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STOCHASTIC STABILITY OF THE DEFORMABLE FORMS AND VIBRATION MODES OF A PARAMETRICALLY EXCITED SANDWICH DOUBLE HEREDITARY BEAM SYSTEM

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Paper is dedicated to the honor of important scientist and nice person, ***Professor S. T. Ariaratnam,*** *University of Waterloo, Canada.*

ABSTRACT

The coupled partial integro-differential equations of transversal stochastic vibrations of a parametrically excited sandwich double hereditary beam system were derived. The beams are graded by a hereditary material with known relaxation kernel, and it is subjected by axial stochastic external excitations. The influence of rotatory inertia of beam cross sections and transverse shear of beam cross sections under the transverse forces, and the corresponding members in the partial integro-differential equations are taken into account. Bernoulli particular integral method and Lagrange method of variation constant are used for the transformation problem of solutions. The asymptotic averaged method Krilov-Bogolyuvov-Mitropolskiy is used for obtaining the first approximation of Itô stochastic differential equations and Stratonovich results. By using idea of Ariaratnam the sets of Lyapunov exponents are obtained.

INTRODUCTION

The transversal vibration beam problem is classical, but in current university books on vibrations, we can find only the Euler-Bernoulli's classical partial differential equation for describing transversal beam vibrations. In monograph [20] we can find a non-linear partial differential equation for describing transversal vibrations of the beam with non-linear constitutive stress-strain relation. By using the asymptotic method of Krilov-Bogolyubov- Mitropolskiy [20, 21], many authors studied one frequency or multi-frequency non-linear oscillation regimes of deformable bodies. Specially, Hedrih [9, 9, 10, 11] studied one-single and two-frequency stationary and non-stationary regimes of non-linear transversal and forced vibration of beams. Transversal vibrations of the beam on the elastic Winckler's foundation under the action of multi-frequency forces with frequencies in the form of the first frequency resonant range of the beam was also studied by Hedrih [9], and some results of transversal vibrations of beams graded by a creep and hereditary material, were presented in References [12,13, 16].

In the university book [22] by Rašković, an extended partial differential equation of transversal ideally elastic beam vibrations was presented considering the inertia rotation of the beam's cross sections and transverse shear of the cross section. Also, in numerous papers, by using the partial differential equation of the transversal ideally elastic beam vibrations with members, by which influences of the inertia rotation of the beam's cross sections and transverse shear of the cross section by transversal forces are taken into account, and based on the monograph [19] by Nowatski as the scientific source, the complex properties of the transversal vibrations of the beam are investigated.

In paper [1] by Ariaratnam stochastic stability of visco-elastic systems under bounded noise excitation was investigated. For small damping and weak random fluctuation, asymptotic expressions are derived for the Lyapunov exponent and the rotation number using the method of stochastic averaging. From the sign of the Lyapunov exponent, the condition for asymptotic stability with probability 1 of the trivial equilibrium state is obtained. The stochastic almost-sure stability of a single degree-of-freedom linear visco-elastic system subjected to random fluctuation in the stiffness

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parameter is investigated by Ariaratnam S. T. [2]. For small damping and weak random fluctuation, asymptotic expressions are derived for the Lyapunov exponent and the rotation number using the method of stochastic averaging. From the sign of the Lyapunov exponent, the condition for asymptotic stability with probability 1 of the trivial equilibrium state is obtained. In the paper [3] by Ariaratnam S. T. and Xie W.C. wave localization of a long continuous beam over several supports is studied. The localization factor is related to the larger of the two Lyapunov exponents associated with a product of 1-1 random wave transfer matrices. By using a theorem- due to Furstenberg- on the asymptotic properties of a product of independent and identically distributed random matrices\ the localization factors are calculated by a combination of analytical and numerical simulation methods.

In the paper [4] by Ariaratnam and Wei-Chau Xijz buckling mode localisation in large randomly disordered one-dimensional structures is studied. Furstenberg's theorem on the limiting behaviour of the product of random matrices is employed to determine the Lyapunov exponent and the localisation factor. Green's function formulation is applied to show that although the buckling loads are different for different sample structures, the buckling loads satisfy a probability distribution which depends only on the disorder parameters and is independent of the specific sample realisations for large structures. Due to the positivity of the Lyapunov exponent it is found that localised modes (bulges) may be visible for an arbitrary value of load close to the buckling loads if there exist perturbations or imperfections. In the paper [5] the dynamic stability of non-gyroscopic viscoelastic systems under multiple parametric excitations is investigated. The largest Lyapunov exponent as an indicator of the almost-sure asymptotic stability of the system is obtained by applying the stochastic averaging method together with Khasminskii's technique. The integral term arising from the viscoelastic effect is averaged by making use of Larianov's method. As an application, the flexural–torsional instability of a deep rectangular viscoelastic beam under stochastically fluctuating central load and end moments applied simultaneously is investigated. Both cases of follower and non-follower central fluctuating load are included in this analysis. Also, in paper [6] Ariaratnam and Abdelrahman presented results about stochastic stability of non-gyroscopic visco-elastic systems.

In the papers [14, 16] the influence of rotatory inertia of beam cross section and transverse shear of beam cross section under the transverse force, and the corresponding members in the partial differential equation are taken into account and by use Ariaratnam's idea [1] the expression for Lyapunov exponents are obtained and the stochastic stability of beam deformable forms and processes are investigated. Bernoulli particular integral method and Lagrange method of variation constant are used for the transformation problem of solutions. The asymptotic averaged method is used for obtaining the first approximation of Itô stochastic differential equations. The sets of Lyapunov exponents are obtained.

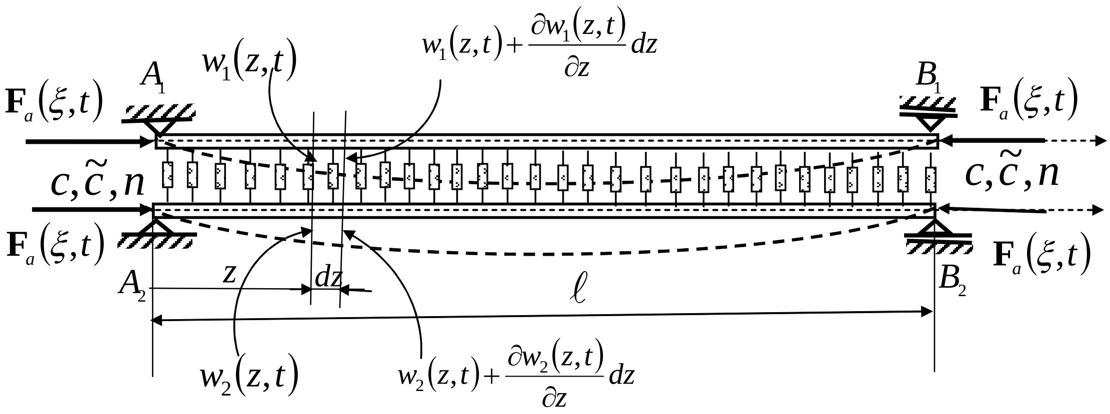
In the paper [16], the stability of a hereditary visco-elastic beam subjected to parametric random bounded excitations described by stochastic processes of small intensity is investigated. The motivation for the study of these problems is the necessity to explain the influence of rotatory inertia of beam cross sections and transverse shear of beam cross section under the transverse forces on the stability of the transversal time vibrations process of the beam, and also on the stability of the deformable beam's forms.

