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TRANSIENT REGIMES IN SYSTEMS WITH INERTIAL EXCITATION OF OSCILLATIONS

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ABSTRACT

A problem of passing the resonance frequency zone at start and run-out of vibrational machine with inertial exciter of oscillations is considered with application of method of direct separation of motions in conjunction with method of sequential approximation. Expression for retarding moment and equation of semi slow oscillations of so called “internal” pendulum has been obtained.

INTRODUCTION

The problem of passing through resonance frequences zone arises in start and run-out periods of vibration machines operation. In particular, sticking of rotor rotating frequency close by one of its own frequencies may occur at starting, that is, Zommerfield’s effect may develop. Passing through resonance zone in these cases involves considerable oscillations in the system and, correspondingly, dynamic loads on the construction elements. Besides, up rated engine power is needed.

Zommerfield’s effect is considered with the application of various methods in a number of works (books [1-6], paper [7], see also works [8-11]). Rigorous investigation of Zommerfield’s effect by Poincare’s method was carried out in work [1]. Book [4] shows that theoretical explanation and numerical description of the known appropriateness of Zommerfield’s effect may be easily obtained by means of the method of direct separation of motions. In [7] the problem for the case of oscillating system with one degree of freedom is solved by the method of successive approximation coupled with the method of direct separation of motions. It is shown that such approach, rougher than in known works, allows to comparatively easier describe the system behavior in both pre- and post- resonance zones of rotor rotation frequencies. Such approach is used in the offered work for systems whose oscillating part is a rigid body with plane-parallel motion.

1. SCHEME OF THE SYSTEM AND MOTION EQUATIONS

Carrying body (vibrating member of machine) is considered to be a rigid body capable to execute small plane-parallel oscillations, that is, it has, in general case, three degrees of freedom (Fig. 1). It is linked with stationary base by the system of elastic and damping elements. An unbalanced rotor, set to rotation by asynchronous electric motor or by d.c. current motor, is mounted on the carrying body.

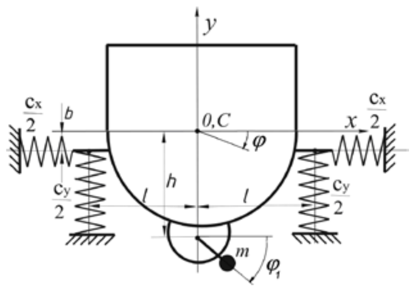

Fig. 1 Scheme of oscillatory system

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Let x and y be masses C centre coordinates in the rest system xOy and φ, φ_1 be correspondingly angles of rotation of carrying body and rotor exciter. Differential equations of motion of the system under consideration are presented as

$$I\ddot{\varphi}_1 = L(\dot{\varphi}_1) - R(\dot{\varphi}_1) + m\varepsilon\left(\ddot{x}\sin\varphi_1 + \ddot{y}\cos\varphi_1 - \ddot{\varphi}h\sin\varphi_1\right), \tag{1}$$

$$M\ddot{x} + \beta_x\dot{x} + c_x x = m\varepsilon\left(\ddot{\varphi}_1\sin\varphi_1 + \dot{\varphi}_1^2\cos\varphi_1\right), \quad M\ddot{y} + \beta_y\dot{y} + c_y y = m\varepsilon\left(\ddot{\varphi}_1\cos\varphi_1 - \dot{\varphi}_1^2\sin\varphi_1\right),$$
$$J\ddot{\varphi} + \beta_\varphi\dot{\varphi} + c_\varphi\varphi = -m\varepsilon h\left(\ddot{\varphi}_1\sin\varphi_1 + \dot{\varphi}_1^2\cos\varphi_1\right), \tag{2}$$

where M is total mass of the system; J is a moment of inertia in respect to the axes passing through the masses centre; I is a total moment of vibro exciter rotor inertia in respect to its axis of rotation; m, ε are, correspondingly, vibro exciter mass and its eccentricity; $\beta_x, \beta_y, \beta_\varphi$ are coefficients of viscous resistance, c_x, c_y are longitudinal rigidities of vertical and horizontal springs; $c_\varphi = c_y l^2 + c_x b^2$; l, b are parameters determining attaching point of the upper spring ends in respect to the masses centre of carrying body; h is the distance from centre the masses of carrying body to exciter rotor axis; $L(\dot{\varphi}_1), R(\dot{\varphi}_1)$ are correspondingly, motor torque and a moment of forces resistant to rotation.

