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THE PHASE PORTRAIT OF THE VIBRO-IMPACT DYNAMICS OF TWO MASS PARTICLE MOTIONS ALONG ROUGH CIRCLE

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ABSTRACT

The paper presents combinations of analytical and numerical results and some visualizations of some kinetic parameters of the non-linear dynamics of a vibro-impact system with two degree of freedom and one side impact limiter of system elongations.
System of two heavy mass particles free oscillations along a rough circle in vertical plane, with Coulomb's type friction and with impacts between mass particles and also, with one side, impact limiter of the angle elongations of one mass particle, is object of the analytical and numerical research. The corresponding system of two ordinary non-linear differential double equations of non-linear dynamic equilibrium states of two mass particles is derived accompanying with corresponding initial conditions and impact conditions, as well as conditions of the direction alternation of the friction forces of the Coulomb's type as reactions to the two mass particles motions. Analytical expressions of the phase trajectory branches of both mass particles in the intervals between two kind impacts are derived with corresponding integral constants depending of initial representative phase point coordinates for each phase trajectory branch.
In considered vibro-impact system dynamics two kind of impacts appear: one kind of impacts are impacts between two mass particles, and second kind of impacts are impacts of the one of mass particles into one left side right impact limiter of its angle elongations. Then it is necessary to calculate the moments of time of each kind of impacts as well as time intervals between two successive impacts and velocities of the mass particles, before and after each of impacts and angular coordinate of the place of each impact. Description of the methodology of problem investigation is possible express by analytical approach, but for each particular case it is necessary to use numerical methods for solutions step by step. MathCAD is applied by us in this vibro-impact system dynamics investigation. Analytical results in combinations with numerical experiment gives to us a set of numerical data for visualizations of the non-linear phenomena of this vibro-impact system with two degree of freedom.
For all of considered cases of the heavy two mass particles motions along rough line we can identify a member in the both differential double equations proportional to the square of the corresponding generalized coordinate derivation with respect to time by which both non-linear differential double equations of the mass particle motions are expressed. This corresponds to the known case of turbulent damping.
Changes of the friction forces directions, as an alternation of the directions of the both mass particle motions, are strong discontinuities and non-linearities followed to the double alternate equilibrium position as a bifurcation of positions of the equilibrium depending of the direction of the mass particles motions.

INTRODUCTION

Non-linear phenomena in dynamics of vibro-impact systems are special types of non-linearity caused by series of impacts, followed by discontinuities of kinetic parameter properties and alternations of the motion and velocity direction, as strong non-linearity. In the case, that the basic system dynamics is pure linear, series of vibro-impacts are source of strong non-linearity

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appearance in the system dynamics. In the case, that the basic system dynamics is non-linear, series of vibro-impacts are source of interaction between two types of non-linearity and in the system appear very complex non-linear vibration regimes.

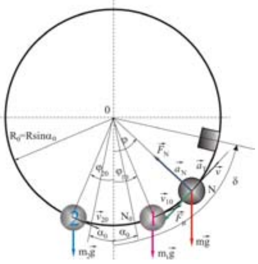
Also, no ideal constraints with Coulomb's type friction forces are source of the strong non-linearity caused by alternations of the friction force directions. Theoretical knowledge and analytical results concerning vibro-impact system dynamics are very valuable in the trends that large numbers of researchers focused to the computation investigations. Also, knowledge of vibro-impact phenomena and vibro-dynamics with impacts is very important for engineering applications, taking into account that working processes of many new engineering system are based on the vibro-impact processes. Series of monographs [3] and [4] by Babickii and Kolovskii, and [2] by Babickii and papers [17], [18] and [19] by Peterka and [5] by Bapat and Popplewell and other contain important scientific and advances to the topic properties of vibro-impact dynamics with corresponding particular methodology applied to the particular classes of the vbro-impact system dynamics.