Paper [15] present an investigation about stochastic dynamics of hybrid systems with thermo-rheological hereditary elements. Tensor of state of the random vibrations was considered in the paper [17].

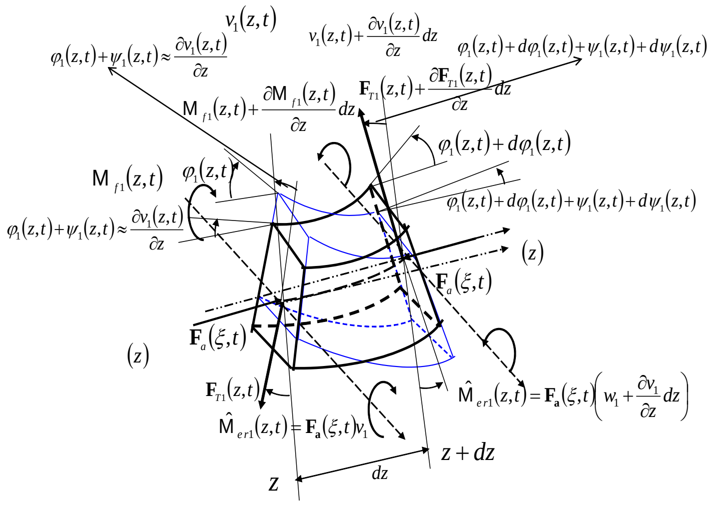
In the present paper transversal vibrations of a parametrically excited sandwich double hereditary beam system and influence of rotatory inertia and transverse shear on stochastic stability of deformable forms and processes are investigated.

1. CONSTITUTIVE RELATION OF THE VISCO-ELASTIC HEREDITARY BEAM

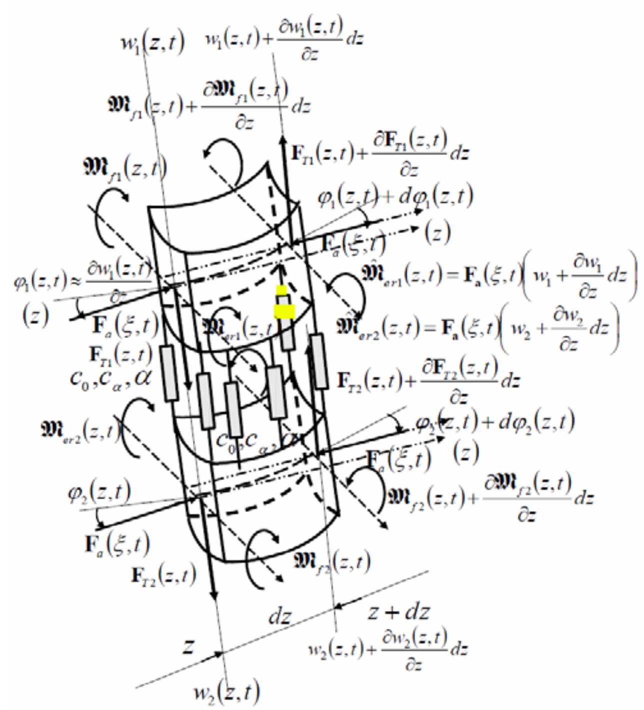
Let suppose that material of the beam is rheological with hereditary property (se Ref. [7]). Parameter of beam material are: n own material clock of the material relaxation or short relaxation time of beam material; E and $\tilde{E}$ modulus of elasticity momentaneous behavior of material and prologeous one in long time period. In Figure 1. a* we can see homogeneous prismatic hereditary beams with two axes symmetry of the beam cross sections with line element in deformed stressed state. For ideal visco-elastic hereditary beam and axially stressed line element at the distance y from neutral beam line, the normal stress component $\sigma_z(z, y, t)$ for the beam cross section on the distance z from left beam end, at the moment t is:



a*



b*



c*

Fig. 1. a Sandwich double beam hereditary system.*
Cross section surface forces and moments acting on a beam element:
The influence of rotatory inertia of beam cross sections and transverse shear under the influence of transversal force in the cross section
b. Cross section displacement and surface forces and moments acting on a beam element - the influence of rotatory inertia of beam cross sections*
c The sandwich double beam hereditary system elements with standard light hereditary connection*

$$\sigma_z(z,y,t) = E\left[\varepsilon_z(z,y,t) - \int_0^t \mathbf{R}(t-\tau)\varepsilon_z(z,y,\tau)d\tau\right] \tag{1}$$

where $\varepsilon_z(z,y,t)$ is dilatation – strain of beam line element and

$$\mathbf{R}(t-\tau) = \frac{E-\tilde{E}}{nE}e^{-\frac{t-\tau}{n}} \tag{2}$$

kernel of relaxation of beam visco-elastic material with hereditary properties.

2. PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS OF THE TRANSVERSAL VIBRATIONS OF PARAMETRICALLY EXCITED DOUBLE HEREDITARY BEAM SYSTEM

The dilatations $\varepsilon_{zi}(z,y,t), i=1,2$ of corresponding beam's stressed and strained line elements at the distance y from neutral beam line and normal to the beam cross section on the distance z from left beam end, at the moment t are:

$$\varepsilon_{zi}(z,y,t) = \frac{ds_i - dz}{dz} = y\frac{\partial\varphi_i(z,t)}{\partial z}, i=1,2 \tag{3}$$

where $\varphi_i(z,t), i=1,2$ are the component inclination angles of the tangent to the visco-elastic bended beam's line in result of pure bending beams by corresponding couple moments. With $\psi_i(z,t)$, $i=1,2$ are denoted the component inclination angle of the tangent to the visco-elastic bended beam's line in result of transverse shear as influence of the transversal forces in corresponding beam's cross section.

