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TRANSPORT AND MIXING ACROSS GULF STREAM

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ABSTRACT

The new mathematical model for a stream function of a meandering jet of Gulf Stream is suggested. It is based upon a modification of the von Kármán vortex street stream function. The suggested modification allows one to approximate experimentally found main patterns in the meandering jet of Gulf Stream. This stream characterizes by the following coherent structure elements in a coordinate frame moving with a speed of the meander: 1) an eastward-propagating meandering jet; 2) regions of recirculating fluid below and above meander crests and troughs; 3) regions of westward-propagating fluid below and above the jet and recirculation regions. The inclusion of eddies above the recirculation regions and the jet enhance transport and mixing across the jet. Calculations show that more than a half of the circular area above hyperbolic points may contain warm fluid from a central area of the jet. To study mixing across the jet we examine deformation of this circular area back in time, so we can determine from which part of the jet that area is composed. Contour line tracking method conserving all topological properties in 2-D flows is used for this procedure.

INTRODUCTION

Transport of warm water from the meandering jet of Gulf Stream into cold water surrounding the jet has been the focus of many recent studies. Mixing across the jet was experimentally shown in works of A. Bower [1]. Bower and Rossby [2] showed that meanders associated with Gulf Stream are responsible for much of the cross-stream motions of RAFOS floats within the jet. However, meanders alone cannot lead to the motion from one side of the jet to another. We expect that the interaction of the jet with a chain of eddies could be important in that respect. To study mixing process A. Bower [1] suggested a simple two-dimensional kinematic model. Her model describes a simple streamfunction that reproduces the kinematic features of an eastward propagating meandering jet and in this model the meander parameters affect the rate and amount of water that propagate downstream. But Bower's model does not allow for any mixing, any movement of fluid particles from one side of the jet to another. It is known that the Gulf Stream das not remain invariant in shape due to growth and diminishing of meanders. Time dependence of the meander's parameters was used by Somelson [3] to increase mixing. Another way that the Gulf Stream will change shape is by interacting with rings. The inclusion of eddies in the simple meandering jet model should enhance the mixing of fluid parcels within the jet.

The Gulf Stream frequently interacts with many rings and it can be expected that these eddies play an essential role in the distribution of tracer properties in the vicinity of the stream [4].

Our intention is to consider the enhancement of the mixing caused by the interaction of a two-dimensional jet, modelled by von Kármán vortex street [5], and Zimmerman eddies [6]. To observe the mixing that occur under interaction with eddies we examine Lagrangian particle dispersion in time.

1. ANALYTICAL MODEL OF THE GULF STREAM WITH EDDIES

The streamfunction in the Bower's model has the form

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$$\psi(x,y,t) = \psi_0 \left\{ 1 - \tanh\left[\frac{y - y_C}{\lambda / \cos(\alpha)} \right] \right\} \tag{1}$$

where ψ_0 -- scale factor, which with λ, determines maximum downstream speed, $\lambda =$ 40 km, the scale width of the jet, $y_c = A \sin\left[k(x - c_x t)\right]$ -- defines center streamline, A -- wave amplitude, $k = 2\pi/L$ -- the wave number, $\alpha = \tan^{-1}\left\{Ak\cos\left[k\left(x - c_x t\right)\right]\right\}$, direction of current. The $\cos(\alpha)$ term is included to give the jet uniform width everywhere. It is convenient to transform the streamfunction field into a reference frame moving with the phase speed c_x, (as it was done by Bower [1]). In the moving frame, the streamfunction has the form

$$\psi'(x', y') \; = \; \psi_0 \left\{ 1 - \tanh\left[\frac{y' - y'_C}{\lambda / \cos(\alpha')} \right] \right\} \; + \; c_x y' \tag{2}$$

where $y'_c = A\sin(kx)$, $\alpha' = \tan^{-1}\left[Ak\cos(kx')\right]$

The streamfunction in this frame is independent of time and streamlines can be interpreted as trajectories of fluid parcels relative to the moving wave.

