Palis Stefan **Germany**
Stamann Mario **Magdeburg**

Otto-von-Guericke University Magdeburg

NONLINEAR CONTROL DESIGN FOR MAGNETIC BEARINGS VIA AUTOMATIC DIFFERENTIATION

Introduction. Active magnetic bearings are getting more and more important in various applications. In contrast to conventional bearings the movable parts are not supported by mechanical contact or a fluid, but by magnetic forces. Therefore they are free of mechanical wear and need less maintenance. However, due to the attractive force between the magnet and the movable part the equilibrium is unstable and has to be stabilized by control. Although electromagnets provide a nonlinear behaviour between current and magnetic force in industrial applications predominantly linear control laws (PID and state feedback) are used [1], [2]. The application of nonlinear controllers has been investigated [3], [4] for pure electromagnets with promising result.

Magnetic Bearing. For real industrial applications however hybrid magnets are used, providing a higher magnetic force. Unfortunately the magnetic force equation $F_{M,hyb}$ (1) for the hybrid magnets in the differential current topology (fig. 1) is much more difficult then for a pure electromagnet $F_{M,elec}$ (1).

$$F_{M,hyb}=a\left[\frac{(i+H_0)^2}{(k_2-2x)^2}-\frac{(-i+H_0)^2}{(k_1+2x)^2}\right] \qquad F_{M,elec}=a\frac{i^2}{(k_2-2x)^2} \tag{1}$$

Fig.1. Prototype of a industrial magnetic bearing with differential configuration of hybrid magnets

Thereby the hybrid magnets configuration is realized as shown in figure 1. Here the magnetic forces F_O and F_U are produced by the upper and lower hybrid magnet. By superposition of the fields of the permanent magnet and the inductor an attractive force is generated. Due to the opposed direction of winding of the upper and lower electromagnet the upper force F_O is strengthened, while the lower force F_U is weakened, when applying a positive current. And vice versa, when a negative current is supplied. Therefore both positive and negative forces are applicable.

In the following for illustrative reasons only the model equations for an one degree of freedom magnetic bearing consisting of two hybrid magnets in differential topology are used. The current is supplied via a separate current control loop, whose characteristics can be approximated by a PT2 transfer function (2).

$$i=\frac{\omega_0^2}{s^2+2D\,\omega_0 s+\omega_0^2}u \tag{2}$$

The overall system equations for a one degree of freedom magnetic levitation system with two hybrid magnets are as follows (3):

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix}=\begin{pmatrix}x_2\\ \frac{1}{m}F_{M,hyb}(x_3,x_1)-g\\ x_4\\ -2D\,\omega_0 x_4-\omega_0^2 x_3+\omega_0^2 u\end{pmatrix} \qquad F_{M,hyb}=a\left[\frac{(x_3+H_0)^2}{(k_2-2x_1)^2}-\frac{(-x_3+H_0)^2}{(k_1+2x_1)^2}\right] \tag{3}$$

Exact Linearization via Feedback. In the following nonlinear control theory, i.e. feedback linearization theory [5], is applied to the magnetic bearing. To this aim a transformation is applied by repeated differentiating the output $y=h(x)$ along the system trajectory as long as the control input u appears. For the time derivatives $y,\dot{y},...,y^{(r-1)}$ new state-variables $z_1,...,z_r$ are introduced (4).

$$z=\begin{pmatrix} z_1 \\ z_2 \\ . \\ . \\ z_r \end{pmatrix}=\begin{pmatrix} y \\ \dot{y} \\ . \\ . \\ y^{(r-1)} \end{pmatrix}=\begin{pmatrix} h(x) \\ L_f h(x) \\ . \\ . \\ L_f^{(r-1)} h(x) \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ . \\ . \\ z_n \end{pmatrix}=\begin{pmatrix} z_1 \\ z_2 \\ . \\ . \\ z_n+f_n(z)+g_n(z)u \end{pmatrix} \tag{4}$$

For a system in Brunovsky normal form (4) a simple nonlinear control law is:

