FACULTY OF MECHANICAL ENGINEERING
UNIVERSITY OF WEST BOHEMIA

DEPARTMENT OF POWER SYSTEM ENGINEERING

Nonlinear vibration analysis of steam turbine bladed disk with friction contact between adjacent blades

Josef Voldřich
Ukraine, November 2018

Проект „Развитие международного сотрудничества с украинскими ВУЗами в областях качества, энергетики и транспорта“
г. Харьков, 11/2018

Степан Прокопович Тимошенко

1878 - 1972

- A grand native of Ukraine, the father of modern engineering mechanics.
- His portrait, as the only one, has displayed with reverence in my workroom for 20 years.
- He was a giant for mathematical modelling of strength problems in mechanical engineering.

Encouragement:

- Let's not be afraid to use MATHEMATICS for solution of our actual problems.
- And it is not inevitable to look up only to commercial FEM software.

Content

- Motivation and introduction
- Methodology
- Calculations of nonlinear vibration for bladed disks
- Contact stiffnesses
- Conclusions

Motivation and introduction

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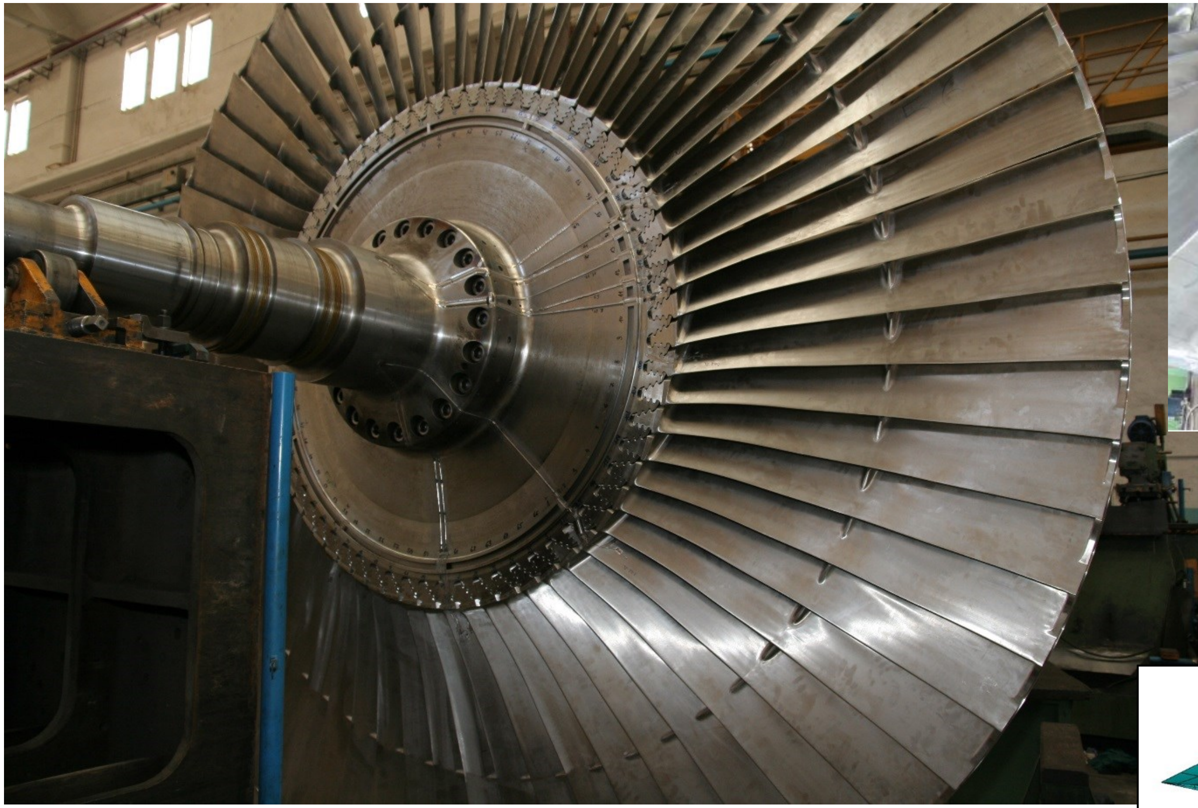
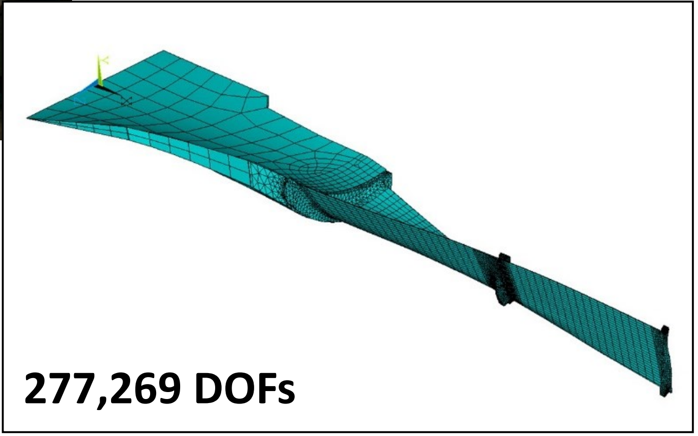
Pilsen (170 000 inhabitants):

- **Steam turbines**
  produced by Doosan Škoda Power Ltd.
- **Nuclear reactors WWER for NPPs**
  produced by Škoda JS, Ltd. (25 so far).
- **Well-known for its brewing**
  The world's first-ever pilsner type blond lager, making it the inspiration for much of the beer produced in the world today, many of which are named *pils*, *pilsner* and *pilsener.*
- **The Škoda company** (established 1859) was one of the biggest European arms factories up to the WW II.

Motivation and introduction





Demand from Doosan Škoda Power Ltd. :

**Vibration analysis of bladed disk
with 66 blades of type LSB48 (1220 mm)**

Motivation and introduction

- **renovation of steam turbines' low-pressure stages in NPP Temelín by Doosan Škoda Power**

Reactor heat power [%]	Before modernization LP [MWe]	Power rise garanted [MWe]	Power rise measured [MWe]
100	1016,9	22	28,8

Methodology – Modal analysis

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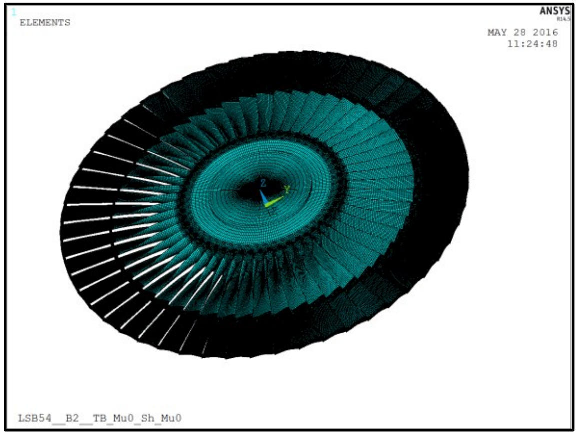
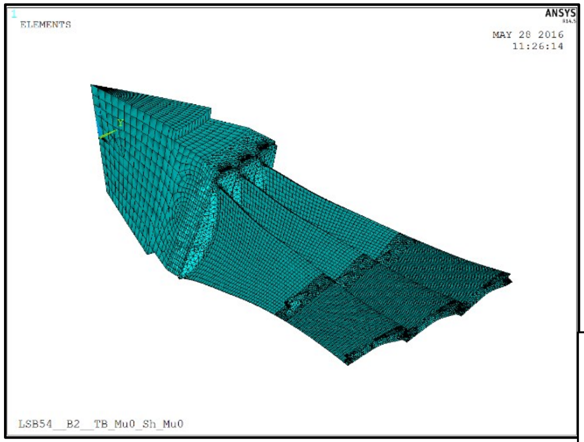


