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ПОСТРОЕНИЕ КИНЕТИЧЕСКОГО УРАВНЕНИЯ ПРОИЗВОДСТВЕННОГО ПРОЦЕССА

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THE CONSTRUCTION A KINETIC EQUATION OF THE PRODUCTION PROCESS

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Аннотация. В статье обсуждаются методы построения кинетического уравнения технологического процесса. Представлена модель взаимодействия предметов труда с технологическим оборудованием, положенная в основу вывода кинетического уравнения. Рассмотрены численные характеристики, описывающие состояние производственной линии.
Ключевые слова: кинетическое уравнение, производственная линия, массовое производство, незавершенное.
Abstract. The paper discusses methods of constructing the kinetic equation of the technology process. The article presents a model of the interaction of objects of labor with technological equipment, which is the basis for the derivation of the kinetic equation. To describe the state of the production line introduced numerical characteristics.
Keywords: kinetic equation, the production line, mass production, work in progress, balance equations, quasi-static process, stochastic process.

Problem statement and analysis of recent publications

Modeling complex dynamic production processes is an effective method of research [1,2]. One of widespread classes form the production systems, in which character of the observable production processes has stochastic nature [2, 3, 4, 5, 6]. Regularities of inherent equilibrium states in production systems in many ways similar to those that take place in the physical (thermodynamic) systems [7, 8]. They appeared so deep and useful, that were proclaimed as some general principles for the thermodynamics and production systems: Le Chatelier-Samuelson, Carnot-Hicks [9]. On the basis of the principles of the functioning of modern mass production it can be represented as a stochastic process, during which the manufacturing system changes from one state to another [6,10]. The production process state is determined by the state of the overall number N of items of work [6]. In transition of the object of labor from one state to another, there is a transformation of resources (raw material, materials, living labor) in the prepared product as a result of purposeful influence of equipment. State j-th object of labor in the phase space will be described by state parameters

$$\vec{S}_j = (S_{j,1}, S_{j,2}, ..., S_{j,\alpha}, ..., S_{j,A}),$$
$$\vec{\mu}_j = (\mu_{j,1}, \mu_{j,2}, ..., \mu_{j,\alpha}, ..., \mu_{j,A}),$$

where $S_{j,\alpha}$ (USD) value of the transferred α - of the technological resource or part thereof for the j-th subject of work, $\mu_{j,\alpha}$ (USD / hour), the intensity of the transfer value of α - of the resource to the j-th subject of work, $0 < j \le N$, $0 < \alpha \le A$ [6]. The state of parameters of production process in some moment of time will be defined, if the parameters of the state of the object of labor are defined $(\vec{S}_1, \vec{\mu}_1, ..., \vec{S}_N, \vec{\mu}_N)$ and the objective function $J(t, \vec{S}_j, \vec{\mu}_j)$ [11,12], and at any other time it is found from the equations of states of objects of labor [6,12]. However, if the number of objects of labor N is much greater than unity, then decide System of N_0 equations second order is practically impossible [13, 14]. The last clarification requires a transition from the object-process description (micro-description) to aggregated streaming (macroscopic) description with the elements of probabilistic nature. The main difficulty in this specification is to highlight the characteristics of the parameters of states [15] objects of labor [1,6], which could be measured in the study of the actual production processes [12, 13]. Instead of considering the state of the production process with the parameters of the state of the objects of labor $(\vec{S}_1, \vec{\mu}_1, ..., \vec{S}_N, \vec{\mu}_N)$, we will enter normalized discrete phase function of distribution of number N objects of labor in the phase space (t, S, μ) [6,16].