2. THE FIRST APPROXIMATION, PECULIARITIES OF ZOMMERFIELD'S EFFECT MANIFESTATION IN THE SYSTEM

To study motion of unbalanced excites rotor at passing through the resonance zone the method of direct separation of motions is used [4], according to the main precondition of the method let us assume that motions under consideration may be presented in the form: $\varphi_1 = \omega t + \psi(t, \omega t)$, $x = x(t, \omega t)$, $y = y(t, \omega t)$, $\varphi = \varphi(t, \omega t)$ where $\omega = \omega(t)$ is slow and ψ and x, y, φ are fast time functions, they are 2π - periodical at $\tau = \omega t$ and they value average equals zero; it is also assumed that $\dot{\psi} << \omega$.

Such presentation of equations (1), (2) at studying the vibroexciter rotor passing through resonance zone, when Zommerfield's effect is taking place and, correspondingly, the frequency of rotor rotation $\dot{\varphi}_1$ changes slowly enough seems to be rightful.

In the capacity of the first approximation let us assume $\psi = \psi^{(1)} = 0$, $\varphi_1 = \varphi_1^{(1)} = \omega t$. Then we come to the equation of slow motions of rotor exciter at passing through resonance zone in the form

$$I\dot{\omega} = L(\omega) - R(\omega) + V(\omega). \tag{3}$$

Here $V(\omega) = -(m\varepsilon\omega)^2\left[\frac{n_x}{MB_x^2} + \frac{n_y}{MB_y^2} + \frac{h}{J}\frac{n_\varphi}{B_\varphi^2}\right]$ (4)

is so called vibrational moment,

$$B_x = \sqrt{\left(1-\lambda_x^2\right)^2 + 4n_x^2}, \quad \lambda_x = \frac{p_x}{\omega}, \quad p_x = \sqrt{\frac{c_x}{M}}, \quad n_x = \frac{\beta_x}{2M\omega}, B_y = \sqrt{\left(1-\lambda_y^2\right)^2 + 4n_y^2}, \lambda_y = \frac{p_y}{\omega},$$
$$p_y = \sqrt{\frac{c_y}{M}}, \quad n_y = \frac{\beta_y}{2M\omega}, \quad B_\varphi = \sqrt{\left(1-\lambda_\varphi^2\right)^2 + 4n_\varphi^2}, \quad \lambda_\varphi = \frac{p_\varphi}{\omega}, \quad p_\varphi = \sqrt{\frac{c_\varphi}{J}}, \quad n_\varphi = \frac{\beta_\varphi}{2J\omega}.$$

At obtaining this system linearization of expressions $L(\dot{\varphi}_1), R(\varphi_1)$, as in [1], close by value $\dot{\varphi}_1 = \omega$ (where ω is frequency of rotor "sticking") is performed, $k = -\left.\frac{d(L-R)}{d\dot{\varphi}_1}\right|_{\dot{\varphi}=\omega} > 0$ being a total damping coefficient.

All components in formula (4) are negative. Thus, as it is for the system with one oscillatory degree of freedom, vibrational moment is always retarding, i.e., it is an additional load upon the engine rotor, its dependency on frequency is of resonance character, and, therefore, its retarding effect manifests itself in comparatively narrow frequency range.