Eigenvalue problem for the whole bladed disk

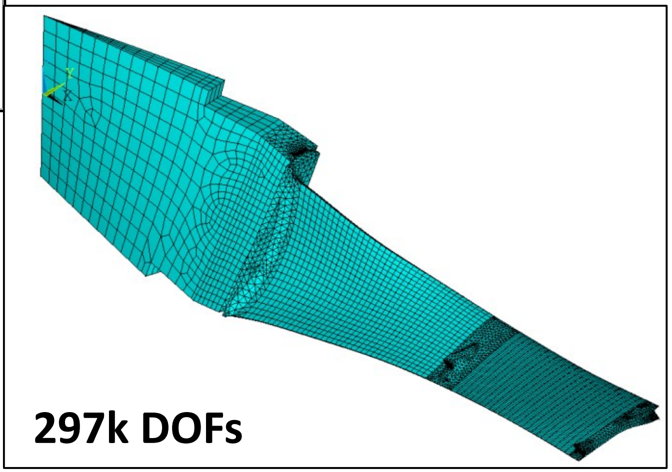
$$(-\omega^2 \boldsymbol{\mathcal{M}} + \boldsymbol{\mathcal{K}})\boldsymbol{w} = \mathbf{0}$$

$$\boldsymbol{\mathcal{K}} = \begin{bmatrix} \boldsymbol{K}_0 & \boldsymbol{K}_1 & \mathbf{0} & & \mathbf{0} & \boldsymbol{K}_{N-1} \\ \boldsymbol{K}_{N-1} & \boldsymbol{K}_0 & \boldsymbol{K}_1 & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{K}_{N-1} & \boldsymbol{K}_0 & & \mathbf{0} & \mathbf{0} \\ & \vdots & & \ddots & \vdots & \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{K}_0 & \boldsymbol{K}_1 \\ \boldsymbol{K}_1 & \mathbf{0} & \mathbf{0} & & \boldsymbol{K}_{N-1} & \boldsymbol{K}_0 \end{bmatrix}$$

$$\boldsymbol{\mathcal{M}} = \begin{bmatrix} \boldsymbol{M} & 0 & \cdots & 0 \\ 0 & \boldsymbol{M} & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{M} \end{bmatrix}$$



It is sufficient to consider only a sector with 1 blade – the reference sector



Eigenvalue problém for the reference sector:

$$\left(-\omega^2 \boldsymbol{M} + \boldsymbol{K}_0 + \boldsymbol{K}_1 \mathrm{e}^{\mathrm{i}k\sigma} + \boldsymbol{K}_{N-1} \mathrm{e}^{-\mathrm{i}k\sigma}\right) \boldsymbol{v} = 0$$

$\boldsymbol{M}, \boldsymbol{K}_0$ **: mass and stiffness matrices**
$\boldsymbol{K}_1, \boldsymbol{K}_{N-1}$ **: sector-to-sector coupling**
N **: number of blades,** $\sigma = 2\pi/N$, k **: harmoni index (nodal diameter)**

Methodology

Vibration of the reference sector

$$\boldsymbol{M}\frac{\mathrm{d}^2\boldsymbol{u}(t)}{\mathrm{d}t^2}+\boldsymbol{B}\frac{\mathrm{d}\boldsymbol{u}(t)}{\mathrm{d}t}+\boldsymbol{K}\boldsymbol{u}(t)+\boldsymbol{f}_{\mathrm{L}}\big(\boldsymbol{u}(t-\Delta t),\boldsymbol{u}(t)\big)+\boldsymbol{f}_{\mathrm{R}}\big(\boldsymbol{u}(t),\boldsymbol{u}(t+\Delta t)\big)=\boldsymbol{p}(t)$$

Linear part

Nonlinear forces between blades (friction contact)

Excitation forces

$$\boldsymbol{p}(t)=\sum_{k=1}^{n}\big(\boldsymbol{P}_k^{\mathrm{c}}\cos(k\omega t)+\boldsymbol{P}_k^{\mathrm{s}}\sin(k\omega t)\big)$$

Steady-state vibration responce can be represented by Fourier series

$$\boldsymbol{u}(t)=\boldsymbol{U}_0+\sum_{k=1}^{n}\big(\boldsymbol{U}_k^{\mathrm{c}}\cos(k\omega t)+\boldsymbol{U}_k^{\mathrm{s}}\sin(k\omega t)\big)=\boldsymbol{U}_0+\sum_{k=1}^{n}\big(\boldsymbol{U}_k\,\mathrm{e}^{-\mathrm{i}k\omega t}+\overline{\boldsymbol{U}}_k\mathrm{e}^{\mathrm{i}k\omega t}\big)$$

Substituing into the equation we obtain

$$\boldsymbol{Z}_k(\omega)\boldsymbol{U}_k+\boldsymbol{F}_k(\boldsymbol{U})-\boldsymbol{P}_k=\boldsymbol{0},\qquad k=0,\dots,n$$

$$\boldsymbol{Z}_k(\omega)=\boldsymbol{K}+\mathrm{i}k\omega\boldsymbol{B}-(k\omega)^2\boldsymbol{M},$$ **Matrix of dynamic stiffnesses for the system without couplings**

Methodology –Nonlinear vibration, Multiharmonic Balance Method

4 important steps for the solution of equation $\boldsymbol{Z}_k(\omega)\boldsymbol{U}_k + \boldsymbol{F}_k(\boldsymbol{U}) - \boldsymbol{P}_k = \boldsymbol{0}, \quad k = 0, \dots, n$

1) **Calculate a multiharmonic FRF matrix of the linear part of the system**

$$\left(\boldsymbol{Z}_k(\omega)\right)^{-1} = \boldsymbol{A}_k \approx \sum_{r=1}^{m} \frac{1}{(1 + \mathrm{i}\eta_{k,r})\Omega_{k,r}^2 - (k\omega)^2} \boldsymbol{\phi}_{k,r}\, \boldsymbol{\phi}_{k,r}^{\mathrm{H}}$$

2) **Separate linear and nonlinear degrees of freedom**

$$\boldsymbol{U}_k = \left[\boldsymbol{U}_k^{\mathrm{ln}}, \boldsymbol{U}_k^{\mathrm{nln}}\right] \qquad \boldsymbol{A}_k = \begin{bmatrix} \boldsymbol{A}_k^{\mathrm{ln/ln}} & \boldsymbol{A}_k^{\mathrm{ln/nln}} \\ \boldsymbol{A}_k^{\mathrm{nln/ln}} & \boldsymbol{A}_k^{\mathrm{nln/nln}} \end{bmatrix}$$

3) **Split the equation into linear and nonlinear part**

$$\boldsymbol{U}_k^{\mathrm{ln}} + \boldsymbol{A}_k^{\mathrm{ln/nln}} \boldsymbol{F}_k^{\mathrm{nln}}\left(\boldsymbol{U}^{\mathrm{nln}}\right) - (\boldsymbol{A}_k \boldsymbol{P}_k)^{\mathrm{ln}} = \boldsymbol{0}^{\mathrm{ln}}$$

$$\boldsymbol{U}_k^{\mathrm{nln}} + \boldsymbol{A}_k^{\mathrm{nln/nln}} \boldsymbol{F}_k^{\mathrm{nln}}\left(\boldsymbol{U}^{\mathrm{nln}}\right) - (\boldsymbol{A}_k \boldsymbol{P}_k)^{\mathrm{nln}} = \boldsymbol{0}^{\mathrm{nln}}$$

using $\boldsymbol{F}_k(\boldsymbol{U}) = \left[0, \boldsymbol{F}_k^{\mathrm{nln}}\left(\boldsymbol{U}^{\mathrm{nln}}\right)\right]$