Each point in the space of states [15, 16, 17] will set the state of the object of labor. It is reasonable to expect that at large N ($N \approx 10$ pc. [18], [13]) function is well approximated by a continuous phase distribution function of the objects of labor $\chi(t, \vec{S}, \vec{\mu})$ by the states [6, 16, 19, 20]. If the production system produces the K kinds of products [21, p.445], K=10, then it will be required to get a distributing function for every k-th species $\chi_k(t, \vec{S}, \vec{\mu})$, $(k = 1..K)$. Before considering the multi-axis (multiresource, $0 < \alpha \le A$) model of the production process, in interoperational reserves of which there are parties of K of types of interactive with each other and with an equipment wares), we examine patterns of processing batches N objects of the labor of one kind $(N >> 1)$. We suppose that processing occurs over successive the M operations generalized production line

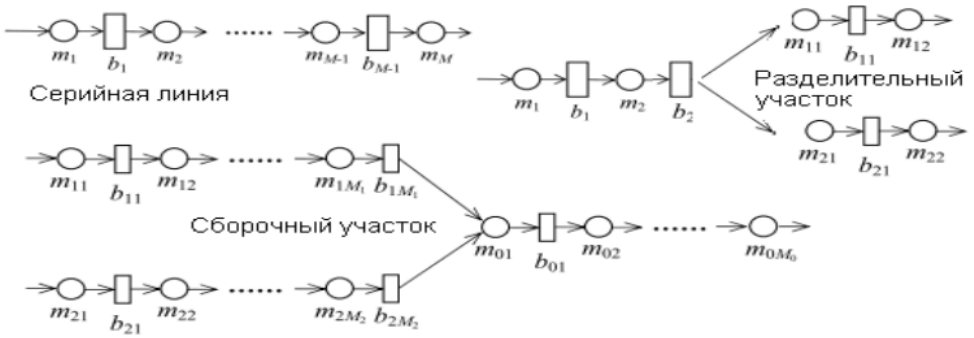

Fig. 1. Scheme of flows of objects of labor (m - equipment and -buffer), [23]

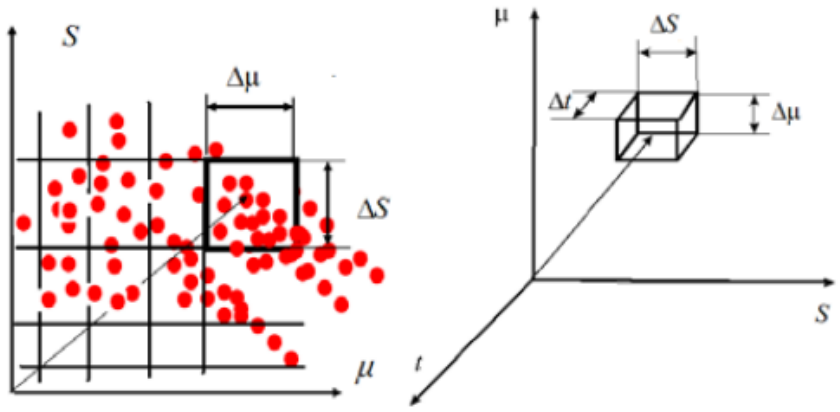

Fig. 2. The unit cell of phase technological space

(*fig. 1*), ($M >> 1$) [23]. The obtained patterns are common for both single-threaded and multi-threaded lines with a wide variety of resources [10, 23].

The construction of a kinetic equation of the production system

We will break up phase space on such number of cells, that sizes of cell $\Delta\Omega = \Delta S * \Delta\mu$ will contain into itself the large number of the objects of labor (*fig. 2*). It is possible to conduct the estimation of amount of wares in a cell $\Delta\Omega = \Delta S * \Delta\mu$. For the process, which consists of $M \approx 100$ [13], $M \approx 250$ [23, p.4589], $M \approx 300$ [21, p. 445] operations with the number of items of work in progress $N \approx 10^5$ [25], $N \approx 10^4$ [13], m-th operation in the reserve contains the average number of items of work $\left(M/N\right) \approx 10^3$. Influence of the number of operations M on the accuracy of calculations is investigated in [6, p.23], where an analysis of the calculation results obtained using DES- and PDE-models for M = 50 ($N \approx 10^4$), operations with satisfactory accuracy. Along with M=50 a comparative analysis is executed for M = 10 operations. It is shown that this amount can be used for a rough approximation. Instead of fixing the exact parameters of objects of labor, we will approximately describe the state of the production process using the number of objects of labor in cells $\Delta\Omega$ with the coordinates of the parameters of the object of labor, $S_j \in [S, S+\Delta S], \mu_j \in [\mu, \mu+\Delta\mu]$ (*fig. 2*).

