Chapter 10

Numerical-Analytical Approach for the Solution to the Wave Scattering by Polygonal Cylinders and Flat Strip Structures

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1. INTRODUCTION

The principal approach for the solution to the wave scattering by perfectly conducting polygonal cylinders (PC) is to formulate the problems in terms of the integral equations of the first kind satisfied by the unknown functions associated with the current induced on the cylinder surface. In general, the integral equations are derived by applying the potential theory together with the Dirichlet or Neumann boundary conditions on the scatterer surface. Numerical techniques [1-11] such as the Galerkin method of moments can often be employed to reduce these integral equations to the system of linear algebraic equations (SLAE). For example, the results based on the method of moments for the diffraction by PC structures are presented in [3,4]. However, the existence of edges on the integration contour can make current density computations less accurate. In order to improve the accuracy of the solution to a certain extent, some approaches accounting for the Meixner edge condition in explicit form, have been proposed in [3,6,8,9], where the current distribution on the PC facets and the far field behavior have been investigated. There is also an attempt [12,13] to seek an asymptotic solution using the geometrical theory of diffraction (GTD).

In this chapter, a new rigorous method is developed for obtaining the solution to the PC diffraction problems with any preassigned accuracy. This method is based on the following foundations. First, a perfectly conducting PC is considered as a composition of key elements such as flat strips (see Fig. 1b). Then the original scattering problem involving the PC is equivalent to the scattering by N key elements. As a consequence, the total scattered field from the PC is expressed as a superposition of scattered fields excited due to the induced currents on facets of the cylinder. Second, the scattered field from each facet of the cylinder is represented using the Fourier transform of the corresponding surface current density, which offers a number of advantages for constructing the solution of the problem. Application of the boundary conditions to the employed model then leads to a coupled system of dual integral equations (DIE) with the kernel in trigonometric functions, satisfied by the unknown Fourier transforms. Finally, a hybrid technique [14] based on the semi-inversion procedure for equation operators [15] and the method of moments [14,16], is used to obtain the desired solution. It should be noted that our approach is essentially different from the other existing methods mentioned above, since this method, as a special case, can produce a rigorous solution

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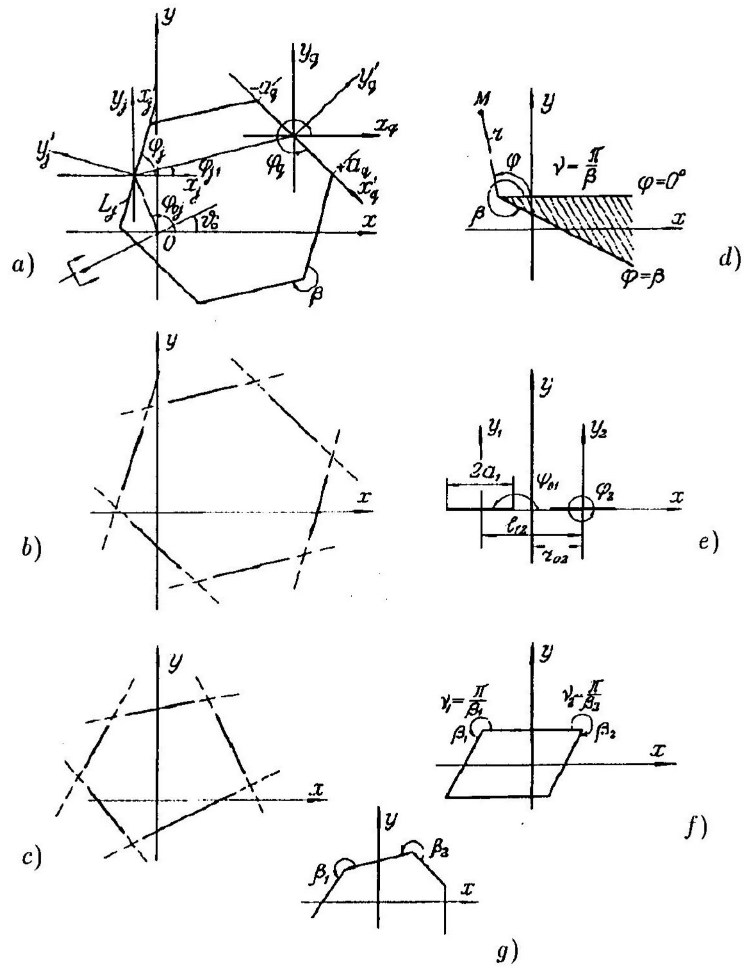