4) **Accomplish effective calculation**

$$\boldsymbol{U}^{\mathrm{nln}} \longrightarrow \boldsymbol{F}_k^{\mathrm{nln}}\left(\boldsymbol{U}^{\mathrm{nln}}\right)$$

Methodology – Mathematical model of contacts

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Dry Coulomb friction model for contactforces is used

	Contact		Separation
	Stick	Slip	
Tangential force, f_t	$f_{t0} + k_t(x - x_0)$	$\mathrm{sgn}(\dot{x})\,\mu f_n$	0
Normal force, f_n	$N_0 + k_n y$		0

μ **friction coefficient**

N_0 **normal preloading force of the contact**

x, y **relative displacement of nodes of the nonlinear contact element in the tangential and the normal directions**

k_t, k_n **tečné a normálové kontaktní tuhosti**

Calculation of harmonic coefficients for nonlinear forces

$$\boldsymbol{U}^{\mathrm{nln}} \rightarrow \boldsymbol{F}_k^{\mathrm{nln}}$$

[Petrov E.P. and Ewins D: *Analytical formulation of friction interface elements for analysis of nonlinear multiharmonic vibrations of bladed disks*. ASME Journal of Turbomachinery 125:364-371, 2003]

Methodology – mathematical model of contacts

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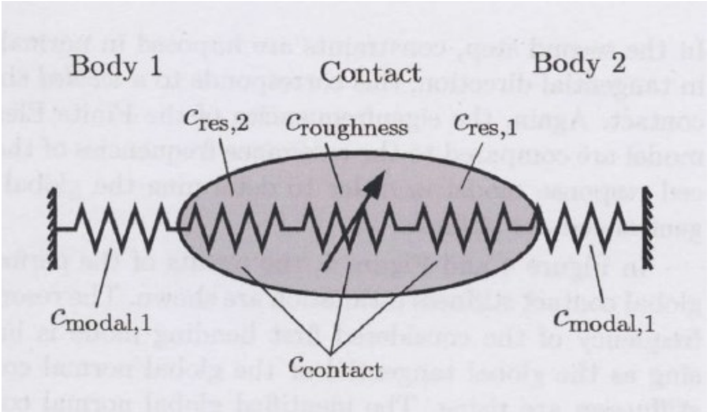
Concept of „contact stiffnesses“

Finite number m of mode shapes

$$A_k \approx \sum_{r=1}^{m} \frac{1}{(1 + \mathrm{i}\eta_{k,r})\Omega_{k,r}^2 - (k\omega)^2} \, \phi_{k,r} \, \phi_{k,r}^{\mathrm{H}}$$



Notion of „contact stiffness“ is misleading, because it does not label only physical properties of contact.

Contact stiffnesses compensate higher mode shapes that are not included in the approximation of A_k .

Calculation of contact stiffnesses:

Linearized contact
stick or μ=0

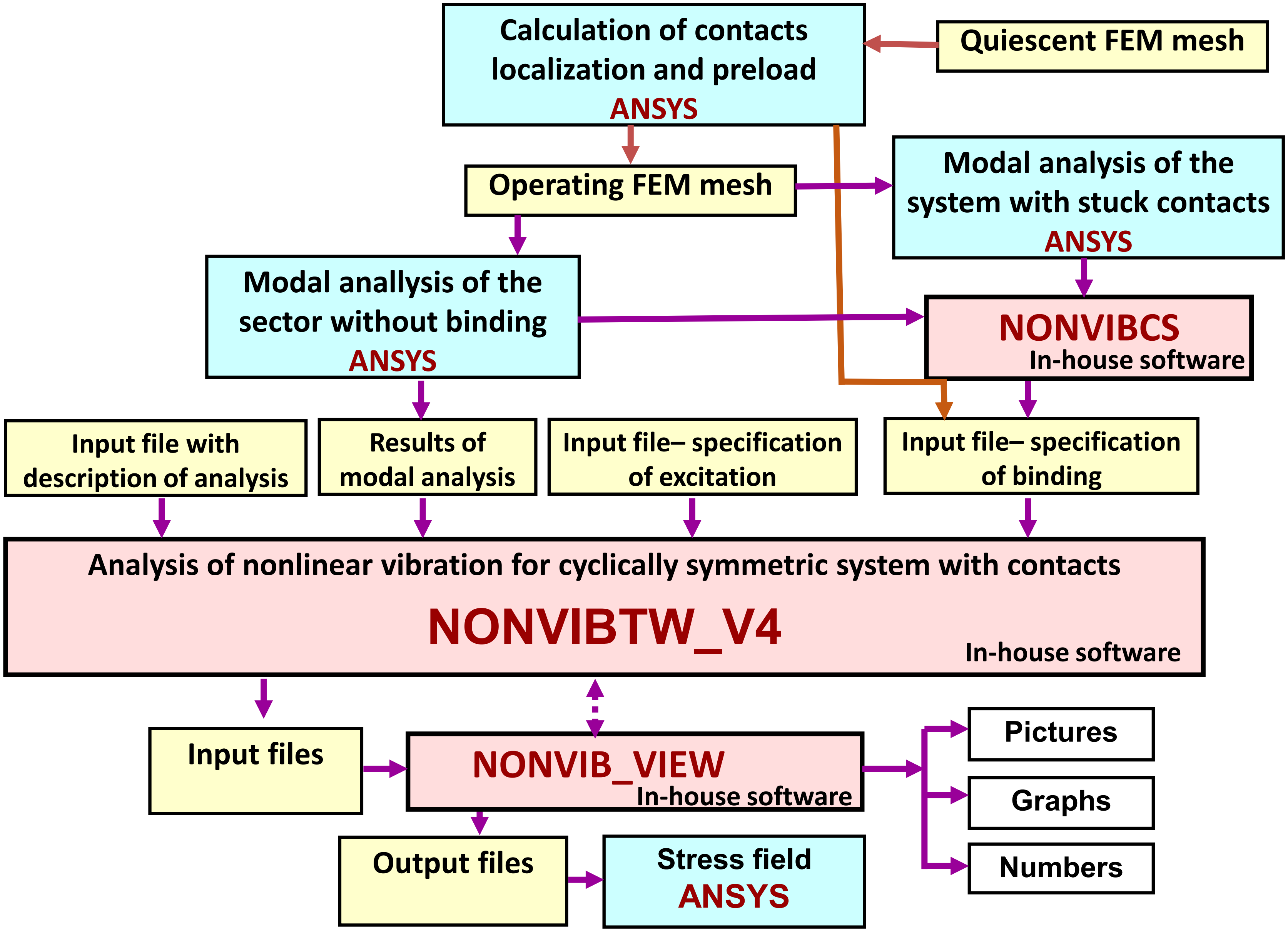
→

Model of nonlinear systém –
resonant frequency

≈

Measurement – resonant frequencies
FEM calculations – natural frequencies

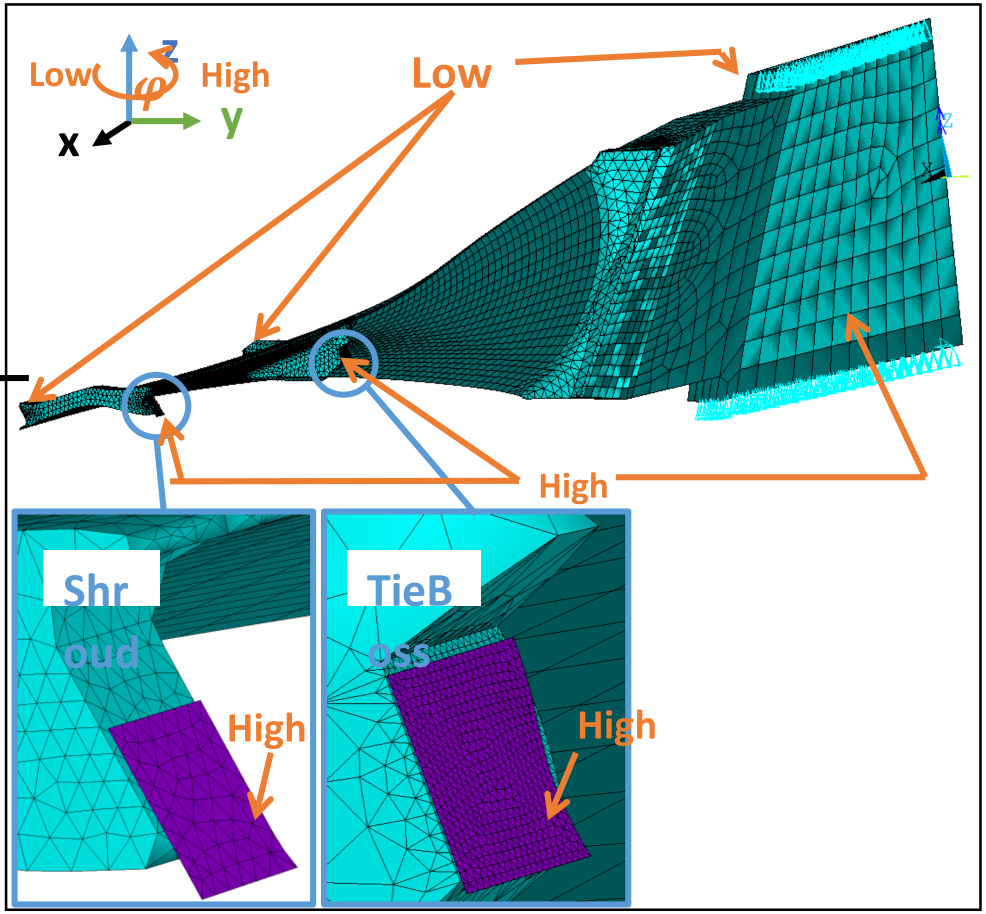
Methodology – Computational diagram



Calculation of nonlinear vibration

LSB 48"
1220 mm, steel

LSB 54"
1375 mm, titanium alloy



Each blade has two integral coupling:
Tie-boss – middle part
Shroud (bandage) – top

Blade geometries of LSB48 and LSB54 are moderately different.

Contact surfaces are plane.

Calculation of nonlinear vibration

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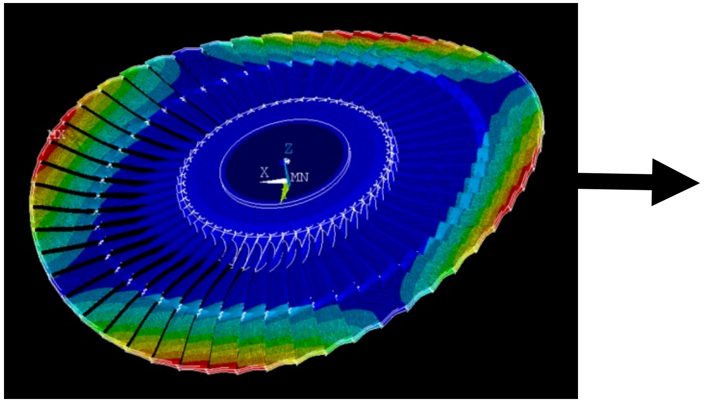
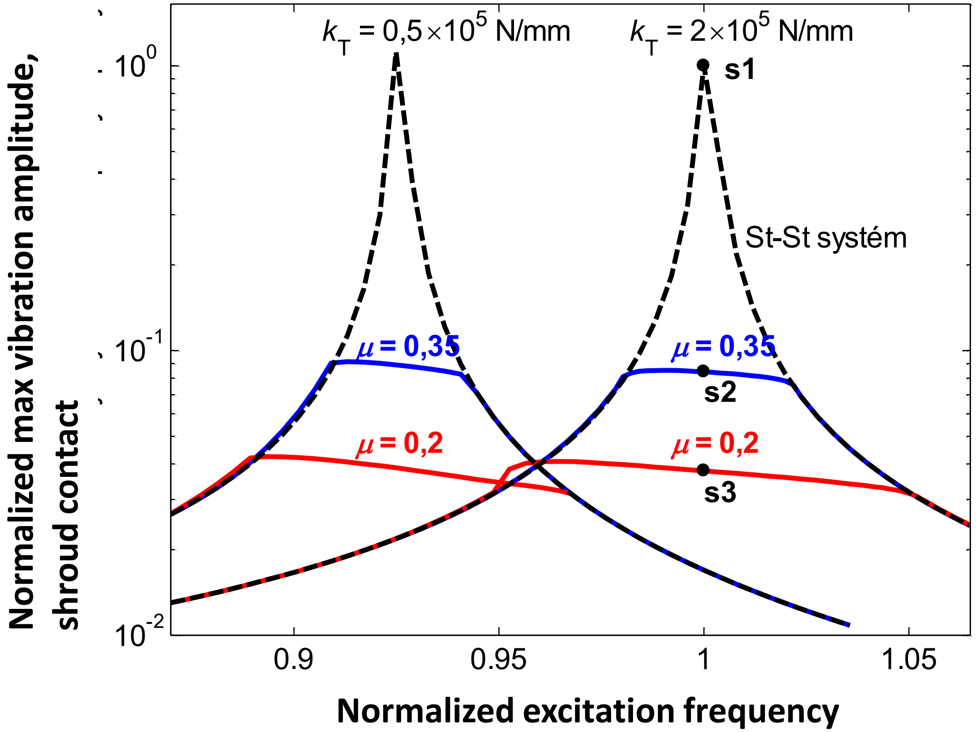
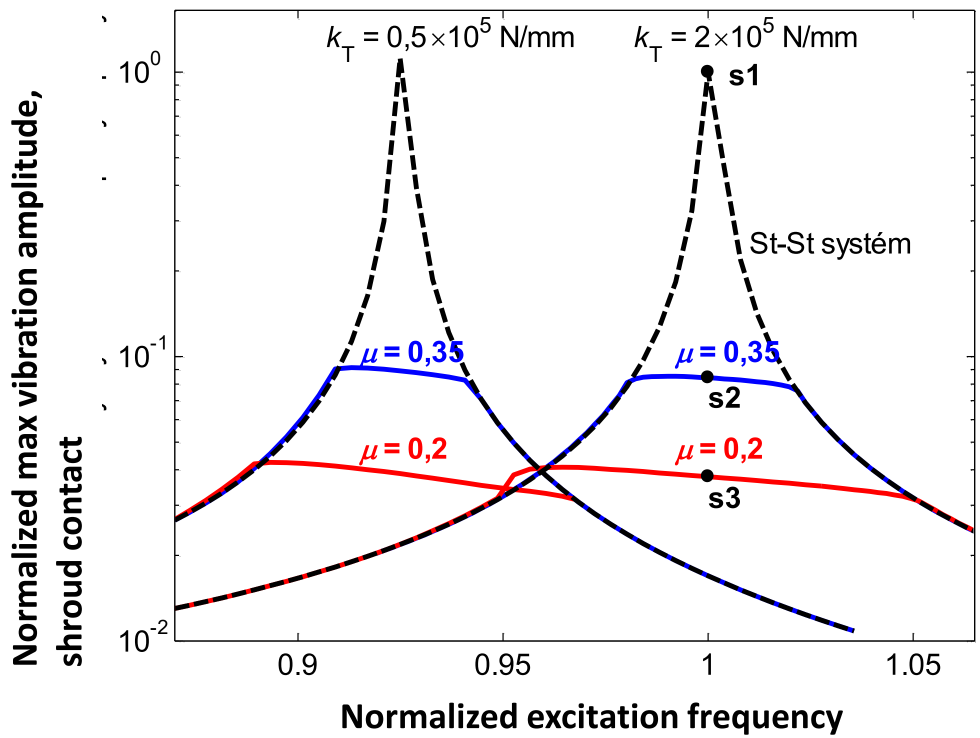
LSB 48″, excitation by traveling waves with 2 ND