If the cell size is small enough, the approximate description will carry the equally detailed information as accurate does. For the technological process, consisting of several hundred operations it is reasonable to go to the continuous description of the parameters, that describe the operation, considering along with [23] the main limit when $N \to \infty$ and the limiting case of $M >> 1$. Due to the fact that the product of $\chi(t,S,\mu) * d\Omega$ is the number of items of work in the cell $d\Omega$ phase space with coordinates $S_j \in [S, S+\Delta S], \mu_j \in [\mu, \mu+\Delta\mu]$, the integration over the volume Ω of the phase space (S, μ), gives the total number N of items of work that are in progress [6, 19]:

$$\int_0^{S_d}\int_0^{\infty} \chi(t,S,\mu)\,d\mu dS = N, \quad \Omega = \int_0^{S_d}\int_0^{\infty} d\mu dS, \qquad (1)$$

where S_d (USD) - the cost of the product. The limits of integration S = 0 and $S = S_d$ specify the range of change in the coordinates $S, S \in [0, S_d]$ which determines the position of the object of labor along the technological route. We will accept (1) as a condition for the normalization of the phase distribution function $\chi(t,S,\mu)$ items of work over the states [6, 16, 19, 20], which is the law of conservation of articles in the production process [6]. We introduce a numerical characteristics

$$\int_0^{\infty} \mu^k * \chi(t,S,\mu)\,d\mu = [\chi]_k \qquad (2)$$

that reflect the essential features of the distribution of the states of the objects of labor that are in progress. Features (2) for the distribution function $\chi(t,S,\mu)$, we will define as moments of k-th order [20, s.906]. Often the problem can be solved using numerical characteristics, leaving aside the laws of distribution.

Numerical characteristics and method of moments associated to them play an important role in the construction of many of the statistical theory of dynamical systems [6]. Essential value in the models of the production process are the zero $[\chi]_0 = [\chi]_0(t,S)$ and the first $[\chi]_1 = [\chi]_1(t,S)$ moments of the distribution function of the states of the objects of labor [19] which determine density of distribution on the positions of objects of labor and the rate of processing operations on objects of labor [26, c.37]. The change of the distribution function $\chi(t,S,\mu)$ of the states of the of objects of labor is due to the stochastic nature of the interaction of objects with the equipment and with each other [23, c.4591]. This interaction is characterized by function $G(t,S,\mu)$

$$\frac{d\chi(t,S,\mu)}{dt} = G(t,S,\mu) \tag{3}$$

which takes into account spatial layout of equipment the laws of impact of equipment on the subject of work for the purpose of transfer of resources and mechanisms of interaction of objects of labor with each other. The interaction of objects of labor with each other is determined by the processing priorities (processing rules FIFO, LIFO ...), and restrictions on the technological modes. The total derivative in (3) means differentiation along the phase trajectory of the object of labor. If the motion of the object of labor in the state space is deterministic and defined by the Euler equations for the objective function of the production system [11, 12], the equation (3) by virtue of the Liouville theorem becomes the identity $G(t,S,\mu) = 0$. We write a total derivative $\chi(t,S,\mu)$ in the form

$$\frac{d\chi(t,S,\mu)}{dt} = \frac{\partial\chi(t,S,\mu)}{\partial t} + \frac{\partial\chi(t,S,\mu)}{\partial \mathrm{S}} * \mu + \\ + \frac{\partial\chi(t,S,\mu)}{\partial\mu} * \frac{d\mu}{dt} \tag{4}$$

and represent the equation (3) as follows:

$$\frac{\partial\chi(t,S,\mu)}{\partial t} + \frac{\partial\chi(t,S,\mu)}{\partial \mathrm{S}} * \mu + \\ + \frac{\partial\chi(t,S,\mu)}{\partial\mu} * \frac{d\mu}{dt} = G(t,S,\mu). \tag{5}$$