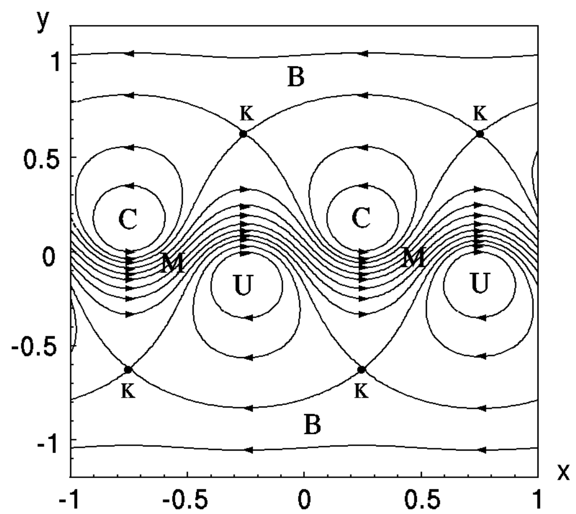

Fig.1. Coherent structure elements (the von Kármán vortex street)

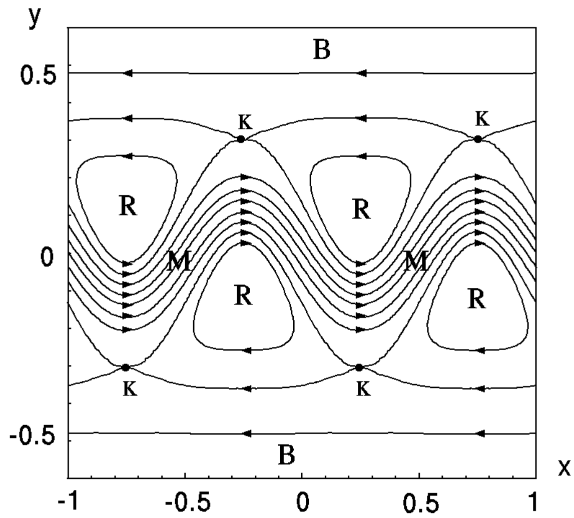

Fig.2. Coherent structure elements (the Somelson model)

The main coherent structure elements of the Gulf Stream in the moving frame are [7]: 1) an eastward-propagating meandering jet; 2) regions of fluid recirculation below and above meander crests and troughs; 3) regions of westward-propagating fluid below and above the jet and recirculation regions. We will study transport of passive particles (tracers) in such moving frame.

To study transport properties of fluid motions we suggest to use the new mathematical model for the stream function of the Gulf Stream. This new stream function is a modification of the von Kármán vortex street streamfunction. The von Kármán vortex street function describes a system of vortices behind a cylinder, which moves with a constant speed. The streamfunction of the vortex street has the same three main coherent structure elements. In the moving coordinate frame, which moves with a constant speed together with vortices, the streamfunction has the form

$$\psi(x,y) = -\frac{\Gamma}{4\pi} \ln\frac{P(x,y)}{Q(x,y)} + cy \tag{3}$$

where c is the vortex speed in the x direction;

$$P(x,y) = \cosh\frac{2\pi}{l}\left(y + \frac{h}{2}\right) + \sin\frac{2\pi x}{l}; \qquad Q(x,y) = \cosh\frac{2\pi}{l}\left(y - \frac{h}{2}\right) - \sin\frac{2\pi x}{l} \tag{4}$$

or the dimensionless variables $\tilde{x} = x/l$; $\tilde{y} = y/l$ the streamfunction can be written as

$$\psi(\tilde{x}, \tilde{y}) = -\frac{1}{2k} \ln \frac{P(\tilde{x}, \tilde{y})}{Q(\tilde{x}, \tilde{y})} + \tilde{c}\tilde{y} \tag{5}$$

where $P(\tilde{x}, \tilde{y}) = \operatorname{csh} k\left(\tilde{y} + b\right) + \sin k\tilde{x}$; $Q(\tilde{x}, \tilde{y}) = \operatorname{csh} k\left(\tilde{y} - b\right) - \sin k\tilde{x}$

$$\tilde{\psi} = \psi/\Gamma; \quad \tilde{c} = cl/\Gamma; \quad b = h/2l; \quad k = 2\pi$$

In the Fig.1 the streamlines of the streamfunction (5) are shown. In the Fig.1 by M is shown meandering jet, by C and U the recirculation regions of cyclonic and anticyclonic rotation, by B regions of westward propagating fluid.