Rotor rotational speed in stationary regimes is determined from equation $L(\omega) = R(\omega) - V(\omega)$. Solutions of this equation are in conformity with cross-points of plots $L(\omega)$ and $M_s = R(\omega) - V(\omega)$, where curves L correspond to static characteristics of the motor (Fig. 2). According to the figure, the presence of several resonance peaks of the vibrational moment curve may lead to the emergence of additional cross-points of curves M_s and L in comparison with the system with rectilinear oscillations of the working head. Thus, several regimes of motion, close to uniform rotation of the rotor and having different average angular velocities are possible in the system. Solution $\omega_1 < p_1$ under conditions of the picture is pre resonance ω_2, ω_3 ($p_1 < \omega_2, \omega_3 < p_3$) is inter resonance, $\omega_4 > p_3$ is post resonance and $\omega_5 >> p_3$ is a post resonance. Inequality $R'(\omega_*) - V'(\omega_*) > L'(\omega_*)$ is a condition of stability of the regime under consideration [4]. Thus, solution ω_1, ω_3, ω_5 and $\omega_5^{(1)}$ are stable and ω_2, ω_4, corresponding to discending branches of the curve M_s are unstable. Characteristic L corresponds to “sticking” of the system with motor of deficient power close to resonance at frequencies ω_1 or ω_3 (motor, on coming to this regime in the process of acceleration would not be able to overcome the resonance peak and reach nominal angular velocity ω_5) and characteristic L_1 of more powerful motor corresponds to coming to post resonance regime of motion with velocity $\omega_5^{(1)}$. Hence, as it is in the system with one oscillatory degree of freedom, only two basically different regimes of motion take place: “sticking” of the system in resonance zone and a post resonance regime, or if motor power is sufficient for acceleration, the system, as a rule, after some retardation, rapidly (“by a leap”) comes to the second stationary regime, corresponding to angular velocity ω_5.

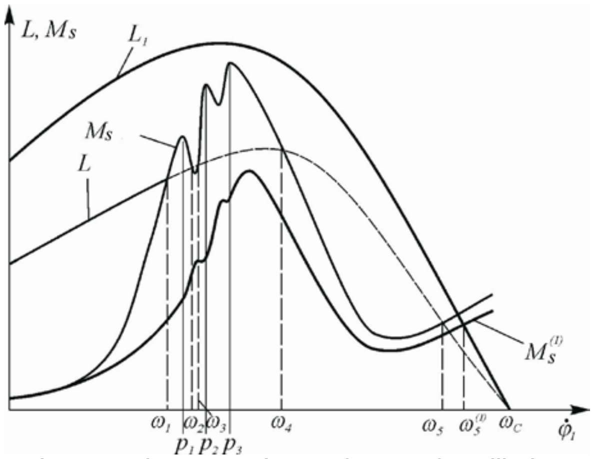

Fig. 2 Stationary regimes of rotor of oscillations rotation

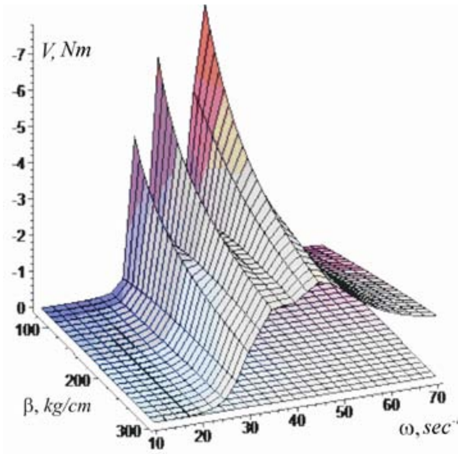

Fig. 3 Dependency of vibrational moment on frequency and resistance coefficient β

Expression (4) for vibrational moment may be considered as the sum $V(\omega) = \sum_{q=x,y,\varphi} v_q$, summands of which $v_q = -(m\varepsilon\omega)^2 \dfrac{n_q}{M_q B_q^2}$, are “particular” vibrational moments characterizing the affect of oscillations exciter upon rotor rotation, corresponding to q generalized coordinate. (Here $q = x$, if $q = y$, то $M_q = M$; if $q = \varphi$, то $M_q = M\dfrac{\rho^2}{h^2}$).