Equation (5) μ has the averaged intensity of transfer resources on all subjects of labor $d\Omega$ in the cell with coordinates $S_j \in [S, S+\Delta S], \mu_j \in [\mu, \mu+\Delta\mu]$ [6]. Replace $d\mu/dt$ in (5) with the equations defining the normative trajectory of the object of labor in the cell $d\Omega$ [6,11,12, 16, 20],

$$\frac{d\mu}{dt} = \frac{\partial}{\partial t}\left(\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right) + \\ + \frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)} \frac{\partial}{\partial S}\left(\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right) = f(t,S), \\ \mu = \frac{dS}{dt}, \tag{6}$$

where $[\chi]_{1\psi}(t,S)$ - the pace of processing of objects of labor equipment at the point of the technological route with coordinate *S*. Equation (6) connects the release of products in place technological route, specified by the coordinate *S* and the amount of necessary costs of technological resources to transform the object of labor. The ratio of the form (6) which provides the connection with the cost of manufacture can be determined as a function of the generalized manufacturing equipment located in the designated area of a technological route. The kinetic equation of technological process allows us to describe the evolution of the transition state of the production process to the equilibrium state:

$$\frac{\partial\chi(t,S,\mu)}{\partial t} + \frac{\partial\chi(t,S,\mu)}{\partial \mathrm{x}} * \mu + \\ + \frac{\partial\chi(t,S,\mu)}{\partial\mu} * f(t,S) = G(t,S,\mu), \\ \chi(x,S,0) = \chi(x,S,\infty) = 0. \tag{7}$$

The kinetic equation of the form (7) is used in [26] (2003) in the construction of balance models of production lines. At the same time the US Ringhofer in the study of the production processes of the company "INTEL" got a kinetic equation for quasi-static description of the production lines

$$\frac{\partial\chi(x,r,t)}{\partial t} + \frac{1}{r}\frac{\partial\chi(x,r,t)}{\partial \mathrm{x}} = G(r,x,t) \tag{8}$$

where the coordinates of the phase space *x*, *r* - the degree of incompleteness of manufacture of the product (corresponding position), and the effective time of processing. Construction of kinetic equations is one of the major problems of statistical theory of production lines management systems [6,19,28]. The kinetic equation (7), (8) takes on real sense after the establishment of the form of function $G(t,S,\mu)$. At motion of the object of labor on a technological route, an equipment has an impact on the object of labor, changing it qualitatively and quantitatively. The stochastic process impact of equipment on the object of labor is described by a distribution density $\psi(t,S,\tilde{\mu}_\psi,\mu_\psi)$ of a random variable μ_ψ, where $\tilde{\mu}_\psi$ and μ_ψ - the intensity of the transfer of resources to the object of labor before and after exposure [27]. The cross-section random process for regulatory trajectory, that characterizes the rate of transfer of resources at time *t* at the point of a technological route with the coordinate *S* is a random variable [12] μ_ψ with the distribution density $\psi(t,S,\tilde{\mu}_\psi,\mu_\psi)$. The probability that the result of the impact of the equipment on the object of labor the value μ_ψ would be in range $(\mu_\psi; \mu_\psi + d\mu_\psi)$ is the value $\psi(t,S,\tilde{\mu}_\psi,\mu_\psi)d\mu_\psi$, and the total probability of transition to any state is unity:

$$\int_0^\infty \psi(t,S,\tilde{\mu}_\psi,\mu_\psi)d\mu_\psi = 1. \tag{9}$$

For regulatory technological trajectory that characterizes the change of state of object of labor by treating the transition from one generalized operation to another, the mathematical expectation of the intensity of the transfer of resources $\langle\mu_\psi\rangle$ for labor can be calculated:

$$\int_0^\infty \mu_\psi * \psi(t,S,\tilde{\mu}_\psi,\mu_\psi)d\mu_\psi = \langle\mu_\psi\rangle. \tag{10}$$