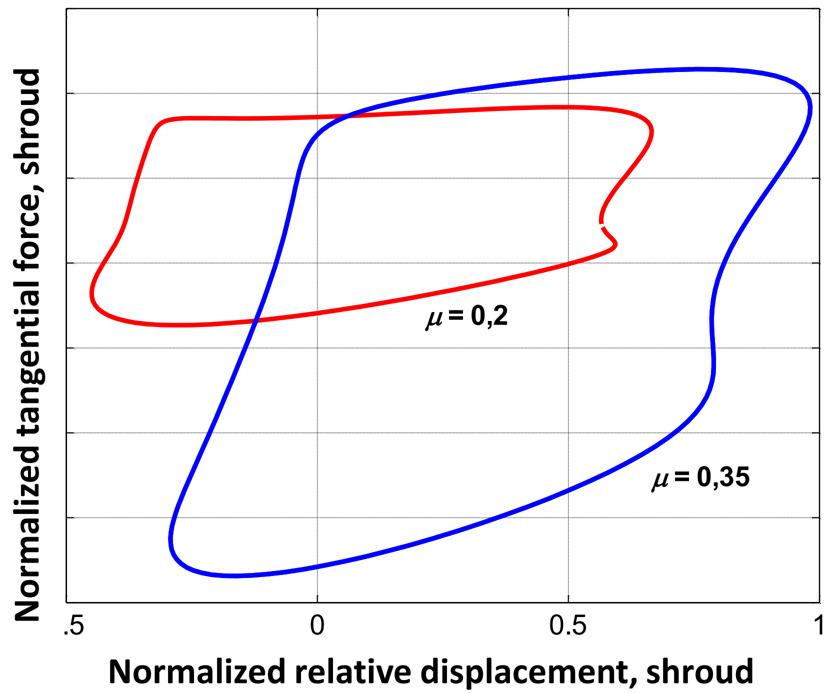




Limited number (10) of mode shapes for approximation of blade dynamic behavior → necessity to fit contact stiffnesses (influenceing resonant frequencies of computational model)

Calculation of nonlinear vibration

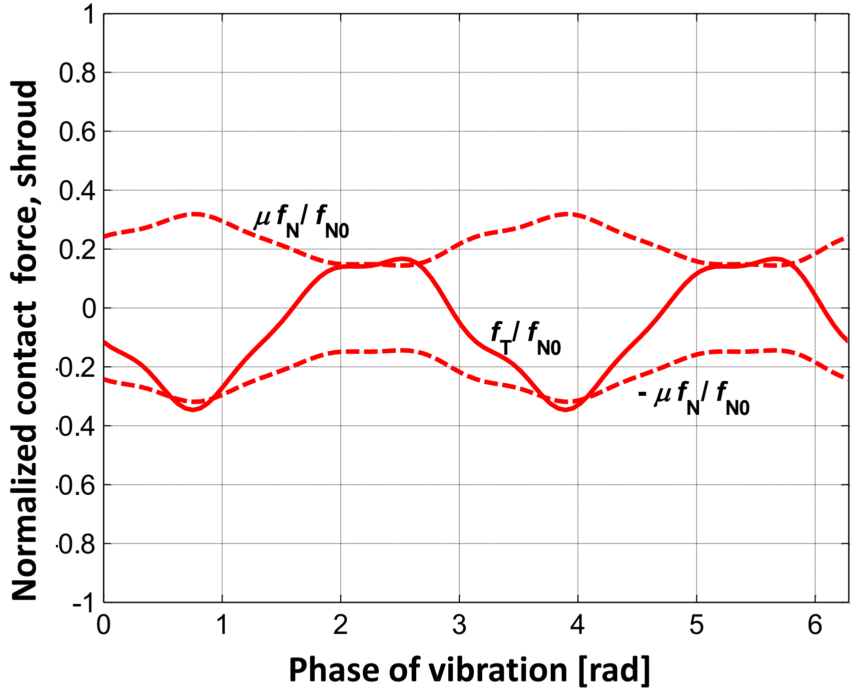


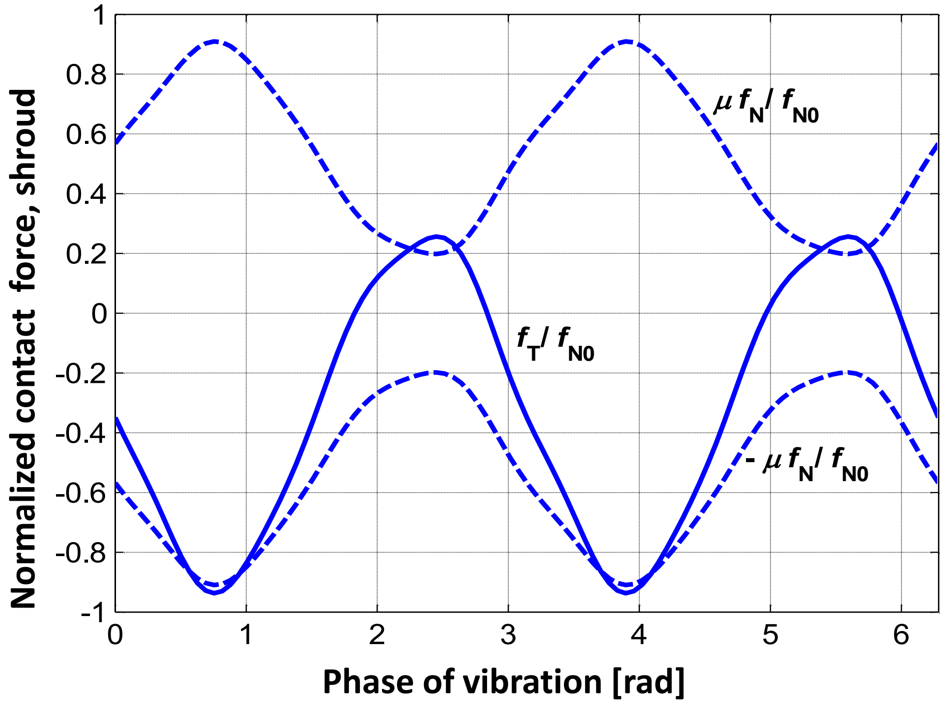


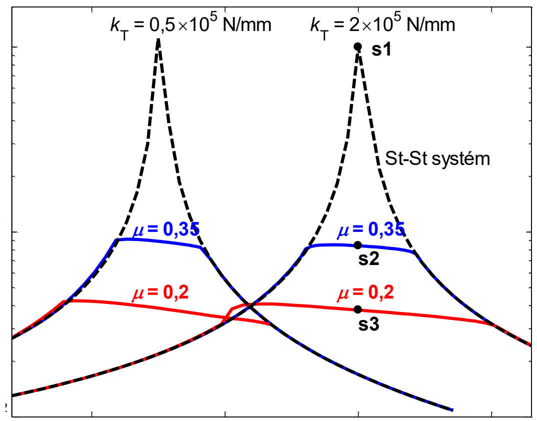
- **Higher friction coeficient, μ → more friction energy dissipated in contact.**
- **Higher energy dissipated in contacts does not lead to drop of displacement aplitudes.**

friction damper ??

Calculation of nonlinear vibration







- Frictional sliding occurs in contact.
- Excitation by travelling wave of 2ND → two cycles within the period (0,2π).