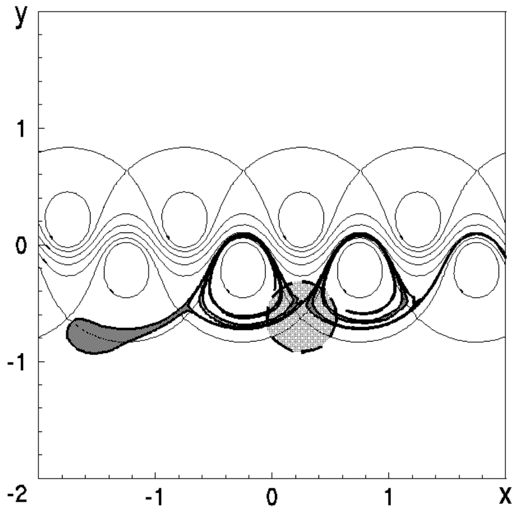

Fig.3 Chaotic advection pattern in the von Kármán model after 18 periods of tidal flow

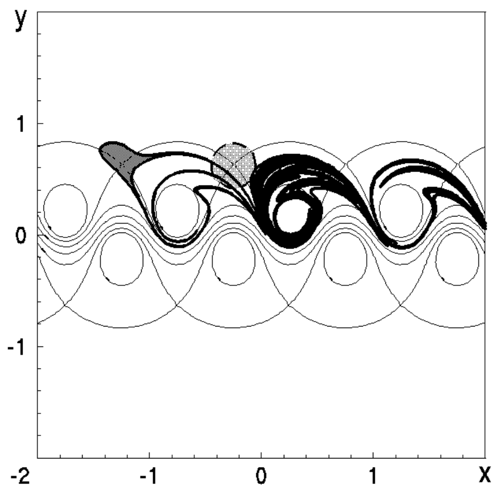

Fig.4. Mixing pattern of circular blobs in the von Kármán model with small variation after 18 periods

To compare the new model streamlines patters with the Samelson's model [3] we represent the streamlines (2) in the Fig.2 for the function (2). The flow fields in the Fig.2 and Fig.1 have hyperbolic points K .

The advection equations for passive tracers have the form

$$\dot{x} = u = -\frac{\partial \psi}{\partial y}; \;\; \dot{y} = v = \frac{\partial \psi}{\partial x} \tag{6}$$

For the streamfunction (5) equations (6) could be written as

$$\begin{cases} \dot{\tilde{x}} = \dfrac{\cosh kb}{PQ}\left(\sinh kb - \sin k\tilde{x} \sinh k\tilde{y}\right) - c \\ \dot{\tilde{y}} = -\dfrac{\cosh kb}{PQ} \cos k\tilde{x} \, \sinh k\tilde{y} \end{cases} \tag{7}$$

To find hyperbolic points we use equations $\dot{x} = 0$; $\dot{y} = 0$ so we have, for example,

$$\tilde{x}_1 = \frac{1}{4}; \;\; \tilde{x}_2 = \frac{3}{4} \quad \text{and} \quad \tilde{y}_{1,2} = \mp \frac{1}{k} \operatorname{Arsinh}\left(\frac{1}{\tilde{c}} \cosh kb - \sinh kb\right) \tag{8}$$

Both Bower's model and von Kármán vortex street model do not allow any movement of fluid particles from one side of the jet to another or cross jet movement. Particles (passive tracers) can exhibit periodic or chaotic trajectories in the recirculation zones or along the meandering jet if we assume that the amplitude of the streamfunction (5) has a small variation in time, say, as $-\frac{1}{2k}(1+0.1\cos\pi t)$.