If should, be noted that expression for “particular” vibrational moment may be presented in the form $v_q = \dfrac{1}{2} F a_q \sin\gamma_q$, where $F = m\varepsilon\omega^2$ is an amplitude of driving force developed by exciter rotor at stationary carrying body, $a_q = \dfrac{m\varepsilon}{M_q\sqrt{\left(1-\lambda_q^2\right)^2 + 4n_q^2}}$ is an amplitude of platform oscillations, corresponding to q oscillatory coordinate.

Both particular and general vibrational moments characterize vibrational link between carrying body oscillatory motions and rotating motions of vibro exciter rotor.

According to formula (4) the retarding effect of vibration at starting is the less, the stronger the resistance of the system in coordinates $x,\ y,\ \varphi$. Fig.3 shows the dependency of vibrational moment on viscous resistance coefficient β ($\beta = \beta_x = \beta_y /1{,}1 = \beta_\varphi / 0{,}05$) at passing through the resonance zone.

It should be emphasized that the value of maximal retarding vibrational moment sufficiently depends, according to (4), on the frequencies of natural oscillations of the system; decreasing natural frequency we may decrease the retarding moment and, in consequence, decrease resonance amplitudes of oscillations as well as the power of the engine necessary for passing through the resonance zone. Taking into account dependency of vibrational moment on natural frequencies we may assume that the most significant retarding effect is exerted by a particular vibrational moment v_φ, whose frequency is p_φ, as a rule, the highest for the range of machines under consideration. Thus, for instance, in the case of damper application for decreasing the level of oscillations at passing through the resonance the mounting of only one damper of rotational oscillations will be enough.

A particular case of the system when the axle of unbalanced rotor passes through the centre of-the carrying body masses has been considered. The exciter axle and attaching chamber spring points are in the same plane ($b \approx 0$). Thus, carrying body performs only transitional motion in plane xOy.

3. SECOND APPROXIMATION. SEMISLOW OSCILLATIONS OF EXCITER ROTOR

For further analysis of rotor motion at passing through the resonance zone we shall use methods offered for investigation of the simplest system in work [7]. We assume $\varphi_1 = \varphi_1^{(2)} = \omega t + \psi$, $x = x^{(1)} + x^{(2)}$, $y = y^{(1)} + y^{(2)}$, $\varphi = \varphi^{(1)} + \varphi^{(2)}$. Then we come to the following system of equations for ψ and $x^{(2)}, y^{(2)}, \varphi^{(2)}$:

$$I\ddot{\psi} + k\dot{\psi} = -m\varepsilon\{\left[\ddot{x}^{(1)} + \ddot{x}^{(2)} - (\ddot{\varphi}^{(1)} + \ddot{\varphi}^{(2)})h\right]\sin(\omega t + \psi) + (\ddot{y}^{(1)} + \ddot{y}^{(2)})\cos(\omega t + \psi)\} -$$
$$-\frac{m\varepsilon\omega}{2\pi}\int_0^{2\pi}[(\ddot{x}^{(1)} - \ddot{\varphi}^{(1)}h)\sin\omega t + \ddot{y}^{(1)}\cos\omega t]d\tau,$$
$$M\ddot{x}^{(2)} + \beta_x\dot{x}^{(2)} + c_x x^{(2)} = m\varepsilon[(\dot{\omega} + \ddot{\psi})\sin(\omega t + \psi) + (\omega + \dot{\psi})^2\cos(\omega t + \psi) - \omega^2\cos\omega t],$$
$$M\ddot{y}^{(2)} + \beta_y\dot{y}^{(2)} + c_y y^{(2)} = m\varepsilon[(\dot{\omega} + \ddot{\psi})\cos(\omega t + \psi) + (\omega + \dot{\psi})^2\sin(\omega t + \psi) + \omega^2\sin\omega t],$$
$$J\ddot{\varphi}^{(2)} + \beta_\varphi\dot{\varphi}^{(2)} + c_\varphi\varphi^{(2)} = m\varepsilon h[(\dot{\omega} + \ddot{\psi})\sin(\omega t + \psi) + (\omega + \dot{\psi})^2\cos(\omega t + \psi) - \omega^2\cos\omega t]. \quad (5)$$