Calculation of nonlinear vibration

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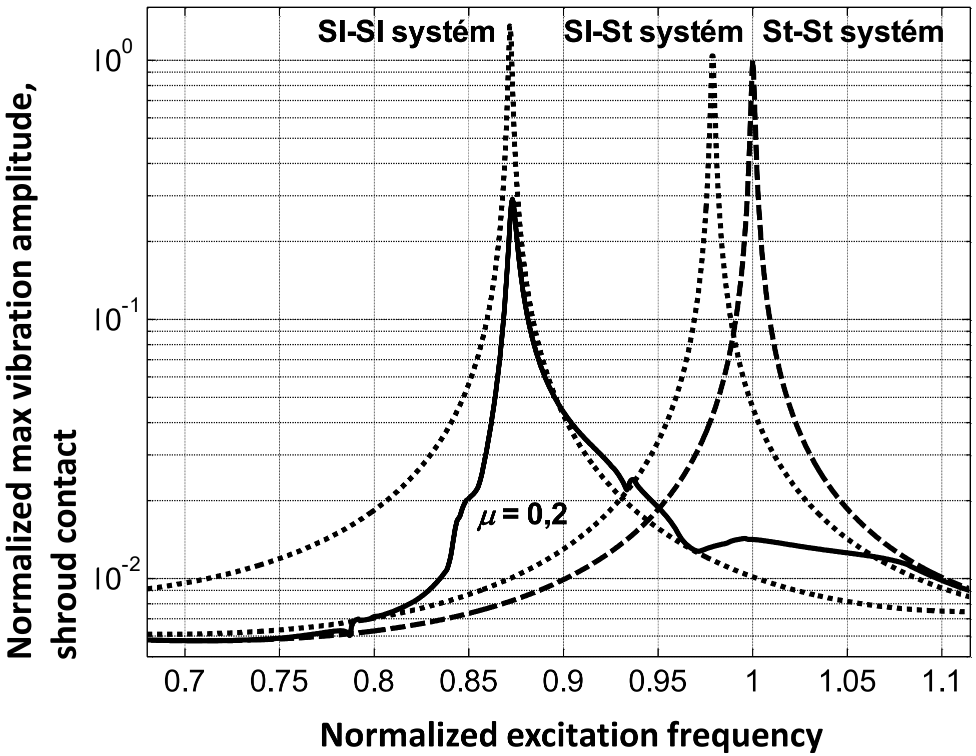
LSB 48″, excitation by harmonic variation of rotor torque moment



- **Linearized problems: Contacts in durable ideal sliding („Slide“) or sticking („Stick“).**
- **„Sl-Sl system“ = sliding without friction for Shroud and Tie-Boss contact.**
- **Dangerous situation: the excitation frequency of variation of torque moment is close to the natural frequency of Sl-Sl system.**
- **Contact couplings could activate a „sleeping resonance regime“.**

Calculation of nonlinear vibration

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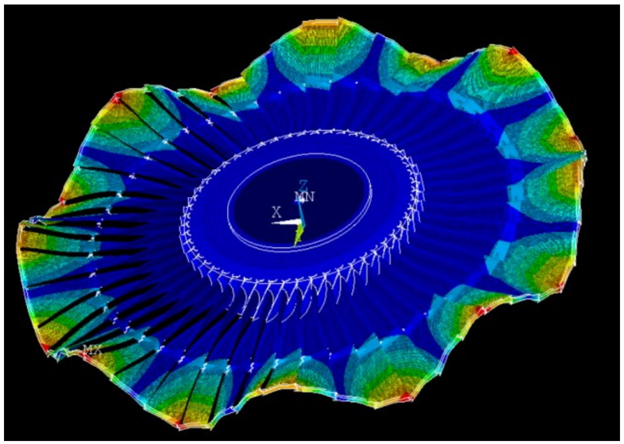
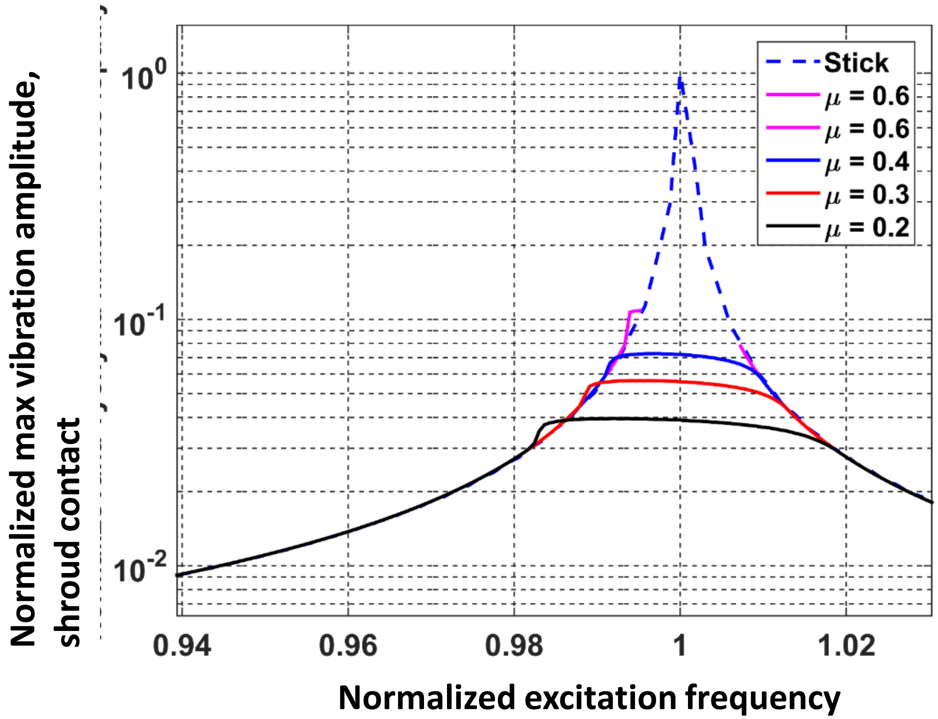
LSB 54″, excitation by traveling waves with 8 ND





- Analogous character as in the case of LSB 48″ excited with 2ND: higher values of μ → higher amplitudes of displacements of forced vibrations
- loss of numerical konvergency for μ > cca 0.6 .

Calculation of nonlinear vibration

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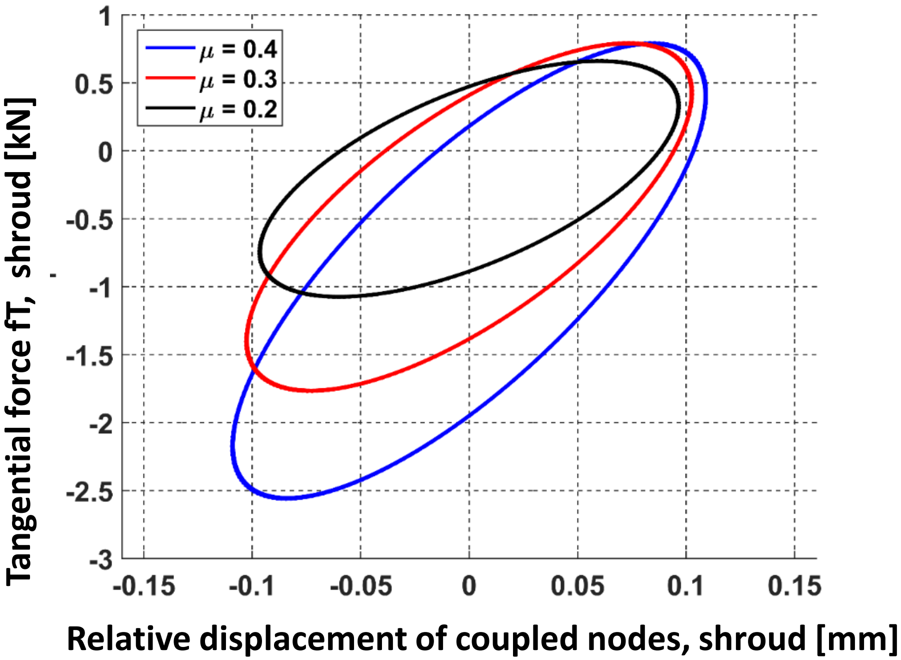