By introducing previous expression (3) for strain into expression of constitutive relation (1), the normal stress component $\sigma_{zi}(z,y,t), i=1,2$ for the corresponding beam cross section on the distance z from left beam end, at the moment t we obtain in the following form:

$$\sigma_{zi}(z,y,t) = Ey\left[\frac{\partial\varphi_i(z,t)}{\partial z} - \int_0^t \mathbf{R}(t-\tau)\frac{\partial\varphi_i(z,\tau)}{\partial z}d\tau\right],\ i=1,2. \tag{4}$$

In Figure 1. b* the beam's element with length dz is presented with transversal displacements. In the results of the elimination of the component inclination angles φ_1 and φ_2 from the system of the four partial integro-differential equations obtained into results of Principle of dynamical equilibrium application to the double hereditary beam element, shown in Figure 1.c*, we obtain two coupled partial integro-differential equations of the transversal vibrations of the two coupled beams of the previous sandwich double hereditary beam system in the form:

$$\begin{aligned}
&\frac{\partial^2 v_1(z,t)}{\partial t^2} + c_1^2\left[\frac{\partial^4 v_1(z,t)}{\partial z^4} - \int_0^t \mathbf{R}(t-\tau)\left[\frac{\partial^4 v_1(z,t)}{\partial z^4}\right]d\tau\right] - a_1^2\frac{\partial^4 v_1(z,t)}{\partial t^2\partial z^2} + b_1^2\frac{\partial^4 v_1(z,t)}{\partial t^4} - \\
&+\left(a_1^2 - i_{x1}^2\right)\int_0^t \mathbf{R}(t-\tau)\left[\frac{\partial^4 v_1(z,t)}{\partial t^2\partial z^2}\right]d\tau + \hat{c}_1[v_1(z,t) - v_2(z,t)] + \hat{c}_1 b_1^2\frac{\partial^2}{\partial t^2}[v_1(z,t) - v_2(z,t)] + \\
&+\hat{c}_1\left(a_1^2 - i_{x1}^2\right)\left[\frac{\partial^2}{\partial z^2}[v_1(z,t) - v_2(z,t)] - \int_0^t \mathbf{R}(t-\tau)\left[\frac{\partial^2}{\partial z^2}[v_1(z,\tau) - v_2(z,\tau)]\right]d\tau\right] - \frac{\partial}{\partial z}\left[\hat{\mathrm{F}}_{a1}(\Xi,z,t)\frac{\partial v_1(z,t)}{\partial z}\right] = 0
\end{aligned} \tag{5}$$

$$\begin{aligned}
&\frac{\partial^2 v_2(z,t)}{\partial t^2} + c_2^2\left[\frac{\partial^4 v_2(z,t)}{\partial z^4} - \int_0^t \mathbf{R}(t-\tau)\left[\frac{\partial^4 v_2(z,t)}{\partial z^4}\right]d\tau\right] - a_2^2\frac{\partial^4 v_2(z,t)}{\partial t^2\partial z^2} + b_2^2\frac{\partial^4 v_2(z,t)}{\partial t^4} - \\
&+\left(a_2^2 - i_{x2}^2\right)\int_0^t \mathbf{R}(t-\tau)\left[\frac{\partial^4 v_2(z,t)}{\partial t^2\partial z^2}\right]d\tau - \hat{c}_2[v_1(z,t) - v_2(z,t)] - \hat{c}_2 b_2^2\frac{\partial^2}{\partial t^2}[v_1(z,t) - v_2(z,t)] - \\
&-\hat{c}_2\left(a_2^2 - i_{x2}^2\right)\left[\frac{\partial^2}{\partial z^2}[v_1(z,t) - v_2(z,t)] - \int_0^t \mathbf{R}(t-\tau)\left[\frac{\partial^2}{\partial z^2}[v_1(z,\tau) - v_2(z,\tau)]\right]d\tau\right] - \frac{\partial}{\partial z}\left[\hat{\mathrm{F}}_{a2}(\Xi,z,t)\frac{\partial v_2(z,t)}{\partial z}\right] = 0
\end{aligned}$$

where:

$$c_i^2 = i_{xi}^2\frac{\mathrm{B}_{xi}}{(\rho \mathrm{I}_x)_i} = \frac{\mathrm{B}_{xi}}{(\rho A)_i} \qquad a_i^2 = i_{xi}^2\left(\frac{\kappa\mathbf{E}}{\mathbf{G}} + 1\right)_i \qquad a_i^2 - i_{xi}^2 = i_{xi}^2\left(\frac{\kappa\mathbf{E}}{\mathbf{G}}\right)_i \qquad b_i^2 = i_{xi}^2\left(\rho\frac{\kappa}{\mathbf{G}}\right)_i$$

$$\hat{c}_i = i_{ix}^2 \frac{c}{(\rho \mathrm{I}_x)_i} = \frac{c}{(\rho A)_i} \qquad \hat{\mathrm{F}}_{ai}(\Xi, z, t) = \frac{\mathrm{F}_a(\Xi, z, t)}{(\rho A)_i} \qquad i_{xi}^2 \left(\frac{\kappa}{GA}\right)_i c = i_{xi}^2 \left(\rho \frac{\kappa}{\mathbf{G}}\right)_i \frac{c}{(\rho A)_i} = \hat{c}_i b_i^2$$

$$c i_{xi}^2 \frac{\mathbf{E}_i}{\rho_i} \left(\frac{\kappa}{GA}\right)_i = i_{xi}^2 \left(\frac{\kappa \mathbf{E}}{\mathbf{G}}\right)_i \frac{c_i}{(\rho A)_i} = \hat{c}_i \left(a_i^2 - i_{xi}^2\right)$$

3. SOLUTION OF THE PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS OF THE TRANSVERSAL VIBRATIONS OF PARAMETRICALLY EXCITED DOUBLE HEREDITARY BEAM SYSTEM

For the solutions of the governing system of the corresponding coupled partial integro-differential equations (5) for free double beam system oscillations, we take in the eigen amplitude function $Z_{i(s)}(z) = Z_{(s)}(z)$, $i = 1,2$, $s = 1,2,3,4,....\infty$ expansion with time coefficients in the form of unknown time functions $T_{i(s)}(t)$, $i = 1,2$, $s = 1,2,3,4,....\infty$ describing their time evolution in the form:

$$v_i(z,t) = \sum_{s=1}^{\infty} \mathbf{Z}_{(\mathbf{s})}(z) \mathbf{T}_{i(s)}(t), \; i = 1,2 \tag{6}$$