For the solution of system (5) we shall again use the method of direct separation of motions assuming that $\psi = \Psi + \gamma$, $x^{(2)} = X + \delta_x$, $y^{(2)} = Y + \delta_y$, $\varphi^{(2)} = \Phi + \delta_\varphi$, where Ψ, X, Y, Φ are slow and γ, δ_x, δ_y δ_φ-fast 2π-periodic in fast time τ components with average zero values.

In the long run we come to the equations of semi slow (or semi fast) oscillations of exciter rotor angular velocity with respect to uniform rotation (equation of "internal pendulum" oscillations) in the form obtained in [7] for the system with rectilinear oscillations of carrying body

$$\ddot{\Psi} + 2n_1\dot{\Psi} + B\sin\Psi - \mathrm{P}\sin^2\frac{\Psi}{2} = 0, \quad (6)$$

here $2n_1 = k/I$, $B = b_x + b_y + b_\varphi$, $P^2 = \rho_x^2 + \rho_y^2 + \rho_\varphi^2$,

$$b_x = \frac{(m\varepsilon\omega^2)^2}{2MI}\frac{p_x^2-\omega^2}{(p_x^2-\omega^2)^2+4n_x^2\omega^4}, \qquad \rho_x^2 = \frac{(m\varepsilon\omega^2)^2}{MI}\frac{2n_x\omega^2}{(p_x^2-\omega^2)^2+4n_x^2\omega^4},$$

$$b_y = \frac{(m\varepsilon\omega^2)^2}{2MI}\frac{p_y^2-\omega^2}{(p_y^2-\omega^2)^2+4n_y^2\omega^4}, \qquad \rho_y^2 = \frac{(m\varepsilon\omega^2)^2}{MI}\frac{2n_y\omega^2}{(p_y^2-\omega^2)^2+4n_y^2\omega^4},$$

$$b_\varphi = \frac{(m\varepsilon\omega^2)^2h^2}{2JI}\frac{p_\varphi^2-\omega^2}{(p_\varphi^2-\omega^2)^2+4n_\varphi^2\omega^4}, \qquad \rho_\varphi^2 = \frac{(m\varepsilon\omega^2)^2h^2}{JI}\frac{2n_\varphi\omega^2}{(p_\varphi^2-\omega^2)^2+4n_\varphi^2\omega^4}. \tag{7}$$

In the case of consideration of small oscillations, having linearized equation (6) we may present it in classical form $\ddot{\Psi}+2n_1\dot{\Psi}+\beta\Psi=0$.

At satisfaction of condition $\omega << \omega^2$ frequency of rotor rotational speed ω changes slowly and value $q=\sqrt{|B|}$ is frequency of small free oscillations of the linearized model of internal pendulum (without account of the force of resistance).

Conclusions, made in work [7], about the validity of equation (6) for the system with one oscillatory degree of freedom apply to the cases with two or three degrees of freedom as well.

It follows from the analysis of equation (6) that at B>0 the solution $\Psi=\Psi_1=0$, corresponding to “lower” position of internal pendulum, is stable and at $B<0$ the solution $\Psi=\Psi_2=\pi$ corresponding to “upper” position is stable. Therefore, solution $\Psi_1=0$ is stable in pre resonance zone of variations of frequency $\omega < p_{\min}$, where $p_{\min}$-is the smallest of values p_x, p_y, p_φ - and in post resonance zone solution $\Psi_2=0$ is stable. So, as in the case of oscillatory system with one degree of freedom, we may say that the internal pendulum turns over in the post resonance zone of frequencies $\omega > p_{\max}$. The fact that in intermediate zone $p_{\min} < \omega < p_{\max}$ pendulum may have time to turn over several times is a sufficient distinction of the system under consideration. In other words, complicated behavior of the system may be expected in the mentioned zone. It is natural, that such effect may take place in the system with any number of oscillatory degrees of freedom.