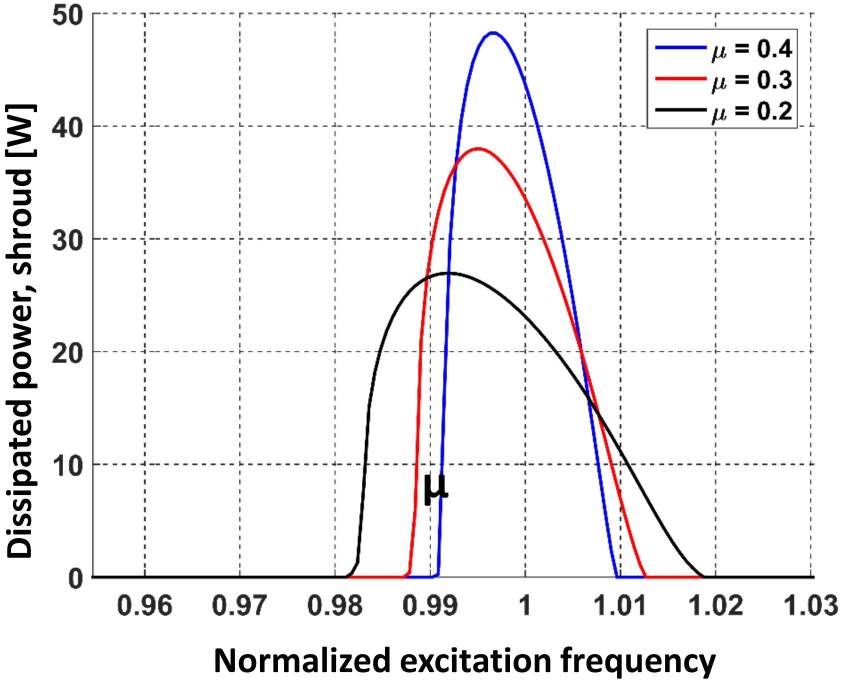

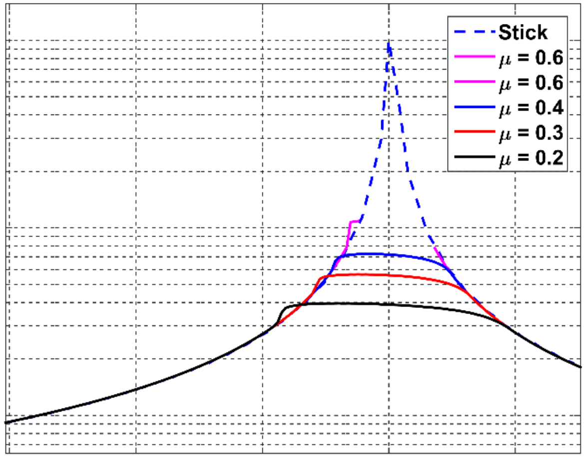

Analysis of nonlinear vibration results:

- Dissipated power in level tens W for each blade, if slipping.
- Higher displacement amplitude rise for higher friction coefficient.

Calculation of nonlinear vibration

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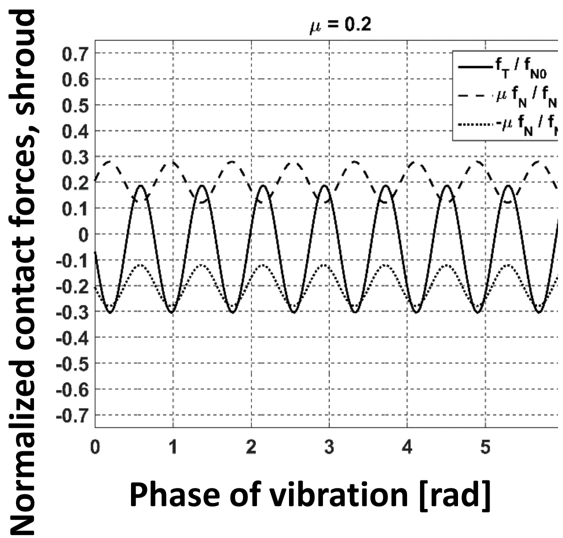
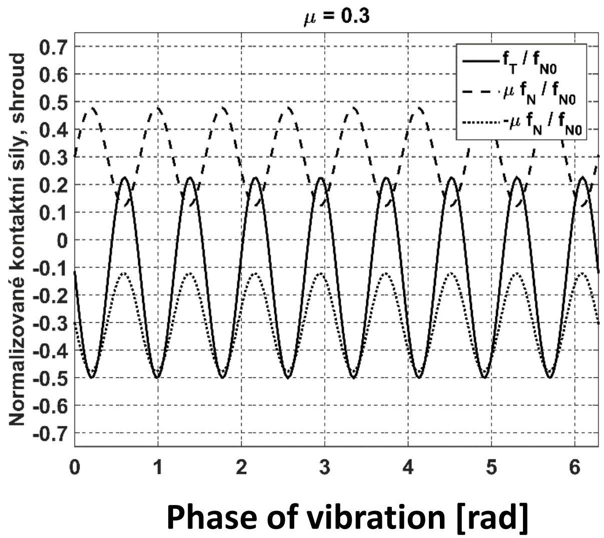
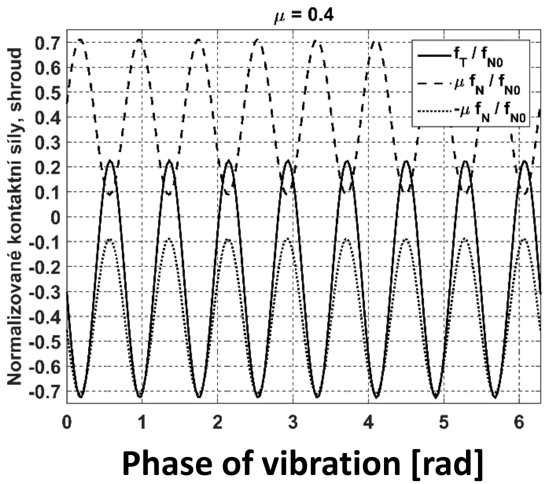
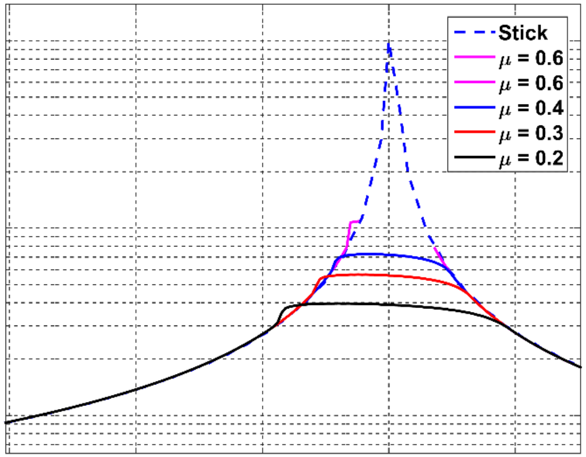
LSB 54″, excitation by traveling waves with 8 ND