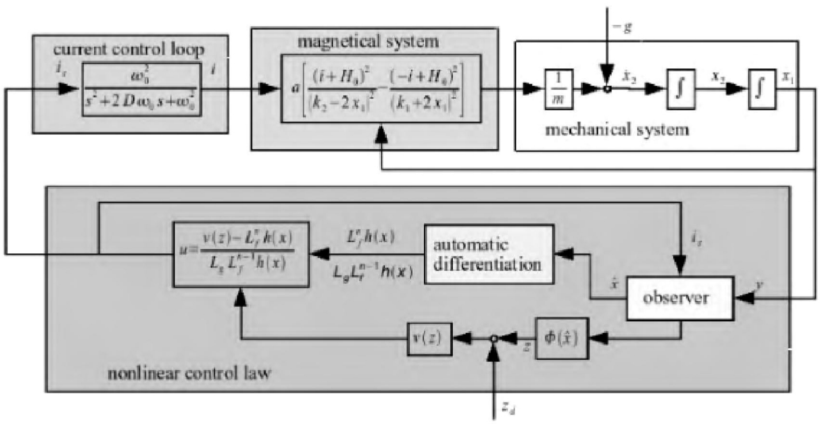
$$u=\frac{-f_n(z)+v(z)}{g_n(z)}=\frac{v(z)-L_f^n h(x)}{L_g L_f^{n-1} h(x)} \tag{5}$$

It can be shown that the system (3) is flat with respect to the output $y = x_1$. Therefore the relative degree r is equal to the order of the system $n = 4$, meaning that the output y has to be differentiated four times.

$$\dot{y}=x_2 \quad , \quad \ddot{y}=\frac{1}{m}F_{M,hyb}-g \quad , \quad y^{(3)}=\frac{1}{m}\left(\frac{\partial F_{M,hyb}}{\partial x_1}x_2+\frac{\partial F_{M,hyb}}{\partial x_3}x_4\right) \tag{6}$$

Because of the increased complexity of the magnetic force equation $F_{M,hyb}$ compared to $F_{M,elec}$ (1) a symbolic derivation of $y^{(3)}, y^{(4)}$ and u becomes error-prone and doesn't give any physical insights. To overcome this problem the Lie derivatives defining the control law (5) will be derived using automatic differentiation.

Automatic Differentiation. Automatic differentiation is based on the application of elementary differentiation rules to elementary functions forming a function $f(x)$. In doing so the intermediate results are immediately evaluated, i.e. the result is not given in form of a symbolic expression but as a real number.




```
void Lie_f(short int tape, double* z,
                              double* dz)
{
    adoublev x(4);
    adoublev dx(4);

    trace_on (tape);
    x<<=z;
    dx[0] = x[1];
    dx[1] = a/m*FM_hyb(x[0],x[2])-g;
    dx[2] = x[3];
    dx[3] = -2*d*x[3] - w02*x[2];
    dx>>=dz;
    trace_off ();
}

  short int tape_f = 0;
  Lie_f(tape_f,x,&dx);

  double **P;
  P = myalloc2 (N,D+1);
  P[0][0] = x[0];
  P[1][0] = x[1];
  P[2][0] = x[2];
  P[3][0] = x[3];
  forode(tape_f,N,D,P);
```

Fig.2. Control scheme and implementation of automatic differentiation using ADOLC-C

Implementation using ADOLC-C. ADOL-C [6] is a C++ library facilitates the evaluation of derivatives of C and C++ functions by automatic differentiation. Using operator overloading only minor changes in the program, which has to be differentiated are required. Therefore the so called active variable is defined representing the independent variables of a function. In the original program all independent variables have to be redefined as active variables. For simulation and a later real time code generation the automatic differentiation has been implemented in a MATLAB C++ S-function.

Conclusion. We have considered the design of a nonlinear control law for magnetic bearings with permanent magnetic force. It has been shown, that the use of symbolic methods for calculating Lie derivatives for hybrid magnetic actuator systems with an additional current control loop leads to complex equations, that may be problematic in real time applications. This drawback can be overcome when automatic differentiation is utilized.

References.

1. G. Schweitzer and A. Traxler. Magnetlager - Grundlagen, Eigenschaften und Anwendungen berührungsfreier, elektromagnetischer Lager. Springer, 1996
2. Thomas Schallschmidt. Regelungstechnische Optimierung eines Magnetlagers. Diplomarbeit, 2003
3. Stefan Palis. Modified optimal control of magnetic levitation systems. APM, 2007
4. S. Palis and M. Stamann and T. Schallschmidt. Nonlinear adaptive control of magnetic bearings. EPE, 2007
5. Hassan K. Khalil. Nonlinear Systems. Prentice-Hall, 1996
6. A. Walther and A. Griewank and O. Vogel. ADOLC-C: Automatic Differentiation Using Operator Overloading in C/C++. Proc. Appl. Math. Mech. 2:41-44, 2003