Some classical problems of mechanical system motion with no ideal constraints and friction as well as an oscillator with Coulomb's type friction are presented in the university books on the level of monographs [20] and [21] written by Rašković contain basic analytical results in this topic. Expellant paper [16] written by Matrosov and Finogenko contain the theory of right solutions of equations for mechanical systems dynamics with sliding friction in one-degree-of-freedom kinematics pairs, which has been developed by the authors. Also, some difficulties bound up with "non-uniqueness" of motion in course of description of such systems, which are known as P. Painlevé's paradoxes are discussed.

New series of published papers by Hedrih (Stevanović) K., present new research results regarding heavy mass particle motions along circles which rotate, as well as hybrid dynamics in the form of the coupled rotations (see References [6-12]). Analysis of the mathematical pendulum dynamics in the field with turbulent damping (see Ref. [22] by Stoker) and papers written by Hedrih (Stevanović) K. [6-12] related to the heavy mass particle dynamics along rotate circle as well as to the heavy mass particle dynamics along rough curvilinear line with Admonton-Coulomb's type frictions are basic inspiration of the series of research results of vibro-impact nonlinear dynamics co-authored by Hedrih (Stevanović) K., Raičević V. and Jović S. and presented in the published co-author papers [13], [14], and [15] in period 2009 and 2010 and listed in the reference list of this paper, as well as in the magistar of science thesis defended by Jović S. in 2009.

1. BASIC SYSTEMS OF THE DOUBLE EQUATIONS OF TWO HEAVY MASS PARTICLES DYNAMICS ALONG ROYGH CIRCLE WITH COULOMB'S TYPE FRICTION

Let consider free vibro-impact dynamics of the two heavy mass particle motions, in vertical plane, along rough circle with Coulomb's type friction and one, one side impact limiter of the angular elongations of the right hand side heavy mass particle. System is shown in Figure 1. The system of two ordinary non-linear differential double equations of non-linear dynamic equilibrium kinetic states of two mass particles is in the following form:



$$\ddot{\varphi}_1 \pm \dot{\varphi}_1^2 tg\alpha_0 + \frac{g}{R\cos\alpha_0}\sin(\varphi_1 \pm \alpha_0) = 0 \begin{cases} za\ \dot{\varphi}_1 > 0 \\ za\ \dot{\varphi}_1 < 0 \end{cases} \quad (1)$$

$$\ddot{\varphi}_2 \pm \dot{\varphi}_2^2 tg\alpha_0 + \frac{g}{R\cos\alpha_0}\sin(\varphi_2 \pm \alpha_0) = 0 \begin{cases} za\ \dot{\varphi}_2 > 0 \\ za\ \dot{\varphi}_2 < 0 \end{cases} \quad (2)$$

where φ_1, φ_2 are independent generalized angular coordinate of the system; $\mu = tg\alpha_0$ is corfficient of Coulomb's type friction.

Fig. 1. The two mass particle vibro-impact system dynamics along rough circke

Let consider case that initial conditions satisy the following relations: $\varphi_{10} > \varphi_{20}$; $\dot{\varphi}_{10} > \dot{\varphi}_{20}$. For full determination of the heavy mass particles it is necessary to add to the each of differential double equations (1) and (2) the following initial conditions: *a** for first mass particle and differential double equation (1): $\varphi_{1(0)} = \varphi_{10}$ and $\dot{\varphi}_{1(0)} = \dot{\varphi}_{10}$; b* for second mass particle and differential double equation (2): $\varphi_{2(0)} = \varphi_{20}$ and $\dot{\varphi}_{2(0)} = \dot{\varphi}_{20}$ and we accapt that is $(\dot{\varphi}_1 > 0, \dot{\varphi}_2 > 0)$.

For describing and determining vibro-impact dynamics of the presented research task it is necessary to the system of differential double equations (1) – (2) with initial conditions to join conditions of the impact limitations of the angular elongations of the second mass particle: $\varphi_{1ul_i} = \delta$,

$\dot{\varphi}_{odl_i} = -k\dot{\varphi}_{ul_i}$, $i = 1,2,3,...,n$, where k is coefficient of the restitution of the second mass particle impacts to the impact limiter of the angular elongations which take values: $k = 0$ for ideally plastic impacts and $k = 1$ for ideally elastic impacts.