After introduce the proposed solution into partial integro-differential equations (5) and taking into account orthogonality conditions (see Refs. [13], [14] and [15]) , we obtain:

$$\begin{aligned} &\ddot{\ddot{\mathrm{T}}}_{(1)s}(t) + \frac{1}{b^2}\left[1 - a^2 n_s\right]\ddot{\mathrm{T}}_{(1)s}(t) + \frac{1}{b^2}\left[c^2 m_s + \hat{c} + n_s\left(\hat{\mathrm{F}}_{a0} - \hat{\mathrm{F}}_{a1}\xi(t)\right)\right]\mathrm{T}_{(1)s}(t) + \\ &+ \frac{a^2 - i_x^2}{b^2} n_s \int_0^t \mathbf{R}(t-\tau)\ddot{\mathrm{T}}_{(1)s}(\tau)d\tau - \frac{c^2 m_s}{b^2}\int_0^t \mathbf{R}(t-\tau)\mathrm{T}_{(1)s}(\tau)d\tau - \frac{\hat{c}}{b^2}\mathrm{T}_{(2)s}(t) = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &\ddot{\ddot{\mathrm{T}}}_{(2)s}(t) + \frac{1}{b^2}\left[1 - a^2 n_s\right]\ddot{\mathrm{T}}_{(2)s}(t) + \frac{1}{b^2}\left[c^2 m_s + \hat{c} + n_s\left(\hat{\mathrm{F}}_{a0} - \hat{\mathrm{F}}_{a1}\xi(t)\right)\right]\mathrm{T}_{(2)s}(t) + \\ &+ \frac{a^2 - i_x^2}{b^2} n_s \int_0^t \mathbf{R}(t-\tau)\ddot{\mathrm{T}}_{(2)s}(\tau)d\tau - \frac{c^2 m_s}{b^2}\int_0^t \mathbf{R}(t-\tau)\mathrm{T}_{(2)s}(\tau)d\tau - \frac{\hat{c}}{b^2}\mathrm{T}_{(1)s}(t) = 0 \end{aligned} \tag{8}$$

where it is introduced the following notations:

$$\int_0^\ell \mathbf{Z}_s(z)\mathbf{Z}_s(z)dz = \tilde{m}_s, \quad s = r; \quad \int_0^\ell \mathbf{Z}_s'(z)\mathbf{Z}_s'(z)dz = \tilde{n}_s, \quad s \neq r$$

$$\int_0^\ell \mathrm{Z}_s''(z)\mathrm{Z}_s(z)dz = \left[\mathrm{Z}_s'(z)\mathrm{Z}_s(z)\Big|_0^\ell - \int_0^\ell \mathrm{Z}_s'(z)\mathrm{Z}_r'(z)dz\right] = m_s\tilde{m}_s \tag{9}$$

$$\int_0^\ell \mathrm{Z}_s^{IV}(z)\mathrm{Z}_s(z)dz = -a_s\left[\mathrm{Z}_s'(z)\mathrm{Z}_s(z)\Big|_0^\ell - \int_0^\ell \mathrm{Z}_s'(z)\mathrm{Z}_s'(z)dz\right] + b_s\int_0^\ell \mathrm{Z}_s(z)\mathrm{Z}_s(z)dz = \tilde{m}_s n_s$$

First, we concentrate our attention to the solution of the coupled ordinary differential equations (7) for the case of free own vibrations. Solution of the basic equations of the previous system (7) are proposed in the following form:

$$\mathbf{T}_{(i)s}(t) = A_{(i)s}\cos\left(\omega_{(s)}t + \alpha_{(s)}\right), \; i = 1,2; \quad s = 1,2,3,..... \tag{10}$$

After introducing this proposed solution (10), we obtain the homogeneous system of two algebra equations with respect to the unknown amplitudes $A_{(i)s}$. The corresponding frequency equation is in the following form: or in the form:

$$f\left(\omega_{(s)}^2\right) = \left[\frac{1}{b^2}\left(c^2 m_s + \hat{c} + n_s\,\hat{\mathrm{F}}_{a0}\right) + \omega_{(s)}^4 - \frac{\omega_{(s)}^2}{b^2}\left[1 - a^2 n_s\right]\right]^2 - \left[\frac{\hat{c}}{b^2}\right]^2 = 0 \tag{11}$$

Circular frequencies are roots of the previous equation and are defined by following expressions:

$$\omega_{(s)1,2}^2 = \frac{1}{2b^2}\left[1 - a^2 n_s\right] \mp \sqrt{\frac{1}{4b^4}\left[1 - a^2 n_s\right]^2 - \frac{1}{b^2}\left(c^2 m_s + n_s\,\hat{\mathrm{F}}_{a0}\right)} \tag{12}$$

$$\omega^2_{(s)3,4} = \frac{1}{2b^2}\left[1 - a^2 n_s\right] \mp \sqrt{\frac{1}{4b^4}\left[1 - a^2 n_s\right]^2 - \frac{1}{b^2}\left(c^2 m_s + 2\hat{c} + n_s\,\hat{\mathrm{F}}_{a0}\right)} \tag{13}$$

Then, we can write the following time functions correspond to the set of the obtained own circular frequencies::

$$\mathbf{T}_{(i)s}(t) = \sum_{r=1}^{4} A^{(r)}_{(i)s} \cos\left(\omega_{(s)r} t + \alpha_{(s)r}\right),\ i = 1,2; \quad s = 1,2,3,..... \tag{14}$$

$$\mathbf{T}_{(i)s}(t) = C_s^{1,2}\cos\left(\omega_{(s)1}t + \alpha_{(s)1}\right) + C_s^{1,2}\cos\left(\omega_{(s)2}t + \alpha_{(s)2}\right) + C_s^{3,4}\cos\left(\omega_{(s)3}t + \alpha_{(s)3}\right) + C_s^{3,4}\cos\left(\omega_{(s)4}t + \alpha_{(s)4}\right)$$

$$\mathbf{T}_{(2)s}(t) = C_s^{1,2}\cos\left(\omega_{(s)1}t + \alpha_{(s)1}\right) + C_s^{1,2}\cos\left(\omega_{(s)2}t + \alpha_{(s)2}\right) - C_s^{3,4}\cos\left(\omega_{(s)3}t + \alpha_{(s)3}\right) - C_s^{3,4}\cos\left(\omega_{(s)4}t + \alpha_{(s)4}\right)$$

These time functions are time component of the solutions for the case of the axial force acting to the sandwich double beam system when these forces are deterministic and constant intensity.

