* are itself α-trajectories of p.

This definition can be viewed as a formalization of a property of independence of future and past in the case of fixed present.

In this paper we consider fuzzy Markov processes with piecewise-constant trajectories.

Definition 1.2 [11]. A fuzzy Markov process p is called a fuzzy jump Markov process if for each trajectory q of p (which have non-zero possibility) the following conditions are satisfied:

1) q is piecewise constant and right-continuous;

2) $P\{x \mid \forall t\, p(t,x) = q(t)\} = \lim_{t^* \to +\infty} P\{x \mid \forall t \le t^*\, p(t,x) = q(t)\}$ (continuity of possibility).

Distribution of a fuzzy jump Markov process $p : T \times X \to I$ (where I is a state space) is uniquely determined by its transition (possibility) distribution – an indexed family $(\varphi_{i,j})_{i,j \in I}$ of functions, defined as $\varphi_{i,j}(t) = P\{x \mid p(t,x) = i, \lim_{\tau \to t+} p(\tau,x) = j\}$, $t \in T$, i.e. $\varphi_{i,j}(t)$ is a possibility of transition from i to j at moment t. The following lemma allows to compute distribution of p:

Lemma 1.1 [11]. $F_p(q) = P\{x \mid \forall t\, p(t,x) = q(t)\} = \inf_{t \in T} \varphi_{q(t),q(t+)}(t)$ for every piecewise constant right-continuous function $q : T \to Y$ (where $q(t+)$ denotes right limit).

Not every family of functions $(\varphi_{i,j})_{i,j \in I}$ can be a transition distribution of some fuzzy jump Markov process. Unlike situation in probability theory, conditions for a family of functions to be a transition possibility distribution are not simple.

Lemma 1.2 [11]. A family $(\varphi_{i,j})_{i,j \in I}$ is a transition distribution of some fuzzy jump Markov process if and only if the following conditions are satisfied:

1) $\sup_{i,j \in I} \varphi_{i,j}(t) = 1$ for all $t \in T$;

2) $\varphi_{i,j}(t_0) = \sup\left\{\inf_{t\in T} \varphi_{q(t),q(t+)}(t) \mid \right.$ a function $q: T \to I$ is piecewise constant, right-continuous and $q(t_0) = i$, $\left. q(t_0+) = j\right\}$, for all $i, j \in I$, $t_0 \in T$.

It is hard to expect that transitions possibility levels derived from expert opinions will satisfy the second condition of this lemma. It is better to derive upper bounds for possibility levels of transitions (or alternatively, lower bounds for necessity levels) from expert opinions, and then find a transition distribution which (optimally) fits these bounds.

Formally, a family of functions $(\psi_{i,j})_{i,j\in I}$ (where $\psi_{i,j} : T \to [0,1]$) is called an upper transition distribution, if there exists a transition distribution $(\varphi_{i,j})_{i,j\in I}$, such that $\varphi_{i,j}(t) \le \psi_{i,j}(t)$ for all $i, j \in I$, $t \in T$. A transition distribution, (optimally) generated by upper transition distribution $(\psi_{i,j})_{i,j\in I}$ is defined as a transition distribution $(\varphi_{i,j})_{i,j\in I}$, such $\varphi_{i,j}(t) \le \psi_{i,j}(t)$, $i, j \in I$, $t \in T$ and $\varphi'_{i,j}(t) \le \varphi_{i,j}(t)$, $i, j \in I$, $t \in T$ for each transition distribution $(\varphi'_{i,j})_{i,j\in I}$, such that $\varphi'_{i,j}(t) \le \psi_{i,j}(t)$, $i, j \in I$, $t \in T$. Existence of generated distributions is guaranteed by the following lemma.

Lemma 1.3. Each upper transition distribution generates (unique) transition distribution.

Still not every family of functions $(\psi_{i,j})_{i,j\in I}$ is an upper transition distribution. However is it expected that expert opinions can be used to from upper transition distribution [11].

Lemma 1.4. A family $(\varphi_{i,j})_{i,j\in I}$ is an upper transition distribution if and only if $\sup\left\{\inf_{t\in T} \varphi_{q(t),q(t+)}(t) \mid q: T \to I \text{ is piecewise constant and right-continuous } \right\} = 1$.