Function $\psi(t,S,\tilde{\mu}_\psi,\mu_\psi)$ is determined by the design and technological documentation or as a result of the experimental study [24] the state of object of labor in the transition from one operation to another. *Fig. 3* shows

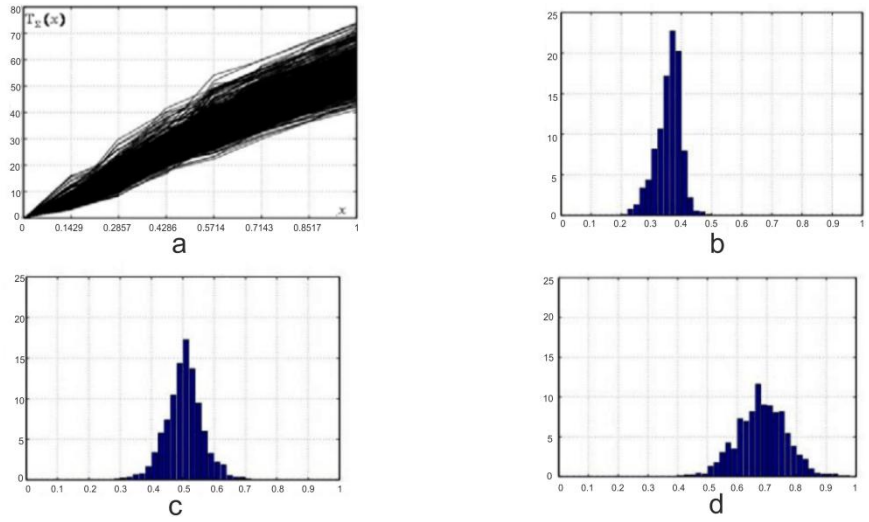

Fig. 3. Experimental results [24]: a – trajectory $T_{\Sigma}(x)$; b, c, d - a histogram of the distribution of the objects of labor

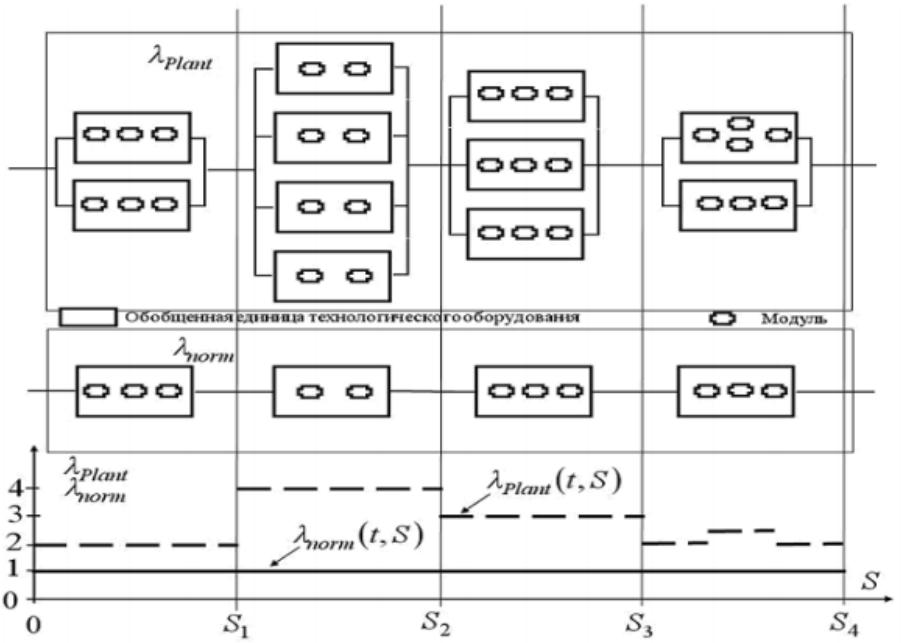

Fig. 4. The density of the arrangement of equipment $\lambda_{norm}(t,S), \lambda_{Plant}(t,S), \Delta S_m = 1$