Also, for full describing and full determining vibro-impact dynamics of the presented research task it is necessary to the system of differential double equations (1) – (2) with initial conditions and conditions of the impact limitations of the angular elongations of the second mass particle to join conditions of the impacts between mass particles, which not determined at initial moment. For each of impacts between mass particles it is necessary to take into account initial positions of the mass particles, as well as initial velocities with corresponding directions of the each of mass particle motions along rough circle and calculate moment of the impact, as well as position of each of the impacts as well as velocities of each mass particle before and after each of the impacts. These values of the kinetic parameters of the mass particles after each of impacts are initial conditions of the next phase trajectory branch with corresponding one of the two signs in the differential double equation (1) for first mass particle and (2) for second mass particle with corresponding sing depending of direction of the first and second mass particle.

Equations of the phase trajectories of the non-linear free dynamics of two mass particles motions along rough circle in analytical forms by integrations of the differential double equations (1) and (2) for the case that right hand side are equal to zeros are obtained and presented in the References [11] and [12] written by Hedrih (Stevanović) K.(2009,2010). In these References an analysis of the bifurcation of the zero equilibrium position into two one side (half) stable equilibrium positions is pointed out. These information about basic system non-linear dynamics and forms of the phase trajectories and integral form of phase portrait as well as of the constant mechanical energy of the basic linearized system in intervals between friction force alternation of direction are important for investigation of the forms of phase trajectory branches for vibro-impact dynamics of the two mass particle motions along rough circle.

Then, by use cited references [11] and [12], the system of two non-linear double equations of the phase trajectories of non-linear dynamic equilibrium kinetic states of two mass particles are in the following form: 1* for first mass particle:

$$\dot{\varphi}_1(\varphi_1)^2 = \frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\varphi_1 \pm \alpha_0) - 2tg\alpha_0 \sin(\varphi_1 \pm \alpha_0)\right] + C_1 e^{\mp 2\varphi_1 tg\alpha_0} \begin{cases} za\ \dot{\varphi}_1 > 0 \\ za\ \dot{\varphi}_1 < 0 \end{cases}; \quad (3)$$

2* for second mass particle:

$$\dot{\varphi}_2(\varphi_2)^2 = \frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\varphi_2 \pm \alpha_0) - 2tg\alpha_0 \sin(\varphi_2 \pm \alpha_0)\right] + C_2 e^{\mp 2\varphi_2 tg\alpha_0} \begin{cases} za\ \dot{\varphi}_2 > 0 \\ za\ \dot{\varphi}_2 < 0 \end{cases} \quad (4)$$

where C_1 and C_2 are integral constant depending of initial conditions of each phase trajectory branche.

Next analysis of two heavy mass particles vibro-impact dynamics along rough circle is realized by use a example with numerical data and through series of intervals of motions between two impacts or between impact and alternations of the Coulomb's type friction force. For corresponding inteval phase trajectroy branchess of the corresponding mass particle motion are obtained by use previous system of the double equations (30-(4) and in combinations of the analytical and numerical approach and by calculations of the same time and equal positions of both mass particles. Corresponding velocities before impacts of the mass particles are obtained numerically by use MatchCad program for graphical presentation of the next phase trajectory branches of the mass particles and for to read comon position of the impact between tnemnad the corresponding time and velocities. By use theory of impacts velocitis of the both masses after impacts are obtained.

2. PHASE TRAJECTORY BRANCHES AND KINETC PARAMETERS OF IMPACTS BETWEEN MASS PARTICLES

Conditions of the first impact of the first mass particle are: $t = t_{ul_1-}$, $\varphi_1(t_{ul_1-}) = \delta$, $\dot{\varphi}_1(t_{ul_1-}) = \dot{\varphi}_{1ul_1-}$. Angular velocity $(\dot{\varphi}_{1ul_1})$ of the first impact of first mass particle into impact limiter of angular elongations we read from phase trajectory branch obtained by double equation (3) defined with upper sign and passing through initial kinetica state by which integral constant is determined in the form