In Fig.3, 4 chaotic advection in the von Kármán model with tidal flow velocity in the y direction (Fig.3) and in the x direction (Fig.4) are shown, when additional components are $v_y = 0.1\cos\pi t;\ v_x = 5\cos\pi t$. Black mixing patterns of circular blobs (with dashed contour line) are shown for different initial positions of blobs. We may see as distinguished circular area (shown by grey colour) was deformed after 18 periods of variations. The whole distinguished fluid parcels will not leave the streamlines of the jet area. To introduce mixing and transport across the boundaries of the jet particles must be allowed to leave the streamlines in Fig.1 and Fig.2. For this purpose we assume that the jet interacts with a chain of topographical eddies, which locations are stationary in time. It means they move westward with constant speed c in the moving frame. For an eddy chain streamfunction we use Zimmerman [6] streamfunction, which in the rectangular non-moving coordinate system $O'x'y'$ can be given by

$$\psi_z = \frac{1}{\pi\sqrt{2}}\sin\pi x'\sin\pi y' \tag{9}$$

It consists from square cells with vortices inside and hyperbolic points in each corner of cells. Let's put the origin of the coordinate system $O'x'y'$ point O' (hyperbolic point of (9)) in the point (1,0) of the moving frame and turn the axis counterclockwise on an angle $\pi/4$. Then in the moving frame the streamfunction (9) has the form

$$\psi_z = \frac{1}{\pi\sqrt{2}}\sin\frac{\pi}{\sqrt{2}}\left[(x-ct)+y-1\right]\sin\frac{\pi}{\sqrt{2}}\left[y-(x-ct)+1\right] \tag{10}$$

The equations for fluid trajectories in the flow field, which is superposition of two streamfunctions (7) and (10) can be written in the form

$$\begin{cases} \dot{x} = \Gamma_1\dfrac{\cosh kb}{PQ}\left(\sinh kb - \sin kx\sinh ky\right) - c - \sin\pi\sqrt{2}y \\ \dot{y} = -\Gamma_1\dfrac{\cosh kb}{PQ}\cos kx\sinh ky + \sin\pi\sqrt{2}\left(x-ct-1\right) \end{cases} \tag{11}$$

where $\Gamma_1 = 1+\varepsilon\cos\omega t$ is an amplitude of the von Kármán vortex street function with small disturbance $\varepsilon\cos\omega t$, when $\varepsilon \ge 0$ and ω is frequency of tidal flow. In order to enhance influence only one vortex located near the hyperbolic point we choose

$$\psi_z = \frac{1}{\pi\sqrt{2}}\frac{A(t)}{B(y)}\left(\cos\pi\sqrt{2}y - \cos\pi\sqrt{2}\left(1-x+ct\right)\right) \tag{12}$$

where $A(t) = 0.2+\varepsilon_1\cos\omega t$; $B(y) = \exp\left(C_2 y\right)^2$. Then, we have the following system:

$$\begin{cases} \dot{x} = \Gamma_1\dfrac{\cosh kb}{PQ}\left(\sinh kb - \sin kx\sinh ky\right) - c - \dfrac{1}{\pi\sqrt{2}}\dfrac{A(t)}{B(y)}\sin\pi\sqrt{2}y + \\ \quad + \dfrac{C_2^2 y}{\pi\sqrt{2}}\dfrac{A(t)}{B(y)}\left(\cos\pi\sqrt{2}y - \cos\pi\sqrt{2}\left(1-x+ct\right)\right) \\ \dot{y} = -\Gamma_1\dfrac{\cosh kb}{PQ}\cos kx\sinh ky + \sin\pi\sqrt{2}\left(1-x+ct\right) + \\ \quad + \dfrac{1}{\pi\sqrt{2}}\dfrac{A(t)}{B(y)}\sin\pi\sqrt{2}\left(1-x+ct\right) + 0.05\cos\left(\omega_1 t\right) \end{cases} \tag{13}$$