the experimental technological trajectories $T_{\Sigma}(x)$, for a party of 920 subjects of work in the phase space *(x, r)*, which show the dependence of the total effective processing time of the object of labor from the place of treatment in the technological route. *Fig. 3* shows the histograms of the distribution of the objects of labor on technological positions for different periods of time (b – t = 20, c – t = 30, d – t = 40) since the beginning of processing of party of wares. Measurements were carried out on the production line for the production of semiconductor products of the company "INTEL" [24] to examine the impact of vibrations of the effective time duration of processing on the duration of the production cycle. This experimental data is sufficient to construct the table function $\psi(t,S,\tilde{\mu}_{\psi},\mu_{\psi})$ which is approximated by a smooth continuous function. At its construction, we consider the technological process which consists of a sequence of generalized operations. In the design of production lines to provide the required performance are using a series-parallel arrangement of generalized items of equipment (*fig. 1*) [23]. To account for the number of generalized equipment within the range of the generalized operation $[S_{m-1},S_m]$- we introduce the notion of density of equipment $\lambda_{Plant}(t,S)$ along the technological route $[0,S_d]$.This is because each generalized equipment unit consists of a plurality of sections, modules, components, subassemblies and others separately provided parts of common equipment distributed along a path of processing the object of labor within the generalized trajectory. The dependence on t in the function $\lambda_{Plant}(t,S)$, suggests the possibility of changing the amount of equipment in the time interval of the production cycle, for example, due to the on / off of reserve equipment units. Then the number of generalized items of equipment at time t within the interval of generalized operation $[S_{m-1},S_m]$(*fig.4*) can be defined as

$$\frac{1}{N_m}\int_0^{\infty}\mu_{\psi}\psi(t,S,\tilde{\mu}_{\psi},\mu_{\psi})d\mu_{\psi}\,. \qquad (11)$$

If the m-th generalized operation corresponds to only one generic unit of equipment, respectively the integral (11) is equal to one.

Often, one generalized equipment can be represented in the form of successive modules, each of which in order to improve the performance of the operation has the ability to duplicate. Suppose that up within the module is done the ratio

$$\int_{S_{b-1}}^{S_b} \lambda_{norm}(t,S)dS = P_{mB}\ ,[S_{b-1},S_b]\in[S_{m-1},S_m]\ ,$$

$$\int_{S_{m-1}}^{S_m} \lambda_{norm}(t,S)dS = 1\ .$$

Then, for the m-th generalized operation with additional number n of modules rightly: $\int_{S_{m-1}}^{S_m} \lambda_{Plant}(t,S)dS = 1+n*P_{mB}$ (*fig.4*) where $\lambda_{Plant}(t,S)dS$ - the number of units of a generalized equipment on the segment $[S,S+dS]$. In unit time a volume element $\Delta\Omega = \Delta S*\Delta\mu$, $S\in[S,S+dS], \mu\in[\mu,\mu+d\mu]$ was visited by $\chi(t,S,\mu)*\mu*d\mu$ objects of labor on average, which have experienced with the $\{\lambda_{Plant}(t,S)*dS\}*\{\chi(t,S,\mu)*\mu*d\mu\}$ acts of impact of equipment at the objects of labor. The probability that the result of the impact of the equipment on the subject of work value of a random variable μ will be in the range of $(\tilde{\mu},\tilde{\mu}+d\tilde{\mu})$ there is the value $\varphi(t,S,\mu,\tilde{\mu})*d\tilde{\mu}$ and the total probability of transition to any state will be.

$$\int_0^\infty \varphi(t,S,\tilde{\mu},\mu)d = 1. \qquad (12)$$

Thus, the number of objects of labor that are experienced per unit time the impact of the technological equipment and took random value in the range of $(\tilde{\mu},\tilde{\mu}+d\tilde{\mu})$ is the product of the transition probabilities $\varphi(t,S,\mu,\tilde{\mu})*d\tilde{\mu}$ on the total number of objects of labor $\lambda_{Plant}(t,S)*dS*\chi(t,S,\mu)*\mu*d\mu$, experienced the impact of the equipment.