$$C_{11}(\varphi_{10},\dot{\varphi}_{10}) = e^{+2\varphi_{10}tg\alpha_0}\left\{\dot{\varphi}_{10}^2 - \frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0}\left[\cos(\varphi_{10}+\alpha_0) - 2tg\alpha_0\sin(\varphi_{10}+\alpha_0)\right]\right\} \quad (5)$$

and graphically presented in Figure 2. a* by use MathCad program, on the coordinate $\varphi_1(t_{ul_1-})=\delta$. Angular velocity of the first impact of first mass particle into angular elongation limiter is:

$$\dot{\varphi}_{1ul_1}=\sqrt{\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\delta+\alpha_0)-2tg\alpha_0\sin(\delta+\alpha_0)\right]+C_{11}(\varphi_{10},\dot{\varphi}_{10})e^{-2\delta tg\alpha_0}}\quad (6)$$

Tme $\left(t_{1ul_1}\right)$ of the first impact of the first mass particle to the to the impact limiter, we calculate by

$$t_{1ul_1}=\int_{\varphi_{10}}^{\delta}\frac{d\varphi_1}{\sqrt{\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_1+\alpha_0)-2tg\alpha_0\sin(\varphi_1+\alpha_0)\right]+C_{11}(\varphi_{10},\dot{\varphi}_{10})e^{-2\varphi_1 tg\alpha_0}}}\quad (7)$$

and take at angular coordinate $\varphi_1(t_{ul_1-})=\delta$. Next, it is necessary to find angular coordinate of the second mass particle position at the moment of the first mass particle impact into angular elongation impact limiter. By use phase trajectory double equation (4) of the second mass particle motion passing through their initial kinetic state with upper sign, and with integral constant

$$C_{21}(\varphi_{20},\dot{\varphi}_{20})=e^{+2\varphi_{20}tg\alpha_0}\left\{\dot{\varphi}_{20}^2-\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_{20}+\alpha_0)-2tg\alpha_0\sin(\varphi_{20}+\alpha_0)\right]\right\}\quad (8)$$

a first phase trajectory branch is presented at Figure 1 b* in the MathCad program. At the end of the time interval t_{1ul_1} of the motion, corresponding angular coordinate of the second mass particle position $\varphi_2\left(t_{1ul_1}\right)$ is obtained numerically by the following expression:

$$t_{21}=\int_{\varphi_{20}}^{\varphi_2}\frac{d\varphi_2}{\sqrt{\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_2+\alpha_0)-2tg\alpha_0\sin(\varphi_2+\alpha_0)\right]+C_{21}(\varphi_{20},\dot{\varphi}_{20})e^{-2\varphi_2 tg\alpha_0}}}\quad (9)$$

and using MathCad program for graphical presentation relation (9).

Second impact in the system vibro-impact dynamics is first impact between mass particles which appear in the second interval of the first mass particle motion and in the first interval of the motion of the second mass particle for considered case of the initial conditions. For different cases of the chosen initial conditions and relation between system parameters are possible different cases.

Second phase trajectory branch and interval of the first mass particle motion, after first impact into angular elongation limiter, is defined by double equation (3) with lower sign and starting through kinetic state after first impact, $t_1=t_{1ul_1+}$, $\varphi_1(t_{1ul_1+})=\delta$, $\dot{\varphi}_1(t_{1ul_1+})=\dot{\varphi}_{1odl_1}=-\dot{\varphi}_{1ul_1-}$, determining integral constant in the form:

$$C_{12}=\frac{1}{e^{2\delta tg\alpha_0}}\left\{\left(-\dot{\varphi}_{1ul_1}\right)^2-\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\delta-\alpha_0)-2tg\alpha_0\sin(\delta-\alpha_0)\right]\right\}\quad (10)$$