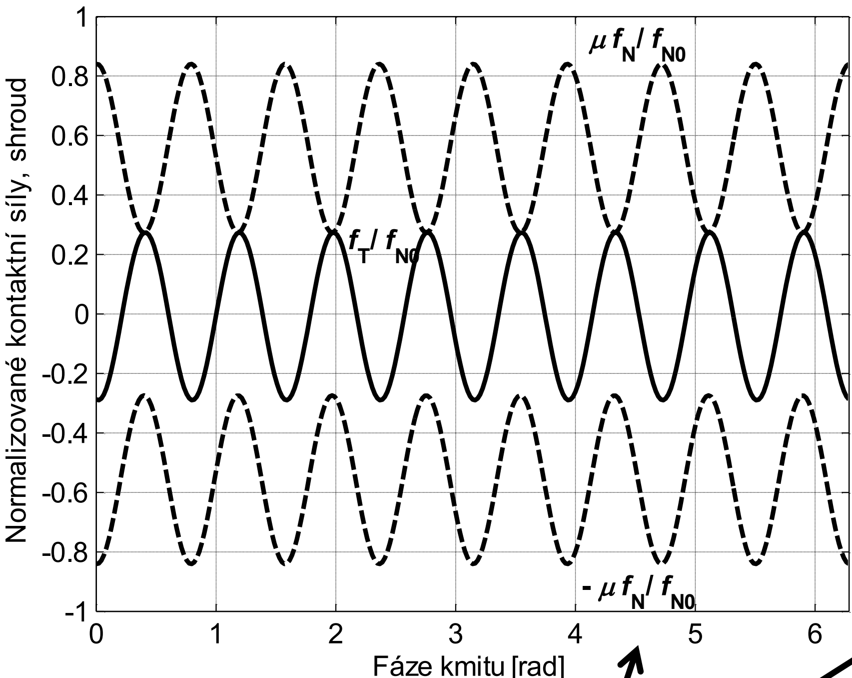




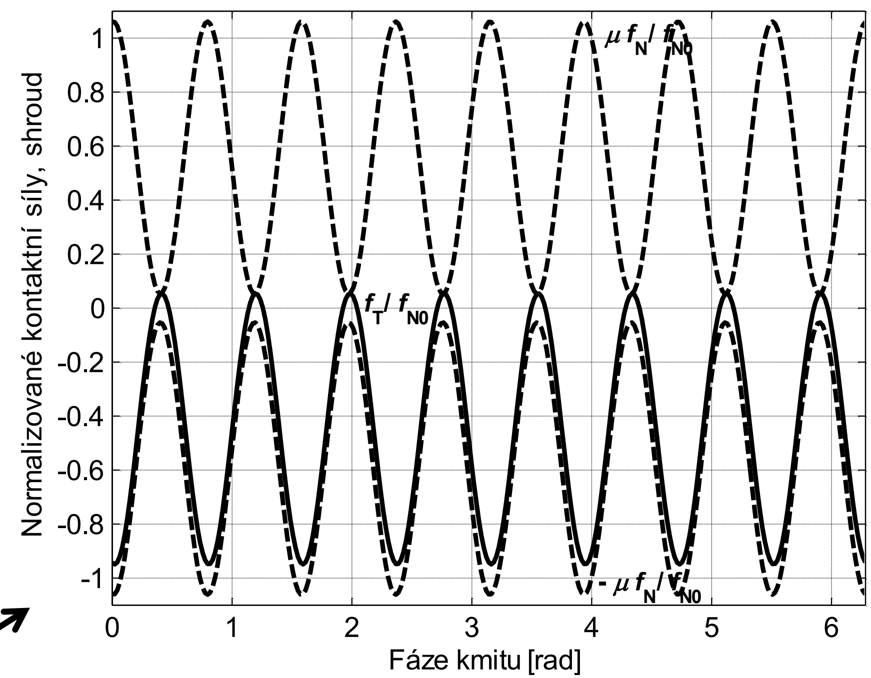
- Excitation by traveling wave with 8 ND → 8 cycles within the period (0,2π).
- Higher friction coefficient → higher amplitude of tangential and normal contact forces

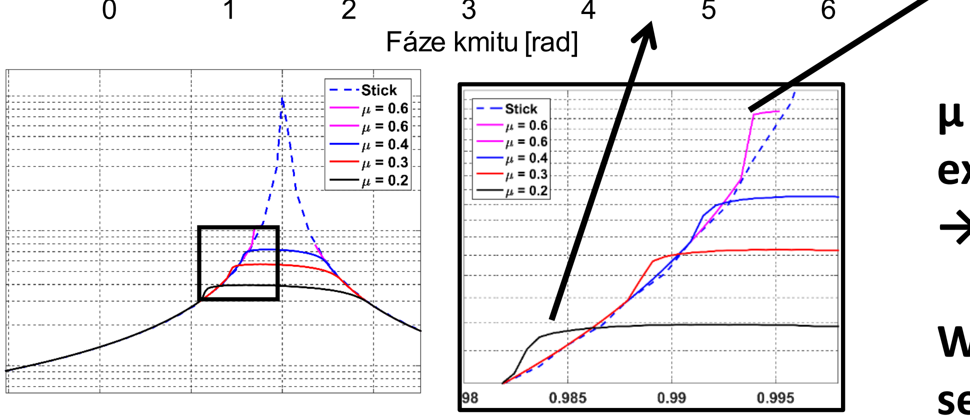
Calculation of nonlinear vibration

0,988 $\omega_{8,1,St}$



0,994 $\omega_{8,1,St}$



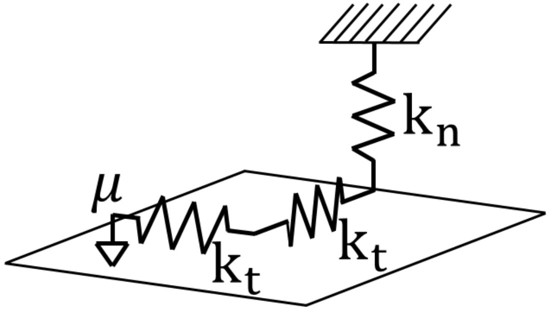


μ = 0.6 : big amplitude of normal forces for excitat. frequencies close to resonance
→ intersection of curves for μF_N and $-\mu F_N$

We can interpreted the intersection as contact separation.

Contact stiffnesses

The compliance of interface contact elements



Related to the deformations of the asperities of the contacting surfaces. May be omitted (an order of magnitude higher).

$$\frac{1}{k_{\mathrm{t}}} = \frac{1}{k_{\mathrm{x}}} + \frac{1}{k_{\mathrm{t,comp}}} \qquad \frac{1}{k_{\mathrm{n}}} = \frac{1}{k_{\mathrm{y}}} + \frac{1}{k_{\mathrm{n,comp}}}$$

Related to the compensation of omitted higher modes.

$$k_{\mathrm{t}} \approx k_{\mathrm{t,comp}} \qquad k_{\mathrm{n}} \approx k_{\mathrm{n,comp}}$$

To our knowledge, up to now, no general method for finding the values of “contact stiffnesses” has been introduced.

We developed a (computational) method that fits the contact stiffnesses effectively, at least in our case of bladed disks (mentioned also in this lecture).

Conclusions

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- **Harmonic Balance Method (HBM)**
  - It is possible to use to calculations of steady state of nonlinear vibration for bladed
  - It is possible to perform the calculations on PC

- **„Contact stiffnesses“**
  - They compensate higher mode shapes omitted in the approximation of dynamic compliance of a calculated system
  - It is necessary to fitted these contact stiffnesses

- **Ne vždy lze třecí vazby považovat pouze za třecí tlumiče:**
  - Větší disipovaný výkon nevede pokaždé k většímu poklesu amplitud výchylek.
  - Prokluz ve třecí vazbě může být příčinou nárůstu amplitud výchylek.
  - Separation of contact surfaces can occur
- **Notion of „sleeping resonant regime“**

Thank You very much