$$\varphi(t,S,\mu,\tilde{\mu})*d\tilde{\mu}*\lambda_{Plant}(t,S)* \\ *dS*\chi(t,S,\mu)*\mu*d\mu. \qquad (13)$$

The distribution density $\varphi(t,S,\mu,\tilde{\mu})$ of a random variable μ can be expressed through the density distribution $\psi(t,S,\tilde{\mu}_\psi,\mu_\psi)$ of random variables μ_ψ. With a uniform transfer of resources for all N_m objects of labor, are in-process backlog of m-th generalized operation, the random variables μ and μ_ψ associated functional dependence [28, p.783], [29, p.117], [29, p.821]:

$$\mu = \mu_\psi * N_m^{-1} \qquad (14)$$

which assumes a linear law of increasing processing time of the party of objects labor with increase of the size of the queue N_m. The distribution density $\varphi(t,S,\tilde{\mu},\mu)$ of the random variable μ can be written as:

$$\varphi(t,S,\tilde{\mu},\mu) = N_m*\psi(t,S,N_m*\tilde{\mu},N_m*\mu)\ . \qquad (15)$$

The moments of a random variable μ we write through the moments of a random variable:

$$[\varphi]_0 = \int_0^\infty \varphi(t,S,\tilde{\mu},\mu)d\mu = [\psi]_0 = 1\ , \qquad (16)$$

$$[\varphi]_1 = \int_0^\infty \mu\varphi(t,S,\tilde{\mu},\mu)d\mu = \frac{1}{N_m}\int_0^\infty \mu_\psi\psi(t,S,\tilde{\mu}_\psi,\mu_\psi)d\mu_\psi = \\ = \frac{[\psi]_1}{N_m} = \frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}, \qquad (17)$$

$$[\varphi]_2 = \int_0^\infty \mu^2\varphi(t,S,\tilde{\mu},\mu)d\mu = \frac{[\psi]_2}{(N_m)^2} = \\ = \left(\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right)^2\left(1+\sigma_\psi^2\Big/\langle\mu_\psi\rangle^2\right), \qquad (18)$$

$$[\varphi]_k = \int_0^\infty \mu^k\varphi(t,S,\tilde{\mu},\mu)d\mu = \frac{[\psi]_k}{(N_m)^k}, \qquad (19)$$

where σ_ψ - is the standard deviation of a random variable μ_ψ. For many practical cases $\sigma_\psi\Big/\langle\mu_\psi\rangle = 0.005...0.1$ that allows you to record approximately

$$[\varphi]_2 = \left(\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right)^2. \qquad (20)$$

Along with the departure of (13) objects of labor from the volume element $dS*d\mu$ in the element of volume $dS*d\mu$ objects of labor come from volume $dS*d\tilde{\mu}$ in the amount of:

$$\varphi(t,S,\tilde{\mu},\mu)*d\mu*\lambda_{Plant}(t,S)*dS*\chi(t,S,\tilde{\mu})\}*\tilde{\mu}*d\tilde{\mu}\ . \qquad (21)$$

After the integration of the difference between (13) and (21) over a range of values of $\tilde{\mu}$, we obtain the change in the number of objects of labor in the volume element $dS*d\mu$ per unit of time

$$dS*d\mu*\lambda_{Plant}(t,S)* \\ *\int_0^\infty\{\varphi(t,S,\tilde{\mu},\mu)*\tilde{\mu}*\chi(t,S,\tilde{\mu})-\varphi(t,S,\mu,\tilde{\mu})*\mu*\chi(t,S,\mu)\}d\tilde{\mu} \qquad (22)$$

from which

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial S}\mu+\frac{\partial\chi}{\partial\mu}f = \\ = \lambda_{Plant}\int_0^\infty\{\varphi(t,S,\tilde{\mu},\mu)\tilde{\mu}\chi(t,S,\tilde{\mu})-\varphi(t,S,\mu,\tilde{\mu})\mu\chi(t,S,\mu)\}d\tilde{\mu}. \qquad (23)$$