Fig. 1. Geometry of the problem in the (x, y) plane.

to the scattering by an aggregate of key elements (see Figs. 1b and 1c as an example of a system of N strips). It is worthwhile to make some remarks here on the previous work related to the foundations of the method developed in this chapter. The proposed modeling for PC geometries sometimes enables us to apply the known rigorous techniques such as the Riemann-Hilbert problem method [15,17]. In fact, the solution for the diffraction by a finite number of flat strips is presented in [17] based on this scheme. However, the previous studies for the PC problems have restrictions since the contours of flat strips and their extensions should neither cross nor touch one another. From a mathematical point of view, it is also necessary to take into account the field behavior near the edges of the PC surface. According to the Meixner condition, it is well known that in free space, there appear stronger and weaker singularities for the current functions at the edges of plane strips and at the strip joints, respectively. Therefore, a correct computation of the current singularities at the PC corners (edges) is one of the important subjects for a successful solution of the scattering problem.

In addition to the above-mentioned remarks on the PC modeling, we should note that the reduction of scattering problems (including internal electromagnetic problems) to integral equations for the Fourier transforms of the unknown functions, also have advantages

[16,18]. In [16], a related approach called the spectral theory of diffraction (STD) has been developed. In this method, the scattered field is first represented as the Fourier transforms of the currents induced by the waves incident on the scatterers. Then, a hybrid approach based on the GTD and the method of moments is employed to solve the scattering problems at high frequencies. In [16], the scattering by a right-angled prism is treated using this method. There are also a number of investigations on the scattering by plane screens [19-22], the radiation from open-ended waveguides [23,24], and internal waveguide problems [25-27], where the method of moments in the Fourier transform domain is employed to solve the integral (algebraic) equations by taking into account the edge singularities in explicit form. The methods proposed in [19-22] are based on the use of properties of the discontinuous Weber-Schafheitlin integrals for the Bessel functions. The properties of these integrals are also used in various methods for solving boundary value problems in elasticity theory [28,29].

In some of the papers mentioned in the previous paragraph, the integral equations are reduced to the infinite SLAE of the first kind, which is solved approximately via a truncation. It is important to note here that the applicability of the truncation method cannot always be justified. In addition, the matrix elements associated with the SLAE decay slowly with an increase of their index. However, these previous works are of interest due to the effective computation algorithms for slowly convergent improper integrals with Bessel functions. On the other hand, the radiation from a plane waveguide with an infinite flange is treated in [30], where the problem is reduced to the solution of the infinite SLAE of the second kind. However, the edge singularities are not taken into account correctly. In [25-27,31,32], the method of partial domains with edge singularity computations is widely employed. In principle, this technique reduces the problem to the infinite SLAE of the first kind but its solvability cannot always be justified. However, if the convergence acceleration procedure [25-27] is incorporated into this method, these SLAE may be solved by the truncation method [32]. Finally in [33,34], the scattering problem for the PC is solved by using the geometry of a flat strip. After expanding the fields due to the currents on the PC facets by the Mathieu theorem, the problem is reduced to a coupled infinte SLAE of the second kind, where the number of equations is equal to that of the PC facets. Nevertheless, neither examination on the solvability of the SLAE nor detailed scattering characteristics are presented in these works.

The main interest in this chapter is the H-polarized electromagnetic wave scattering by the PC, which is one of the complicated but important problems. For a rectangular (or square) cylinder, the infinite SLAE is investigated in detail to verify the existence and uniqueness of the solution as well as the applicability of the truncation method. The characteristics of the current distribution on the facets of the cylinder, the total scattering cross-section, and the far field patterns are discussed via illustrative numerical examples. The time variation for all the field quantities is assumed to be $e^{-i\omega t}$ and suppressed throughout the remaining part of this chapter.