In this paper we do not consider problems of checking of condition of lemma 1.4 and finding generated transition distribution. We only note that for certain classes of functions $(\psi_{i,j})_{i,j\in I}$, generated upper transition distribution can be effectively computed by iterative numerical methods. The following definition describes one important example of such class.

Definition 1.3. A transition distribution $(\varphi_{i,j})_{i,j\in I}$ is called piecewise-monotone if for every $t_0 \in T$ there exists a relatively open (in T) neighborhood $O(t_0)$ of t_0, such that every function $\varphi_{i,j}$, $i, j \in I$ is monotone on sets $O \cap [0, t_0)$ and $O \cap (t_0, +\infty)$ (if they are non-empty).

Note that the character of monotonicity (increasing or decreasing) of functions on these sets can be different for the same or different $i, j \in I$.

2. SYSTEMS WITH FUZZY STRUCTURE

Let I be a non-empty finite set of states, $T = [0, +\infty)$, $p: T \times X \to I$ – a fuzzy jump Markov process, $f_i : T \times \mathbf{R}^d \to \mathbf{R}^d$, $i \in I$ – a family of functions.

Definition 2.1. A system with fuzzy structure (SFS) is an equation of the form

$$\dot{y}(t,x) = f_{p(t,x)}(t, y(t,x)) \tag{1}$$

Definition 2.2. A fuzzy process $y: T \times X \to \mathbf{R}^d$ is called a solution of SFS (1) if for any (fixed) $x \in X_0$, a trajectory $t \mapsto y(t,x)$ satisfies equation in sense of Caratheodory (i.e. is absolutely continuous on every compact segment in T and satisfies (1) almost everywhere with respect to Lebegue measure).

Definition 2.3. Let $\alpha \in [0,1)$. A total function $\bar{y}: T \to \mathbf{R}^d$ is called a (complete) α-trajectory of SFS (1) if $\bar{y}$ is an α-trajectory of some solution of (1).

Consider initial condition

$$y(0,x) = y_0 \text{ for every } x \in X_0 \tag{2}$$

We say that a problem (1), (2) has unique solution (up to trajectories of possibility zero) if every two solutions of (1) satisfying (2) coincide on $T \times X_0$.

The following theorem is an adaptation of Caratheodory existence theorem to SFS.

Theorem 2.1 [11]. Suppose that the following conditions are satisfied:

1) for each $i \in I$ and $t \in T$, a function $y \mapsto f_i(t,y)$ is defined and continuous on $\mathbf{R}^d$, and for each $y \in \mathbf{R}^d$, a function $t \mapsto f_i(t,y)$ is measurable; $t \in T$

2) for every $i \in I$ there exists a function $h_i : T \to \mathbf{R}_+$, bounded on every bounded segment in $\mathbf{R}$, such that $\|f_i(t,y)\| \le h_i(t)(1+\|y\|)$ for all $t \in T$, $y \in \mathbf{R}^d$ (here $\mathbf{R}_+ = [0,+\infty)$, $\|\cdot\|$ denotes Euclidean norm);

3) for every $i \in I$ there exists a function $L_i : T \to \mathbf{R}_+$ (Lipschitz constant), bounded on every bounded segment in $\mathbf{R}$, such that $\|f_i(t,y_1) - f_i(t,y_2)\| \le L_i(t)\|y_1 - y_2\|$ for all $y_1, y_2 \in \mathbf{R}^d$.

Then for every $y_0 \in \mathbf{R}^d$ the problem (1), (2) has a unique solution.

Denote by $(\varphi_{i,j})_{i,j\in I}$ a transition distribution of the process p. The following theorem is a consequence of the lemma 1.1.

Theorem 2.2. Suppose that conditions of the theorem 2.1 are satisfied. Then a function $\overline{y} : T \to \mathbf{R}^d$ is an α-trajectory of (1) if and only if there exists a piecewise constant and right-continuous function $q : T \to I$ such that $\inf_{t\in T} \varphi_{q(t),q(t+)}(t) > \alpha$, and $\overline{y}$ satisfies equation $\dot{y}(t) = f_{q(t)}(t, y(t))$ on T in sense of Caratheodory.