Next step is to find time of the impact between mass particles and common positions of the mass particles in this interval of motions in which first impact between mass particles appear and corresponding velocities before and after impacts (arrival velocity and leaving velocity of the impact). This task can be realized numerically by use the following expressions with two unknown, time t_{sud_1} of first impacts between mass particles and common position φ_{sud1} of this their firs impact:

$$t_{sud_1}=\int_{\delta}^{\varphi_{sud1}}\frac{d\varphi_1}{-\sqrt{\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_1-\alpha_0)-2tg\alpha_0\sin(\varphi_1-\alpha_0)\right]+C_{12}\left(\delta,\dot{\varphi}_{1odl_1}\right)e^{2\varphi_1 tg\alpha_0}}}\quad (11)$$

$$t_{sud_1}=\int_{\varphi_{2ud1}}^{\varphi_{sud1}}\frac{d\varphi_2}{\sqrt{\frac{2g}{\left(1+4tg^2\alpha_0\right)R\cos\alpha_0}\left[\cos(\varphi_2+\alpha_0)-2tg\alpha_0\sin(\varphi_2+\alpha_0)\right]+C_{21}(\varphi_{2ud1},\dot{\varphi}_{2ud1})e^{-2\varphi_2 tg\alpha_0}}}\quad (12)$$

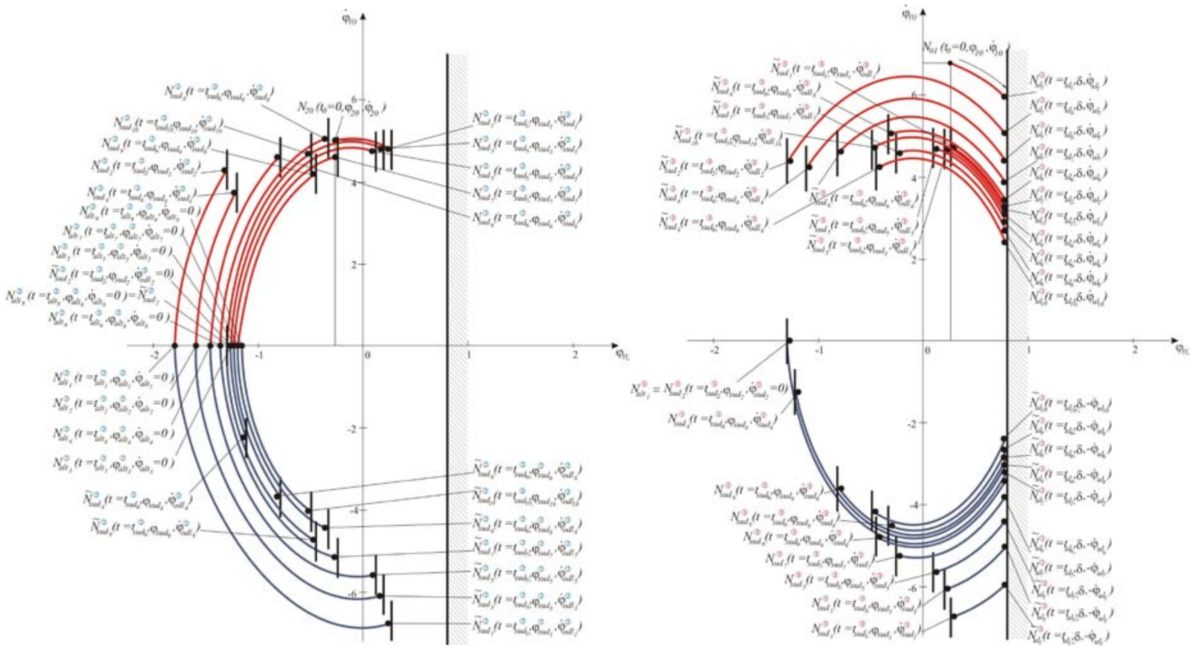
After using MathCad program, and obtained time t_{sud_1} and position φ_{sud1} of the first impact between mass particles it is easy to obtain arrival velocities unreachable before their first impact, $\dot{\varphi}_{1sud_1,ul}$ and $\dot{\varphi}_{2sud_1,ul}$ using expressions (3) with lower sign with corresponding integral constant $C_{12}\left(\delta,\dot{\varphi}_{1odl_1}\right)$ and (4) with upper sign and corresponding integral constant $C_{21}(\varphi_{2ud1},\dot{\varphi}_{2ud1})$. Then, expression for the leaving velocities $\dot{\varphi}_{2sud_1,odl}$ and $\dot{\varphi}_{1sud_1,odl}$ of the mass particles after first impact between mass particles are in the following form:

$$\dot{\varphi}_{2sud_1,odl}=\frac{m_1(1+k)}{m_1+m_2}\dot{\varphi}_{1sud_1,ul}-\frac{m_2-km_1}{m_1+m_2}\dot{\varphi}_{2sud_1,ul},\quad \dot{\varphi}_{1sud_1,odl}=\frac{km_2-m_1}{m_1+m_2}\dot{\varphi}_{1sud_1,ul}+\frac{m_2(k+1)}{m_1+m_2}\dot{\varphi}_{2sud_1,ul}\quad (14)$$

Coordinate of position φ_{sud1} of the first impact between mass particles and leaving velocities $\dot{\varphi}_{2sud_1,odl}$ and $\dot{\varphi}_{1sud_1,odl}$ of the mass particles after first impact between mass particles are starting (initial) coordinates and velocities of the next phase trajectory branches of mass particles for next interval of the their motions. Methodology to build next phase trajectory branches in the next intervals of the mass particle motions between impacts and friction force alternation of direction is clear visible from previous explanations and taking into account limitation of this paper pages no possible to present all details of this used methodology. From previous presented methodology is not difficult to applied to the next intervals of the mass particle motion and numerically determine kinetic parameters of the next impacts up to the rest of the mass particles.

3. NUMERICAL ANALYSIS OF THE VIBRO-IMPACT DYNAMICS –AN EXAMPLE

For numerical investigation we use a vibro-impact dynamics of two heavy mass particle motions along rough circle with following kinetic and geometrical data: $m_1 = 0{,}2[kg]$, $m_2 = 0{,}2[kg]$, $R = 0{,}5[m]$, $\alpha_0 = 0.05$, $g = 9{,}81\left[\frac{m}{s^2}\right]$, $\delta = \frac{\pi}{4}[rad]$, $\varphi_{10} = \frac{\pi}{12}[rad]$, $\dot{\varphi}_{10} = 7\left[\frac{rad}{s}\right]$, $\varphi_{20} = -\frac{\pi}{12}[rad]$, $\dot{\varphi}_{20} = 5\left[\frac{rad}{s}\right]$.



b* **a***

Fig. 2. Phase portrait $(\varphi,\dot{\varphi})$ of the two mass particle vibro-impact system dynamics along rough circle with Coulomb's type friction and one side right impact limiter of the angular elongations of the second mass particle. (a) Phase trajectory branches of the first mass particle and (b*) phase trajectory branches of the second mass particle, each between two succesive impacts .*

Graphical visualizations of the numerical expreiment of the two mass particle vibro-impact dynamics, by use, in previous part described metodology for obtaining phase trajectory branches and for the defined system data, are presented in Figure 2 a* and b*. In Fig.1.a* for first mass particle vibro-impact dynamics, phase trajectiry branches are presented. In Figure 6, for second mass particle vibro-impact dynamics, phase trajectiry branches are presented.

CONCLUSIONS

Non-linear properties in the considered vibro-impact two mass particles motions along rouhg circle are coused by three type of the nonlinearities, which are:

a* first is the basic system non-linearity of the cutvilinear mass particle path in circle form indused non-linear dependence of the proper weigh components of both heavy mass particles, as a primary non-linearity;

b* second is the strong non-linearity induced in the system by no ideal circle line and Coulomb's type friction force with alterantions of the friction force directions in the form of the discontinuity and also a member in the differential double equations expresed by members containing square of the mass particles velocities, $\dot{\varphi}_1^2, \dot{\varphi}_2^2$;

c* third is the strong non-linearity induced in the system by impacts first mass particle into impact limiter of the angular elongations and by impeacts between mass particles caused discontinuties of the mass particles velocities before and after impacts.

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