Now, following the idea presented by S.T. Ariaratnam (1995) in Reference [1], for solving the previous equations (7), we can propose that random, bonded noise axial excitation $\xi(t)$ is taken in the following form:

$$\hat{\mathrm{F}}_{a1}(\mathrm{t}) = \hat{\mathrm{F}}_{a1}\,\xi(t) = \hat{\mathrm{F}}_{a1}\,\mu \sin\left[\Omega t + \sigma B(t) + \gamma\right] \tag{15}$$

where $B(t)$ is the standard Wiener process, and γ is a random uniformly distributed variable in interval $[0, 2\pi]$, then $\xi(t)$ is a stationary process having autocorrelation function and spectral density function:

$$\mathrm{R}(\tau) = \frac{1}{2}\mu^2 e^{-\frac{\sigma^2\tau}{2}} \cos\Omega\tau \tag{16}$$

and

$$\mathrm{S}(\omega) = \int_{-\infty}^{+\infty} \mathrm{R}(\tau) e^{i\omega\tau} d\tau = \frac{1}{2}\mu\sigma^2 \frac{\omega^2 + \Omega^2 + \frac{\sigma^2}{4}}{\left[\left(\omega^2 - \Omega^2 - \frac{\sigma^2}{4}\right)^2 + \sigma^2\omega^2\right]} \tag{17}$$

Stochastic process $|\xi(t)| \le 1$ is bounded for all values of time t.

Next idea of Ariaratnam is to apply the averaging method, and for that reason we must to introduce the amplitudes $C_s^1(t), C_s^2(t), C_s^3(t), C_s^4(t)$ and phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ i $\Phi_{(s)4}(t)$, which are time unknown functions, by means of the transformation relation of $\mathbf{T}_{(i)s}(t)$, $i = 1,2$, $s = 1,2,3,4,....,\infty$ from the case of free vibrations (14) to the case of the perturbed stochastic vibrations, in the following form:

$$\mathbf{T}_{(1)s}(t) = C_s^1(t)\cos\Phi_{(s)1}(t) + C_s^2(t)\cos\Phi_{(s)2}(t) + C_s^3(t)\cos\Phi_{(s)3}(t) + C_s^4(t)\cos\Phi_{(s)4}(t)$$

$$\mathbf{T}_{(2)s}(t) = C_s^1(t)\cos\Phi_{(s)1}(t) + C_s^2(t)\cos\Phi_{(s)2}(t) - C_s^3(t)\cos\Phi_{(s)3}(t) - C_s^4(t)\cos\Phi_{(s)4}(t) \tag{18}$$

in which amplitudes $C_s^1(t), C_s^2(t), C_s^3(t)$ and $C_s^4(t)$ and full phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ and $\Phi_{(s)4}(t)$ are unknown functions of the time. It is necessary to fined solutions which correspond to parametric resonant state, for which full phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ and $\Phi_{(s)4}(t)$ are functions of time in the proposed forms:

$$\Phi_{(s)k}(t) = \frac{\Omega}{2}t + \phi_{(s)k}(t)\ ,\ \Delta_{(s)k} = \omega_{(s)k} - \frac{\Omega}{2},\ k = 1,2,3,4,\ s = 1,2,3,4,....,\infty \tag{19}$$

We suppose that the corresponding first, second and third derivatives with respect to time of the time functions $\mathbf{T}_{(i)s}(t)$, $i = 1,2$, $s = 1,2,3,4,....,\infty$ are same as in case when the amplitudes $C_s^1(t), C_s^2(t), C_s^3(t), C_s^4(t)$ and phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ i $\Phi_{(s)4}(t)$ are constants and correspond to the solutions of the unperturbed case. For that reason, we obtain sixth conditions – equations with respect to the unknown first derivatives of the unknown amplitudes $\dot{C}_s^1(t), \dot{C}_s^2(t), \dot{C}_s^3(t), \dot{C}_s^4(t)$ and phases $\dot{\Phi}_{(s)1}(t)$, $\dot{\Phi}_{(s)2}(t)$, $\dot{\Phi}_{(s)3}(t)$ and $\dot{\Phi}_{(s)4}(t)$ with respect to the time. Substituting these time derivatives together with four derivative into system differential equations (7), we obtain system of eight equations, bat in the form of homogeneous system of series of unknown expressions with sub-system determinant different them zero, then as follow we obtain simpler system with eight equations. Now, obtained simpler system of the eight equations with unknown time functions which represent unknown amplitudes $C_s^1(t), C_s^2(t), C_s^3(t)$ and $C_s^4(t)$ and unknown full phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ and $\Phi_{(s)4}(t)$ or $\phi_{(s)1}(t)$, $\phi_{(s)2}(t)$, $\phi_{(s)3}(t)$ and $\phi_{(s)4}(t)$ which are the time functions, is not difficult to solve. By solve these equations along first time derivatives of the amplitudes $\dot{C}_s^1(t), \dot{C}_s^2(t), \dot{C}_s^3(t)$ and $\dot{C}_s^4(t)$ and phases $\dot{\phi}_{(s)1}(t)$, $\dot{\phi}_{(s)2}(t)$, $\dot{\phi}_{(s)3}(t)$ and $\dot{\phi}_{(s)4}(t)$, we obtain the system of the eight, first order, integro-differential equations. We can conclude that this full system of the first order integro-differential equations contain eight coupled integro-differential equations. These integro-differential equations of the first order with respect to unknown amplitudes $C_s^1(t), C_s^2(t), C_s^3(t)$ and $C_s^4(t)$ and unknown phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ and $\Phi_{(s)4}(t)$, or in difference of the phases $\phi_{(s)1}(t)$, $\phi_{(s)2}(t)$, $\phi_{(s)3}(t)$ and $\phi_{(s)4}(t)$, represent example of the eight Itô stochastic integro-differential equations.