2. SOLUTION TO THE PROBLEM OF H-POLARIZED ELECTROMAGNETIC WAVE SCATTERING BY A POLYGONAL CYLINDER

2.1 Statement of the problem: Derivation of a system of dual integral equations

A class of two-dimensional (cylindrical) scatterers of rectangular or regular polygonal cross-section is the subject of the present study. Let an H-polarized plane electromagnetic wave ($\vec{H}$ vector is parallel to OZ axis) be incident on a polygonal cylinder of such cross-section at an angle θ_0 to the OX axis, as shown in Fig.1a. The scatterer is composed of N perfectly conducting flat strips joined at the edges to construct a closed ribbed surface. A general coordinate system (XYZ) and a local coordinate system $(X_sO_sY_s)$ are introduced, where the OZ axis of the (XYZ) system is parallel to the cylinder surface. The origin O_s of each local coordinate system is in the center of the s-th facet of the cylinder, and the O_sY_s axis is normal to the plane of the s-th facet. The total field is considered as a superposition of the incident and scattered fields:

$$H_z^n = H_z^i + H_z^p \tag{1}$$

The total field has to satisfy the wave equation, the Neumann boundary condition at the scatterer's surface, and the condition that the energy in any bounded volume of space is finite. Besides, the Sommerfeld radiation condition holds for the scattered field. Let $H_z^n(x,y) = \Psi^t(x,y)$. For volumes without any incident sources, it follows that $(\Delta + k^2)\Psi^t(x,y) = 0$, where Δ is the Laplacian. The second Green's formula yields

$$\begin{aligned}&\int\limits_V [G(\vec{r},\vec{r}\,')\nabla^2\Psi^t(\vec{r}\,') - \Psi^t(\vec{r}\,')\nabla^2 G(\vec{r},\vec{r}\,')]\,dv \\ &= \oint\limits_{S=S_1+S_2} \left(G(\vec{r},\vec{r}\,')\frac{\partial}{\partial n'}\Psi^t(\vec{r}\,') - \Psi^t(\vec{r}\,')\frac{\partial}{\partial n'}G(\vec{r},\vec{r}\,')\right) dS\end{aligned} \tag{2}$$

where $G(\vec{r},\vec{r}\,')$ is Green's function representing the solution of the equation $(\Delta + k^2)G(\vec{r},\vec{r})$ $= -\delta(\vec{r} - \vec{r}\,')$; V is the region between the scatterer contour S_1 and a certain circle S_2 of large radius; the normal $\vec{n}$ is directed to the interior of the region V. If the scatterer contour S_1 has sharp edges, then an additional condition enforced on the behavior of the function Ψ^t is necessary to validate Green's formula. Such a condition is described by $\int\limits_{S_0} \{|\Psi^t|^2 + |\mathrm{grad}\,\Psi^t|^2\}\,ds < \infty$, which follows from the condition of finite energy in any bounded volume of space. Here, S_0 is a bounded region containing the edge.

Using the homogeneous Neumann boundary condition, the following representation is derived from (2):

$$\Psi^t(\vec{r}) = -\oint\limits_{S_1} \Psi^t(\vec{r}\,')\frac{\partial}{\partial n}G(\vec{r},\vec{r}\,')\,dS' + \oint\limits_{S_2}\left[G(\vec{r},\vec{r}\,')\frac{\partial}{\partial n'}\Psi^t(\vec{r}\,') - \Psi^t(\vec{r}\,')\frac{\partial}{\partial n}G(\vec{r},\vec{r}\,')\right] dS'$$

If the radius of circle S_2 tends to infinity and if we let $\Psi^t = \Psi^i + \Psi^s$, then

$$\Psi^t(\vec{r}) = -\oint\limits_{S_1} \Psi^t(\vec{r}\,')\frac{\partial}{\partial n'}G(\vec{r},\vec{r}\,')\,dS' + \Psi^i(\vec{r})$$

Hence an integral representation of the total field is given by

$$\begin{aligned}H_z^n &= H_z^i + \int\limits_{S_1} \mu(x',y')\frac{\partial}{\partial n'}G(\vec{r},\vec{r}\,')\,dS' = H_z^i + \sum_{s=1}^{N} H_z^s \\ &= H_z^i + \frac{i}{4}\sum_{s=1}^{N}\int\limits_{-a_s}^{a_s} \mu_s(x_s')\frac{\partial}{\partial y_s}H_0^{(1)}\left(k\sqrt{(x_s - x_s')^2 + y_s^2}\right)dx_s'\end{aligned} \tag{3}$$

where $H_0^{(1)}(x)$ is the zero-order Hankel function of the first kind, corresponding to the two-dimensional free-space Green's function; $\mu(x',y') = -H_z^n(x',y')$ with $(x',y') \in S$ is the surface-current density function; $\{\mu_s(x'_s)\}_{s=1}^N$ are the surface-current density functions on the s-th sides of the PC; $\{H_z^s\}_{s=1}^N$ are the fields scattered by the s-th sides of the PC. According to the proposed model of the PC, the total field may be represented as a sum of the incident field and the fields scattered by each PC facet, as shown in (3).