2. NUMERICAL EXPERIMENT

Our study of transport and mixing across the jet is based on the description of paths of dyed blob individual particles, so we will use Lagrangian description. We will investigate of the motion of a mathematical points that move at each instant with the velocity corresponding to point instant position. Thus, the dyed particle is supposed to be inertialess, it is not subjected to diffusion. We will examine deformation of distinguished circular area back in time, so we can determine from which part of the flow that area is composed. Contour line tracking method conserving all topological properties in 2-D flows is used for this procedure. Any algorithm of contour line tracking based on the tracking of points distributed along the initial blob boundary and after this point tracking connect neighbouring points. Because of non-uniform stretching and folding of the line, two neighbouring points may appear far away from each other at some future time. The obvious way to overcome this problem is to increase the number of point. It should not be done uniformly -- but only at those parts of the initial line where considerable stretching or folding occurs. The essence of our algorithm is clear: i) if it appears that some distance Δl_{k} between two neighbouring points becomes larger than some initially prescribed value l_{dis}, insert an additional point on the initial contour in the middle between points k and k+1 solve the system (13) for that one point and renumber correspondingly the initial and final arrays of points. ii) Take in any turn three points m-1, m and m+1 find the angle γ_{m}. If angle γ_{m} appears to be smaller than some prescribed value γ {usually $\gamma = 120^{\circ}$), insert additional points at the initial contour line between points m-1, m and m+1 such a way that, finally, distance between all old and new points do not exceed the value l_{cur} or the angles in the polygon are larger than γ. An additional and important check of the proposed algorithm is the accuracy of fulfilling the area conservation condition.

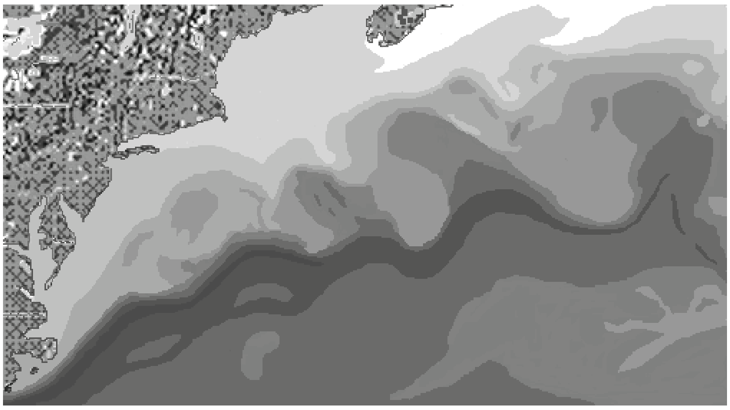

Fig.5. Satellite image of averaged sea surface temperature.

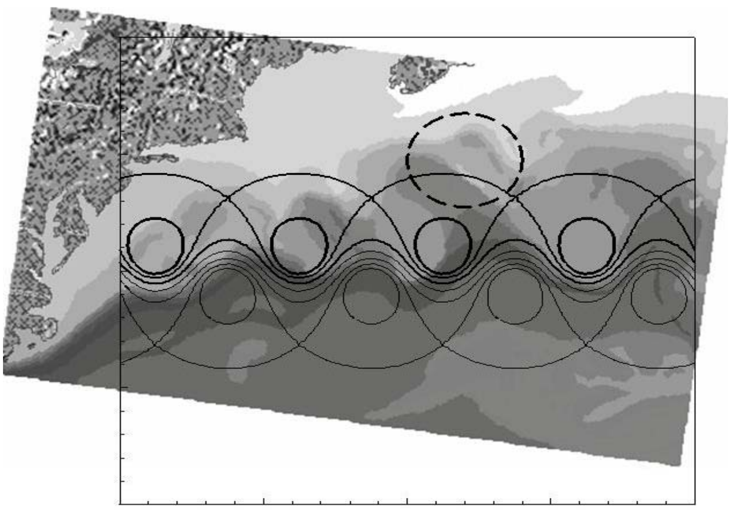

Fig.6. Satellite image of averaged sea surface temperature together with streamlines of von Kármán street.

3. RESULTS AND DISCUSSION

The results presented here correspond to numerical simulations of advections equations (11)-(12) $\varepsilon = 0$; $\varepsilon_1 = 0.033$; $C_2 = 2$; $\omega = 39.77$; $\omega_1 = \omega / 120 \approx 0.33$ and different initial locations of circular blob, radius with the centre in the point (0.45, 0.95). In Fig.5 satellite image of averaged sea surface temperature is shown [8]. In the black and white graph we show only warm surface of fluid (the warmer the darker). So that meanders of Gulf Stream are shown by the darkest grey continious color. Shore is indicated by spotted like colour.