In the previous obtained system of the eight integro-differential equations, $\xi(t)$ is the excitation of the stochastic-random process and it is taken in the form (see Ref. [1] by Ariaratnam):

$$\xi(t) = \mu \sin[\Omega t + \sigma B(t) + \gamma] = \mu \sin[\Omega t + \psi(t)] \tag{20}$$

where $B(t)$ is the standard Wiener process and γ is a random variable. If γ is uniformly distributed in the interval $[o, 2\pi]$, then $\xi(t)$ is a stationary process having autocorrelation function and spectral density. We introduce the following notations:

$$\psi(t) = \sigma B(t) + \gamma \,. \qquad \text{as well as} \qquad \dot{\psi}(t) = \sigma \dot{B}(t) \tag{21}$$

Substituting the $\xi(t)$ in obtained system equations, we obtain system of the stochastic, first order integro-differential equations, with respect to the unknown amplitudes as time functions $C_s^1(t), C_s^2(t), C_s^3(t)$ and $C_s^4(t)$, and unknown phases $\phi_{(s)1}(t)$, $\phi_{(s)2}(t)$, $\phi_{(s)3}(t)$ and $\phi_{(s)4}(t)$ in the transformed form. Now, we must apply the method of averaging to the right-hand sides of obtained equations with respect to the full phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ and $\Phi_{(s)4}(t)$. After averaging the right-hand sides of all other equations with respect to the full phases $\Phi_{(s)1}(t)$, $\Phi_{(s)2}(t)$, $\Phi_{(s)3}(t)$ and $\Phi_{(s)4}(t)$, we obtain the system of averaged differential equations of the first approximation.

The averaging method for integro-differential equations developed by Krilov Bogolyubov Mitropolskiy and also Larionov (1969) is applied to obtain the so called averaged equations. Thus we assume that are:

$$\begin{gathered}
\frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} = O(\varepsilon) \,, \quad \frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} = O(\varepsilon) \,, \quad 0 < \varepsilon < 1, \\
\frac{c^2 m_s}{2b^2} \frac{1}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} = O(\varepsilon) \,, \quad \frac{c^2 m_s}{2b^2} \frac{1}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} = O(\varepsilon), \quad \Delta_{(s)k} = O(\varepsilon) \\
\frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} = O(\varepsilon) \,, \quad \frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} = O(\varepsilon) \,, \quad 0 < \varepsilon < 1, \\
\frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} = O(\varepsilon) \,, \quad \frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} = O(\varepsilon)
\end{gathered} \tag{22}$$

The assumption of the detuning parameter $\Delta_{(s)k} = O(\varepsilon)$ effectively restricts, the analysis to those excitation frequencies Ω that are in the vicinity of the frequency 2Ω of fundamental parametric resonance. After averaging the members in the right hand sides of previous stochastic Itô differential equations, we obtain the averaged differential equations in the following form:

$$\begin{aligned}
\dot{C}_s^1(t) = &-\frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} C_s^1(t) \cos\left(\psi - 2\phi_{(s)1}(t)\right) - \\
&- \frac{a^2}{2b^2} n_s \frac{1}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \left[\omega_{(s)1}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)2}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] - \\
&- \frac{c^2 m_s}{2b^2} \frac{1}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]
\end{aligned}$$

$$\dot{\phi}_{(s)1}(t) = \Delta_{(s)1} - \frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} \sin\left[\psi - 2\phi_{(s)1}(t)\right] -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{C_s^1(t)\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \left[\omega_{(s)2}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)1}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] + \tag{23}$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{C_s^1(t)\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\dot{C}_s^2(t) = -\frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} C_s^2(t) \cos\left(\psi - 2\phi_{(s)2}(t)\right) -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \left[\omega_{(s)1}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)2}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] +$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\dot{\phi}_{(s)2}(t) = \Delta_{(s)1} - \frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} \sin\left[\psi - 2\phi_{(s)2}(t)\right] -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{C_s^2(t)\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \left[\omega_{(s)2}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)1}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] dt +$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{C_s^2(t)\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] dt$$

$$\dot{C}_s^3(t) = -\frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} C_s^3(t) \cos\left(\psi - 2\phi_{(s)3}(t)\right) -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \left[\omega_{(s)3}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)4}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] -$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\dot{\phi}_{(s)3}(t) = \Delta_{(s)3} - \frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} \sin\left[\psi - 2\phi_{(s)3}(t)\right] -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{C_s^3(t)\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \left[\omega_{(s)4}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)3}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] +$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{C_s^3(t)\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)3}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\dot{C}_s^4(t) = -\frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} C_s^4(t) \cos\left(\psi - 2\phi_{(s)4}(t)\right) -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \left[\omega_{(s)4}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)3}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] -$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\dot{\phi}_{(s)4}(t) = \Delta_{(s)4} - \frac{1}{4\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \frac{n_s}{b^2} \hat{\mathrm{F}}_{a1} \sin\left[\psi - 2\phi_{(s)4}(t)\right] -$$

$$- \frac{a^2}{2b^2} n_s \frac{1}{C_s^4(t)\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \left[\omega_{(s)3}^2 \mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)4}^2 \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] +$$

$$- \frac{c^2 m_s}{2b^2} \frac{1}{C_s^4(t)\left(\omega_{(s)4}^2 - \omega_{(s)3}^2\right)\omega_{(s)4}} \left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

where corresponding members are obtained by following expression:

$$\int_0^{+\infty} \mathsf{R}(\tau) e^{i\omega\tau} d\tau = \mathrm{H}_c(\omega) + i\,\mathrm{H}_s(\omega),\ i = \sqrt{-1} \tag{24}$$

The change of variables in the following ways and in the following forms: $\rho_s^k(t) = \ln C_s^k$, $\vartheta_s^k(t) = \phi_s^k(t) - \frac{1}{2}\psi$, $s = 1,2,3,4,...$, $k = 1,2,3,4$ where $\psi = \sigma B(t) + \gamma$ and transforming the averaged system of differential equations into the system of averaged stochastic differential equations with respect to the unknown amplitudes $C_s^1(t), C_s^2(t), C_s^3(t)$ and $C_s^4(t)$, and unknown phases $\phi_{(s)1}(t)$, $\phi_{(s)2}(t)$, $\phi_{(s)3}(t)$ and $\phi_{(s)4}(t)$, results in the following forms:

$$d\rho_s^k(t) = \frac{1}{C_s^k}\dot{C}_s^k dt\,, \qquad d\vartheta_s^k(t) = \dot{\phi}_s^k(t)dt - \frac{1}{2}d\psi \;,\; k = 1,2,3,4 \tag{25}$$

4. LYAPUNOV EXPONENTS AND STOCHASTIC STABILITY OF THE TRANSVERSAL VIBRATIONS OF PARAMETRICALLY EXCITED DOUBLE HEREDITARY BEAM SYSTEM

Let us consider the following expressions [1]:

$$\ln\left\{\left[T_s^k(t)\right]^2 + \frac{1}{\omega_{(s)k}^2}\left[\dot{T}_s^k(t)\right]^2\right\} = \ln\left\{\left[C_s^k(t)\right]^2\right\} = 2\ln\left[C_s^k(t)\right] = 2\rho_s^k(t),$$
$$s = 1,2,3,4,...,\; k = 1,2,3,4 \tag{26}$$

where

$$\mathbf{T}_s(t) = \sum_{k=1}^{k=4} C_s^k(t)\cos\Phi_{(s)k}(t) = \sum_{k=1}^{k=4} T_s^k(t) \tag{27}$$

with the first derivatives in the forms

$$\dot{\mathbf{T}}_s(t) = -\sum_{k=1}^{k=4}\omega_{(s)k}C_s^k(t)\sin\Phi_{(s)k}(t) = \sum_{k=1}^{k=4}\dot{T}_s^k(t) \tag{28}$$

where we introduce the time modes as „new time component coordinates“ $\mathrm{T}_s^k(t)$.