For brevity we use the notation $\chi=\chi(t,S,\mu)$, $[\chi]_k=[\chi]_k(t,S)$, $f=f(t,S)$, $\lambda_{Plant}=\lambda_{Plant}(t,S)$. Taking into account (16), equation (23) can be written as:

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial S}\mu+\frac{\partial\chi}{\partial\mu}f = G(t,S,\mu) = \\ = \lambda_{Plant}\left\{\int_0^\infty[\varphi(t,S,\tilde{\mu},\mu)*\tilde{\mu}*\chi(t,S,\tilde{\mu})]*d\tilde{\mu}-\mu*\chi\right\}. \qquad (24)$$

For multidimensional description equation (24) can be written as

$$\frac{\partial}{\partial t}\chi(t,\vec{S},\vec{\mu})+\vec{\mu}\frac{\partial}{\partial \vec{S}}\chi(t,\vec{S},\vec{\mu})+\vec{f}(t,\vec{S})*\frac{\partial}{\partial \vec{\mu}}\chi(t,\vec{S},\vec{\mu})=$$
$$\lambda_{Plant}(t,\vec{S})\left\{\int_0^{\infty}[\varphi(t,\vec{S},\vec{\tilde{\mu}},\vec{\mu})\vec{\tilde{\mu}}\chi(t,S,\vec{\tilde{\mu}})]d\vec{\tilde{\mu}}-\vec{\mu}\chi(t,\vec{S},\vec{\mu})\right\}. \quad (25)$$

The most interesting, from a practical point of view, cases the density distribution $\varphi(t,S,\tilde{\mu},\mu)$ does not depend on the status of objects of labor, before exposure to the process equipment. Then integration of the right-hand side of (24) leads to a simplification of integro-differential equations:

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial \mathrm{s}}*\mu+\frac{\partial\chi}{\partial\mu}*f=\lambda_{Plant}*\{\varphi(t,S,\mu)*[\chi]_1-\mu*\chi\}. \quad (26)$$

The conclusions

Integro-differential equation (24), (26) is a kinetic equation that describes the processing of objects of labor during their movement on the technological route, it was first obtained in [26]. In the case where the intensity μ is slowly varying with time, $\mu=\mu_0\cong const$ (quasi-static process), the kinetic equation (24) takes the form:

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial \mathrm{s}}*\mu=\lambda_{Plant}*$$
$$*\{\int_0^{\infty}[\varphi(t,S,\tilde{\mu},\mu)*\tilde{\mu}*\chi(t,S,\tilde{\mu})]*d\tilde{\mu}-\mu*\chi\}; \quad (27)$$
$$\frac{d\mu}{dt}=f(t,S)\cong 0,$$

which is used in the quasi-static description of the technological process [19,28]. From equation $f(t,S)\cong const$ (6) follows $[\chi]_{1\psi}(t,S)\cong const*[\chi]_0(t,S)$, which requires compliance with performance proportional to the number of items of work the equipment in its in-process storage. It is difficult to realize in practical terms, except for the case of synchronization the equipment $\frac{\partial}{\partial S}[\chi]_{1\psi}(t,S)\cong 0$. The kinetic equation of the form (27) can be used to construct models of synchronized production lines. It should be noted that the equation

$$\frac{\partial\chi(x,r,t)}{\partial t}+\frac{1}{r}\frac{\partial\chi(x,r,t)}{\partial \mathrm{x}}=T(t,r)\int_0^{\infty}[\omega(t,\tilde{r})*\chi(t,x,\tilde{r})]*d\tilde{r}- \quad (28)$$
$$-\omega(t,r)*\chi(t,x,r),$$

the right part of which is constructed using the method of random phases, it was obtained in [27, p. 788]. Built on the basis of its balance equations is used to study the synchronized production lines for the production of semiconductor products of "INTEL" company. Where $T(t,r)$ - distribution density of the random variable r, $\omega(t,r)$ - the auxiliary phase function [27, p.785]. The right side of the kinetic equation (28) in [27, p.788] is written in general terms, it requires further in-depth study of the effect as the features of processing technology as well as layouts of components and assemblies inside the unit of generalized equipment.

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