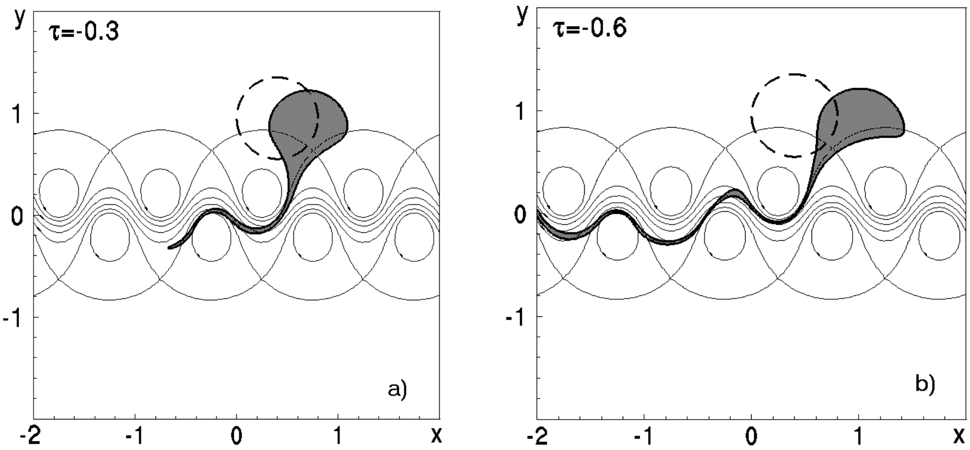

Fig.7. Deformation of circular area back in time

Above the third troughs (the third meander) a big warm fluid area is clearly seen and is shown by dark grey colour. How it was created, from which part of the jet? To answer those questions we study deformation of circular area back in time. So we study motion of particles (passive tracers) on initial contour line of the circular blob. In Fig.6,7 the locations of the circular blob are shown by dashed contour line together with streamlines of von Kármán street. In Fig.7a) the location of that blob fluid parcel at $\tau = -0.3$ (approximately 1 days before) is shown as black area. And in Fig.7b) blob fluid parcel at $\tau = -0.6$ (2 days before) is also shown as black area.

To estimate cross jet transport of fluid parcels we compare area of cold fluid parcel (black spot above the streamlines in the Fig.7b) and area of initial blob (with dashed contour line in the Fig.7a,b). The black area is approximately twice smaller, what means that after 2 days the circular blob will have the half of it's area warm and the half could.

CONCLUSION

In this study we have considered the von Kármán vortex street model of a meandering jet when it interacts with a stationary chain of eddies [4], which results in cross jet transport of fluid parcels and intensive chaotic mixing.

REFERENCES

[1] A. Bower, A simple kinematic mechanism for mixing fluid parcels across a meandering jet. *J. Phys. Oceanography*, Vol.21 N 1, pp.173-180, 1991.

[2] A. Bower, H. Rossby, Evidence of cross-frontal exchange in the Gulf Stream based isopycnal RAFOS float data *J. Phys. Oceanography*, Vol.19 N 9, pp.173-180, 1989.

[3] R. Somelson, Fluid exchange across a meandering jet *J. Phys. Oceanography*, Vol.22 N 4, pp.431-440, 1992.

[4] H. Dijkstra, *Nonlinear Physical Oceanography*, Springer, New York, 2005.

[5] T. Krasnopolskaya, V. Il'chenko, Kinematic model of Gulf Stream. – *Applied Hydromech.*, Vol.10 N 4, pp.41-53, 2008.

[6] J. Zimmerman, The tidal whirpool: a review of horizontal dispersion by tidal and residual currents
Neth. J. Sea Res, Vol.20 N 2/3, pp.133-154, 1986.

[7] R. Somelson, S.Wiggins, *Lagrangian Transport in Geophysical Jets and Waves*, Springer, New York, 2005.

[8] C. Jones, S.Winkler *Invariant manifold and Lagrangian dynamics in the ocean and atmosphere.* In: Handbook of Dynamics system (Ed.B.Fiedler), Vol.2,Elsevier Science B. V., Amsterdam, pp.55-92, 2002.