The Lyapunov exponents of the system mode processes λ_s^k, $s = 1,2,3,4,...$, $k = 1,2,3,4$, [1] may be introduced by using the time modes as „new time component coordinates“ $\mathrm{T}_s^k(t)$ and which by making use of the averaged equations becomes:

$$\lambda_s^k = \lim_{t\to\infty}\frac{1}{2t}\ln\left\{\left[T_s^k(t)\right]^2 + \frac{1}{\omega_{(s)k}^2}\left[\dot{T}_s^k(t)\right]^2\right\} = \lim_{t\to\infty}\frac{1}{t}\rho_s^k(t), k = 1,2,3,4\,. \tag{29}$$

Now, each separate Lyapunov exponent is a measure of the average exponential growth of the amplitudes $C_s^1(t), C_s^2(t), C_s^3(t)$ and $C_s^4(t)$ component processes of the corresponding *„new time component coordinates“* $\mathrm{T}_s^k(t)$ of beam transversal vibrations in the s-th form of the perturbed parametric resonance process. The Lyapunov exponents λ_s^k, $s = 1,23,4,...$, $k = 1,2,3,4$ are the deterministic numbers with probability one (w.p.1) for the system given by averaged equations. Solutions of the averaged differential equations depend on initial values $T_s^k(t_0)$ and $\dot{T}_s^k(t_0)$, and in general will be four values of the Lyapunov exponent λ_s^k, $s = 1,2,3,4,...$, $k = 1,2,3,4$ in the corresponding s-th form of perturbed parametric resonance process. If both Lyapunov exponents are negative, the trivial solution in the corresponding s-th form of perturbed parametric resonance of a two-frequency process is a stable process with probability 1.

In order to calculate the expression and values for both Lyapunov exponents $\lambda_s^k, k = 1,2$, $s = 1,2,3,4,...,$, we integrate both sides of two stochastic differential equations of the system (25) and we obtain the following system::

$$\begin{aligned}
\rho_s^1(t) - \rho_s^1(0) = &-\frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}}\frac{n_s}{b^2}\hat{\mathrm{F}}_{a1}\int_0^t\cos 2\vartheta_s^1 dt - \\
&-\frac{a^2}{2b^2}n_s\frac{t}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}}\left[\omega_{(s)1}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)2}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] + \\
&-\frac{c^2 m_s}{2b^2}\frac{t}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)1}}\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] \\
\rho_s^2(t) - \rho_s^2(0) = &-\frac{1}{4\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}}\frac{n_s}{b^2}\hat{\mathrm{F}}_{a1}\int_0^t\cos 2\vartheta_s^2 dt - \\
&-\frac{a^2}{2b^2}n_s\frac{t}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}}\left[\omega_{(s)1}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \omega_{(s)2}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right] - \\
&-\frac{c^2 m_s}{2b^2}\frac{t}{\left(\omega_{(s)2}^2 - \omega_{(s)1}^2\right)\omega_{(s)2}}\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right) + \mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]
\end{aligned} \tag{30}$$

$$\rho_s^3(t)-\rho_s^3(0)=-\frac{1}{4\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)3}}\frac{n_s}{b^2}\hat{\mathrm{F}}_{a1}\int_0^t\cos 2\vartheta_s^3 dt-$$
$$-\frac{a^2}{2b^2}n_s\frac{t}{\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)3}}\left[\omega_{(s)3}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\omega_{(s)4}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]+$$
$$-\frac{c^2m_s}{2b^2}\frac{t}{\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)3}}\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$
$$\rho_s^4(t)-\rho_s^4(0)=-\frac{1}{4\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)4}}\frac{n_s}{b^2}\hat{\mathrm{F}}_{a1}\int_0^t\cos 2\vartheta_s^4 dt-$$
$$-\frac{a^2}{2b^2}n_s\frac{t}{\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)4}}\left[\omega_{(s)4}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\omega_{(s)3}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]+$$
$$-\frac{c^2m_s}{2b^2}\frac{t}{\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)4}}\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

so that, from expressions for Lyapunov exponents λ_s^k, $s=1,2,3,4,...$, $k=1,2,3,4$, with previous obtained system (30), we can write the following series of the Lyapunov exponent expressions:

$$\lambda_s^1=-\frac{1}{4}\mathbf{L}_s^1\hat{\mathrm{F}}_{a1}\mathbf{F}\left(\frac{L_s^1}{\sigma^2},\frac{4\Delta_{(s)1}}{\sigma^2}\right)-$$
$$-\frac{a^2}{2}\mathbf{L}_s^1\left[\omega_{(s)1}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\omega_{(s)2}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]+$$
$$-\frac{c^2m_s}{2n_s}\mathbf{L}_s^1\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\lambda_s^2=-\frac{1}{4}\mathbf{L}_s^2\hat{\mathrm{F}}_{a1}\mathbf{F}\left(\frac{L_s^2}{\sigma^2},\frac{4\Delta_{(s)2}}{\sigma^2}\right)-$$
$$-\frac{a^2}{2}\mathbf{L}_s^2\left[\omega_{(s)1}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\omega_{(s)2}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]-$$
$$-\frac{c^2m_s}{2n_s}\mathbf{L}_s^2\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

with probability 1. . (37)

$$\lambda_s^3=-\frac{1}{4}\mathbf{L}_s^3\hat{\mathrm{F}}_{a1}\mathbf{F}\left(\frac{L_s^3}{\sigma^2},\frac{4\Delta_{(s)3}}{\sigma^2}\right)-$$
$$-\frac{a^2}{2b^2}n_s\mathbf{L}_s^3\left[\omega_{(s)3}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\omega_{(s)4}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]+$$
$$-\frac{c^2m_s}{2b^2}\mathbf{L}_s^3\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