3. STABILITY OF EQUILIBRIUM POINTS OF SYSTEMS WITH FUZZY STRUCTURE

In this section we assume that conditions of theorem 2.1 are satisfied and hence the problem (1), (2) has unique solution for every initial value. Also we assume that transition distribution of fuzzy jump Markov process p in piecewise-monotone.

Denote by $\mathbf{0}$ a fuzzy process $T \times X \to \mathbf{R}^d$ which is identically equal to null vector. We say that SFS (1) has trivial equilibrium point if $\mathbf{0}$ is a solution of (1).

Suppose that (1) has trivial equilibrium point.

Definition 3.1. Let $\alpha \in [0,1)$. Trivial equilibrium point of SFS (1) is called stable with necessity $1-\alpha$ if there exist an open neighborhood of the origin $O \subseteq \mathbf{R}^d$ and a function $h : \mathbf{R}_+ \to \mathbf{R}_+$ of class K (i.e. continuous, strictly increasing and $h(0) = 0$) such that

$$N\{x \mid \|y(t,x;y_0)\| \le h(\|y_0\|) \text{ for all } y_0 \in O \text{ and } t \in T\} > 1-\alpha .$$

This definition means that $\|y(t,x;y_0)\| \le h(\|y_0\|)$ holds for all atomic events x which have possibility greater then α. Note that the property of stability with given necessity level is possibilistic, i.e. it depends on distribution of the process p.

Let us introduce the following notation: $y(t,x;y_0)$ is a value of trajectory of solution of (1) which satisfies $y(0,x) = y_0$ for all $x \in X_0$. It follows from conditions of theorem 2.1 and Caratheodory existence theorem that initial value problem $\dot{y}(t) = f_i(t, y(t))$, $y(t_0) = y_0$ has a unique solution on $[t_0,+\infty)$ for every t_0, y_0. Denote by $y_i(t;t_0;y_0)$ a value of this solution at time moment t.

Now we are going to investigate stability with necessity $1-\alpha$ of trivial equilibrium point of SFS. The first step is to characterize points reachable by α-trajectories of SFS (1).

For any set $Y_0 \subseteq \mathbf{R}^d$ and $\overline{t} \in T$ let us define a closure of α-reach set:

$$cReach^{\alpha}(Y_0,\overline{t}) = cl(\{\overline{y}(\overline{t}) \mid \overline{y} : T \to \mathbf{R}^d \text{ is an } \alpha\text{-trajectory of SFS (1) and } y(0) \in Y_0\}),$$ where $cl(\cdot)$ denotes closure of a subset of $\mathbf{R}^d$, i.e. $cReach_*^{\alpha}(Y_0,\overline{t})$ is a closure of the set of points reachable by α-trajectories of SFS (1) from Y_0 at moment of time $\overline{t}$.

We will use the following lemma to describe $cReach_*^{\alpha}(Y_0,\overline{t})$:

Lemma 3.1. Let I be a finite set, $(\varphi_{i,j})_{i,j\in I}$ be a piecewise-monotone transition distribution and $\alpha \in [0,1)$. Then there exists a sequence of moments of time $\tau_0, \tau_1, \tau_2, ... \in T$ such that for every $k = 0,1,2,...$ and $i, j \in I$ a function $\varphi_{i,j}$ is monotonous on (τ_k, τ_{k+1}) and either $\varphi_{i,j}(t) > \alpha$ for all $t \in (\tau_k, \tau_{k+1})$ or $\varphi_{i,j}(t) \le \alpha$ for all $t \in (\tau_k, \tau_{k+1})$.

In this paper we do not discuss computation of the sequence $\tau_0, \tau_1, \tau_2, ...$ from given transition distribution. However we note that if functions $\varphi_{i,j}$ are defined symbolically by suitable lattice terms, then a general expression for members of sequence $\tau_0, \tau_1, \tau_2, ...$ can also be computed symbolically.