The obtained results are corroborated by numerical experiment. Fig.3 shows “sticking” of the system I pre-and inter resonance zones with motor of deficient power. Fig.4 shows the effect of emergence of semi slow oscillations of exciter rotor angular velocity close to the resonance zone in the case of rotor “sticking” for the system with one and two degrees of oscillatory freedom.

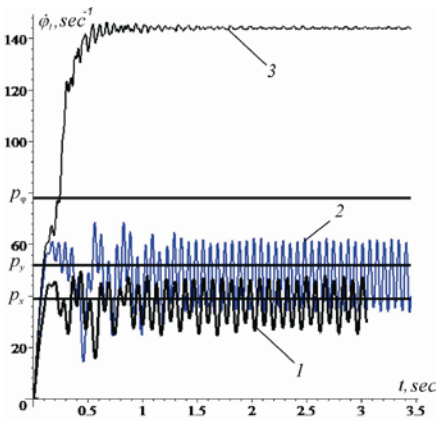

Fig. 4

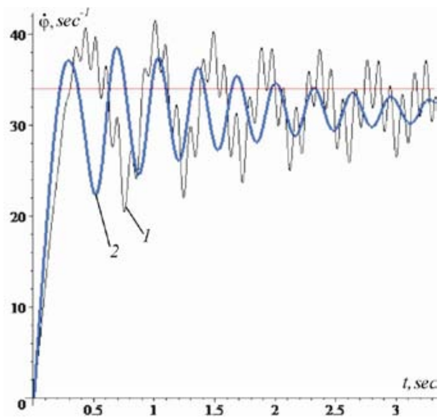

Fig. 5

Fig. 4. Dependency of the vibro exciter rotor rotation frequency time: 1- rotor “sticking” in pre resonance zone, 2- rotor “sticking” in inter resonance zone, 3- acceleration with coming to post resonance regime

Fig. 5. Change of vibroexciter rotor rotation frequency in case of “sticking” in the resonance zone: 1- system with one; 2- system with two oscillatory degrees of freedom.

It should be noted that simulation was performed with account of dynamic response of asynchronous motor. According to the presented plots the ratio of frequencies of semi slow free oscillations of exciter rotor velocity for such oscillatory systems makes up 1.4, as it should be according to the formula $q = \sqrt{(B)}$.

CONCLUSIONS

The work deals with the problem of passing the resonance frequency zone at start and run-out of vibrational machine with inertial exciter of oscillations. The case have been studied when oscillatory part of the system is linear and is a plane-parallely oscillating rigid body. As in the simplest case of the system with one oscillatory degree of freedom, the problem is comparatively simply solved by application of the method of direct separation of motions coupled with the method of successive approximations.

Expression for the retarding vibrational moment which must be overcome by the motor at passing through the resonance zone consists in the considered case with three components, corresponding to each of three frequencies of free oscillations of the body. These components are of pronouncedly manifested resonance character. Accordingly, the obtained expression for the square of the frequency of semi slow oscillations of the internal pendulum (rotor "swinging") also consists of tree components. As in the simplest system, this pendulum as if turns over at passing through resonance frequency: its "lower" position is stable in pre resonance zone and its "upper" position is stable in post resonance zone. Stable positions may alternate in the interval between the smallest and the greatest resonance frequencies. A complicated behavior of the system may be expected in this interval. Absence of fast oscillations of rotor with doubled frequency of rotation in the case of symmetry of the oscillatory part of the system is a peculiar feature of the considered system.

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