$$\lambda_s^4=-\frac{1}{4}\mathbf{L}_s^4\hat{\mathrm{F}}_{a1}\mathbf{F}\left(\frac{L_s^4}{\sigma^2},\frac{4\Delta_{(s)4}}{\sigma^2}\right)-$$
$$-\frac{a^2}{2b^2}n_s\mathbf{L}_s^4\left[\omega_{(s)4}^2\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\omega_{(s)3}^2\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]+$$
$$-\frac{c^2m_s}{2b^2}\mathbf{L}_s^4\left[\mathbf{H}_{re}\left(\frac{\Omega}{2}\right)+\mathbf{H}_{im}\left(\frac{\Omega}{2}\right)\right]$$

with probability 1, where

$$L_s^1(t)=\frac{1}{\left(\omega_{(s)2}^2-\omega_{(s)1}^2\right)\omega_{(s)1}}\frac{n_s}{b^2},\qquad L_s^2=\frac{1}{\left(\omega_{(s)2}^2-\omega_{(s)1}^2\right)\omega_{(s)2}}\frac{n_s}{b^2}$$
$$L_s^3(t)=\frac{1}{\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)3}}\frac{n_s}{b^2},\qquad L_s^4=\frac{1}{\left(\omega_{(s)4}^2-\omega_{(s)3}^2\right)\omega_{(s)4}}\frac{n_s}{b^2}$$
$$\Delta_{(s)k}=\omega_{(s)k}-\frac{\Omega}{2}\ s=1,2,3,4,...,\ k=1,2,3,4: \tag{31}$$
$$\omega_{(s)1,2}^2=\frac{1}{2b^2}\left[1-a^2n_s\right]\mp\sqrt{\frac{1}{4b^4}\left[1-a^2n_s\right]^2-\frac{1}{b^2}\left(c^2m_s+n_s\hat{\mathrm{F}}_{ca0}+\hat{c}\right)}$$

where the random processes $\vartheta_s^k(t)$, $k=1,2,3,4$ given by previous system of the stochastic differential equations (25) can be shown to be ergodic, in which case, we can write:

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t\cos 2\vartheta_s^k dt=\mathbf{E}\left[\cos 2\vartheta_s^k\right],\ k=1,2,3,4,\quad \text{with probability 1.}$$

where $\mathrm{E}[.]$ denotes the expectation operator.

To this end, we set up the Fokker-Planck equation governing the invariant (or stationary) probability density functions $p_s^k=p\left(\vartheta_s^k\right)$, $k=1,2,3,4$ of the processes (see [1], [25] and [26]) $\vartheta_s^k(t)=\phi_s^k(t)-\psi/2$, $k=1,2,3,4$ defined by the second and fourth equations from system (25) of the stochastic differential equations Itô-type:

$$\sigma^2 \frac{d^2 p\left(\vartheta_s^k\right)}{d\left(\vartheta_s^k\right)^2} - \frac{d}{d\vartheta_s^k}\left\{\left[4\Delta_{(s)k} - L_s^k \sin 2\vartheta_s^k\right] p\left(\vartheta_s^k\right)\right\} = 0, \; k = 1,2,3,4 \tag{32}$$

The solutions of the previous series of equations satisfying the periodicity conditions in the form $p_s^k = p\left(\vartheta_s^k\right) = p\left(\vartheta_s^k + 2\pi\right)$, was obtained by Stratonoviich (1967) [24]:

$$p\left(\vartheta_s^k\right) = \frac{1}{C_s^k} \exp\left[\left(8\Delta_{(s)k}\vartheta_s^k + L_s^k \cos 2\vartheta_s^k\right)/\sigma^2\right] \cdot \int_{\theta_n}^{\pi+\vartheta_s^k} \exp\left[\left(8\Delta_{(s)k}\tau + L_s^k \cos 2\tau\right)/\sigma^2\right] d\tau, \tag{33}$$

$k = 1,2,3,4$, where normalizing constants are:

$$C_s^k = 2\pi^2 \exp\left(-\pi \frac{4\Delta_{(s)k}}{\sigma^2}\right) \left| \mathrm{I}_{iq}\left(\frac{L_s^k}{\sigma^2}\right)\right|^2 \quad , \; k = 1,2,3,4 \tag{34}$$

where $\left|\mathrm{I}_{iq}\left(\frac{L_s^k}{\sigma^2}\right)\right|$ is the Bessel function of the imaginary argument and imaginary order and $q = 4\frac{\Delta_{(s)K}}{\sigma^2}$, $k = 1,2,3,4$.

Using equations and results by Ariaratnam [1] and previous stochastic differential equations, the values of the mathematical expectation $\mathrm{E}\left[\cos 2\vartheta_s^k\right]$, for the processes $\vartheta_s^k(t) = \phi_s^k(t) - \frac{1}{2}\psi$, $s = 1,2,3,4,...$, $k = 1,2,3,4$ and $\psi = \sigma B(t) + \gamma$ are found to be in the form of expressions:

$$\mathrm{E}\left[\cos 2\vartheta_s^k\right] = \mathbf{F}\left(\frac{L_s^k}{\sigma^2}, \frac{4\Delta_{(s)k}}{\sigma^2}\right), \quad k = 1,2,3,4 \tag{35}$$

where

$$\mathbf{F}(z,q) = \frac{1}{2}\left[\frac{\mathbf{I}_{1+iq}(z)}{\mathbf{I}_{iq}(z)} + \frac{\mathbf{I}_{1-iq}(z)}{\mathbf{I}_{-iq}(z)}\right] \tag{36}$$

CONCLUSIONS

Hence, by using previous expressions for the infinite sets of the Lyapunov exponents λ_s^k, $s = 1,2,3,4,...$, $k = 1,2,3,4$, in the forms of expressions (37) with probability 1 for evaluation of the stability or instability, we must find the Lyapunov exponent with maximal values between Lyapunov exponents from defined sets, and determine kinetic parameters of the hereditary beam vibration such that this Lyapunov exponents are with negative values. This is not simple, because we need investigation of the $\max \lambda_s^k < 0$, $s = 1,2,3,4,..., k = 1,2,3,4$. Also, we can consider the case when only one of the $\Delta_{(s)k} = \omega_{(s)k} - \frac{\Omega}{2}$, $s = 1,2,3,4,..., k = 1,2,3,4$: is equal to zero, and all other different from zero; this analysis needs a large discussion.

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