Suppose that $\alpha \in [0,1)$ is fixed and a sequence $\tau_0, \tau_1, \tau_2, ...$ described in lemma 3.1 is given. For all $i, j \in I$ and $t_0, t_1 \in T$, such that $t_0 < t_1$, denote:

$LS^{\alpha}_{i,j}(t_0,t_1) = \{ii_1i_2...i_n \in I^+ \mid n \ge 1,\ \ \varphi_{i,i_1}(t_0) > \alpha,\ \ \varphi_{j,j}(t_1-) > \alpha,\ \ i_n = j,\ \ (i_l,i_{l+1}) \in H^{\alpha}(t_0,t_1)$ for all $l = \overline{1,n-1}\}$, where $H^{\alpha}(t_0,t_1) = \{(i,j) \in I^2 \mid \forall t \in (t_0,t_1)\, \varphi_{i,j}(t) > \alpha\}$.

Note that $LS^{\alpha}_{i,j}(t_0,t_1)$ is a regular language in alphabet I.

For any formal language $L \subseteq \mathbf{R}^d$, moments $t_0 < t_1$ and set $Y_0 \subseteq \mathbf{R}^d$ let us define: $reach(L,t_0,Y_0,t_1) = \{\overline{y}(t_1) \mid \ \overline{y}:[t_0,t_1] \to \mathbf{R}^d$ is a function such that $\overline{y}(t_0) \in Y_0$ and there exists a piecewise-constant function $q:[t_0,t_1] \to I$ and time moments $\overline{t_0},...,\overline{t_n} \in T$ such that $t_0 = \tau_0 < \tau_1 < ... < \tau_{n-1} < \tau_n = t_1$, $q(t) = i_{k+1}$ for all $t \in (\tau_k,\tau_{k+1})$, $k = \overline{0,n-1}$, and $\overline{y}$ satisfies equation $\dot{y}(t) = f_{q(t)}(t,y(t))$ in sense of Caratheodory$\}$, i.e. a set of points reachable from Y_0 by means of switching sequences described by words in L.

Also let us define an indexed family of sets:

$Y^0_{i,j} = Y_0$ if $i = j$, and $Y^0_{i,j} = \varnothing$ if $i \ne j$;

$Y^k_{i,j} = \bigcup_{l \in I} reach(LS^{\alpha}_{l,j}(\tau_{k-1},\tau_k),\tau_{k-1},Y^{k-1}_{i,l},\tau_k)$, $i,j \in I$, $k \ge 1$.

The following theorem describes $cReach^{\alpha}_{*}$.

Theorem 3.1. Let $\overline{t} \in [\tau_n,\tau_{n+1})$ for some n. Denote $J_0(\overline{t}) = \{j \in I \mid \max_{j' \in I} \varphi_{j,j'}(\overline{t}) > \alpha\}$.

1) If $\overline{t} = \tau_n$, then $cReach^{\alpha}(Y_0,\overline{t}) = \bigcup_{i \in I, j \in J_0(\overline{t})} cl(Y^n_{i,j})$;

2) If $\overline{t} \in (\tau_n,\tau_{n+1})$, then $cReach^{\alpha}(Y_0,\overline{t}) = \bigcup_{i,l \in I, j \in J_0(\overline{t})} cl(reach(LS^{\alpha}_{l,j}(\tau_n,\overline{t}),\tau_n,Y^n_{i,l},\overline{t}))$.

This theorem characterized reachable points of phase space. But we also need to characterize reachable (discrete) states.

To compute reachable states let us denote $nxt_k(I_0) = \{j \in I \mid LS_{i,j}(\tau_k,\tau_{k+1}) \ne \varnothing\}$ for any $I_0 \subseteq I$, $k \ge 0$ and build the following sequence of state sets: $I_0 = J_0(0)$ and $I_{k+1} = nxt_k(I_k)$, $k \ge 0$. Note that this sequence becomes periodic after some k due to finiteness of I. Then the sequence of sets states reachable at some time moment in $[\tau_k,\tau_{k+1})$ can be computed as follows: $I^{\alpha}_k = \{i \in I \mid w_1 i w_2 \in LS^{\alpha}_{i',j'}(\tau_k,\tau_{k+1}), i' \in I_k, j' \in I_{k+1}, w_1 \in I^*, w_2 \in I^+\}$, $k \ge 0$.

Now we can formulate a theorem which gives sufficient condition for stability of SFS.

Let $O \subseteq \mathbf{R}^d$ be some open neighborhood of the origin and $V:\mathbf{R}^d \to \mathbf{R}_+$ be a continuously differentiable positive-definite (Lyapunov-like) function. Let $g_k:\mathbf{R}_+ \times \mathbf{R} \to \mathbf{R}$, $k \ge 0$ be a sequence of functions such that $\max_{i \in I^{\alpha}_k} \dfrac{dV(y)}{dy} f_i(t,y) \le g_k(t,V(y))$ for all $t \in T$, $y \in \mathbf{R}^d$, $k \ge 0$ and initial value problem $\dot{z}(t) = g_k(t,z(t))$, $z(t_0) = z_0$ has unique solution in sense of Caratheodory on $[t_0,+\infty)$ for all $t_0 \in T$, $z_0 \in \mathbf{R}_+$, $k \ge 0$. Denote by $z_k(t;t_0;z_0)$ the value of this solution at moment t.

Theorem 3.2. Let $\alpha \in [0,1)$. Suppose that there exists an indexed family of monotonically non-decreasing functions $r^{\alpha}_k:\mathbf{R}_+ \to \mathbf{R}_+$, $k \ge 0$ such that $r^{\alpha}_k(0) = 0$ for all $k \ge 0$, $\max_{t \in [\tau_k,\tau_{k+1}]} z_k(t;\tau_k;v) \le r^{\alpha}_k(v)$ for all $v \in \mathbf{R}_+$, and 0 is a Lyapunov-stable equilibrium point of recurrence relation $v_{k+1} = r^{\alpha}_k(v_k)$, $k \ge 0$. Then the trivial equilibrium point of SFS (1) is stable with necessity $1-\alpha$.

Proof. From conditions of the theorem it follows that there exists $\varepsilon > 0$ and a function $h:\mathbf{R}_+ \to \mathbf{R}_+$ of class K such that $v_k \le h(v_0)$ for all $k \ge 0$ if $v_0 \in [0,\varepsilon)$. The function V is positive definite and continuous, therefore there exists an open neighborhood of the origin $O_1 \subseteq \mathbf{R}^d$, and a function $h_1:\mathbf{R}_+ \to \mathbf{R}_+$ of class K such that $\|y\| \le h_1(\|y_0\|)$ if $y_0 \in O_1$ and $V(y) \le h(V(y_0))$ (we can define h_1 to be a function of class K such that $h_1(v) \ge \max\{\|y\| \mid y$ belongs to a connected component of the set $\{y \in O_1 \mid V(y) \le h(\max_{\|y_0\| \le v} V(y_0))\}$, which contains null vector$\}$).

Let $\overline{y}:T \to \mathbf{R}^d$ be some α-trajectory of SFS (1). Then there exists an α-trajectory $q:T \to \mathbf{R}^d$ of the process p such that $\overline{y}$ satisfies equation $\dot{y}(t) = f_{q(t)}(t,y(t))$ in sense of Caratheodory. For any $k \ge 0$ an inclusion $\{q(t) \mid t \in [\tau_k,\tau_{k+1})\} \subseteq I^{\alpha}_k$ holds. Therefore

$\frac{dV(\bar{y}(t))}{dy} f_{q(t)}(t, \bar{y}(t)) \le g_k(t, V(\bar{y}(t)))$ for all $t \in [\tau_k, \tau_{k+1})$. The function V is continuously differentiable and $\bar{y}$ is absolutely continuous on every bounded segment. From this it follows that that the function $t \mapsto V(\bar{y}(t))$ is also absolutely continuous on every bounded segment. Also the function $t \mapsto V(\bar{y}(t))$ satisfies $\frac{dV(\bar{y}(t))}{dt} = \frac{dV(\bar{y}(t))}{dy} f_{q(t)}(t, \bar{y}(t))$ almost everywhere. From the comparison theorem [12] for Caratheodory differential inequality $\frac{dV(\bar{y}(t))}{dt} \le g_k(t, V(\bar{y}(t)))$ and differential equation $\dot{z}(t) = g_k(t, z(t))$ we conclude that $V(\bar{y}(t)) \le z_k(t; \tau_k; V(\bar{y}(\tau_k))) \le r_k^\alpha(V(\bar{y}(\tau_k)))$ for all $k \ge 0$ and $t \in [\tau_k, \tau_{k+1}]$. In particular case an inequality $V(\bar{y}(\tau_{k+1})) \le r_k^\alpha(V(\bar{y}(\tau_k)))$ holds. Suppose that $v_0 = V(\bar{y}(\tau_0)) = V(\bar{y}(0)) < \varepsilon$. Then from monotonicity of r_k^α we conclude that $V(\bar{y}(\tau_k)) \le v_k \le h(v_0) = h(V(\bar{y}(0)))$ for all $k \ge 0$, and therefore $V(\bar{y}(t)) \le h(V(\bar{y}(0)))$ for all $t \in T$. Hence $\|\bar{y}(t)\| \le h_1(\|\bar{y}(0)\|)$ if $\bar{y}(0) \in O_1$. Note that O_1 and h_1 does not depend on chosen α-trajectory $\bar{y}$. Therefore trivial equilibrium point of SFS (1) is stable with necessity $1-\alpha$. Theorem is proved.

This theorem reduces the problem of determining stability of trivial equilibrium point of SFS to a known problem of determining Lyapunov stability of equilibrium point of a recurrence relation.

CONCLUSIONS

On the basis of possibility theory we have constructed a class of continuous-discrete dynamical systems with switching at uncertain moments of time – a class of systems with fuzzy structure. We have introduced a notion of stability with given necessity level and have studied stability properties of systems with fuzzy structure. Obtained results can be useful for modeling of dynamical systems with uncertainty in cases when application of probabilistic models is hard or impossible due to lack of statistical information.

REFERENCES

[1] Liberzon D. *Switching in systems and control*, Birkhauser, Boston, 2003.
[2] Henzinger T.A. The theory of hybrid automata *Proceedings of the 11th Annual Symposium on Logic in Computer Science*, pp. 278-292, 1996.
[3] Kats I.Y., Martynyuk A.A. *Stability and stabilization of nonlinear systems with random structure*, Taylor and Francis, London, 2002.
[4] Pyt'ev Yu.P. *Possibility as an alternative of probability*, Fizmatlit, Moscow, 2007.
[5] Cooman G. Possibility theory I: the measure- and integral-theoretic groundwork *Int. J. of General Systems*, Vol. 25, pp. 291-323, 1997.
[6] Belov Yu.A., Bychkov O.S., Merkuryev M.G., Chulichkov A.I. About an approach to modeling of fuzzy dynamics *Reports of the National Academy of Sciences of Ukraine*, Vol. 10, pp. 14-19, 2006 (in Ukrainian).
[7] Zadeh L.A. Fuzzy sets as a basis for a theory of possibility *Fuzzy Sets and Systems*, Vol. 1, pp. 3-28, 1978.
[8] Bychkov A.S., Ivanov Ie.V. On convergence of sequences of fuzzy perceptive elements defined on different possibility spaces *Cybernetics and systems analysis*, Vol. 2, pp.142-151, 2010 (in Russian).
[9] Bychkov O.S., Ivanov Ie.V. Existence of coherent k-additive extension of measures in possibility theory *Control systems and computers*, Vol. 6, pp. 35-41, 2009 (in Ukrainian).
[10] Matrosov V. M.; Malikov A. I. The development of the ideas of A. M. Lyapunov over one hundred years: 1892-1992. *Izv. Vyssh. Uchebn. Zaved. Mat.*, Vol. 4, pp. 3-47, 1993 (in Russian).
[11] Ivanov Ie.V. Modeling of systems of fuzzy structure *Proceedings of the International Scientific Conference "Simulation-2010"*, Kyiv, pp. 78-85, 2010 (in Russian).
[12] Lakshmikantham V., Leela S. *Differential and integral inequalities*, Vol. 1, Academic Press, New York, 1969.