By using the integral representation of the Hankel function

$$H_0^{(1)}(k\sqrt{(x_s - x'_s)^2 + y_s^2}) = \frac{1}{\pi} \int_{-\infty}^{\infty} e^{ik[(x_s - x'_s)\alpha + y_s\sqrt{1-\alpha^2}]} \frac{d\alpha}{\sqrt{1-\alpha^2}}, \quad y_s \geq 0$$

the scattered field from each cylinder facet is expressed as

$$H_z^s(x_s, y_s) = -\frac{ka_s}{4\pi} \int_{-\infty}^{\infty} h_s(\alpha) e^{ik(\alpha x_s + \sqrt{1-\alpha^2} y_s)} d\alpha \tag{4}$$

where the branch of $\sqrt{1-\alpha^2}$ is chosen so that $\mathrm{Im}\sqrt{1-\alpha^2} > 0$ if $|\alpha| \to \infty$ along the real axis. In (4), $h_s(\alpha)$ is defined as follows:

$$h_s(\alpha) = \frac{1}{a_s} \int_{-a_s}^{a_s} \mu_s(x'_s) e^{-ikx'_s\alpha} dx'_s$$

If the current density function $\mu_s(x'_s)$ on the s-th facet vanishes outside its definition interval $[-a_s, a_s]$, then the function $h_s(\alpha)$ is the Fourier transform of the function $\mu_s(x'_s)$, i.e.,

$$h_s(\alpha) = \frac{1}{a_s} \int_{-\infty}^{\infty} \mu_s(x'_s) e^{-ikx'_s\alpha} dx'_s; \qquad \mu_s(x'_s) = \frac{\varepsilon_s}{2\pi} \int_{-\infty}^{\infty} h_s(\alpha) e^{ikx'_s\alpha} d\alpha \tag{5}$$

where $\varepsilon_s = ka_s$ is a frequency parameter. The scattered fields $\{H_z^s\}_{s=1}^N$ are now expressed by Fourier transforms of the unknown functions of the current densities. It is worth noting the validity of the following identity:

$$H_z^s(x_s, 0) = -\frac{1}{2}\mu_s(x_s), \qquad x_s \in [-a_s, a_s] \tag{6}$$

For determining the unknowns $\{h_s(\alpha)\}_{s=1}^N$, let the total field be subjected to the Neumann boundary condition on the cylinder surface:

$$\frac{\partial}{\partial n}(H_z^i + \sum_{s=1}^N H_z^s)\Big|_L = 0, \qquad L = \bigcup_{s=1}^N L_s \tag{7}$$

where L_s is the contour of each cylinder facet in the $X_sO_sY_s$ plane, and the normal vector $\vec{n}$ is directed along the positive O_sY_s axis. To satisfy (7), it is convenient to rewrite the fields H_z^i and $H_z^q (q \neq s)$ in terms of the coordinate referred to the s-th facet (see Fig.1a).

Using the local coordinate system $(X_s O_s Y_s)$, we obtain the following expressions:

$$H_z^i = e^{i\varepsilon_s[\eta_s B_s(\alpha_0)+\zeta_s A_s(\alpha_0)]+ikr_{0s}D_s(\alpha_0)]},$$

$$H_z^q = \frac{\varepsilon_q}{4\pi}\int_{-\infty}^{\infty} h_q(\alpha)e^{\varepsilon_s[B_{sq}(\alpha)\eta_s+\zeta_s A_{sq}(\alpha)+ikl_{sq}D_{sq}(\alpha)}d\alpha, \qquad \zeta_s < 0$$

where the following notation has been introduced:

$$A_{sq}(\alpha) = -\alpha\sin(\varphi_s - \varphi_q) - \sqrt{1-\alpha^2}\cos(\varphi_s - \varphi_q);$$
$$A_s(\alpha_0) = -\alpha_0\sin\varphi_s + \sqrt{1-\alpha_0^2}\cos\varphi_s;$$
$$B_{sq}(\alpha) = \alpha\cos(\varphi_s - \varphi_q) - \sqrt{1-\alpha^2}\sin(\varphi_s - \varphi_q);$$
$$B_s(\alpha_0) = \alpha_0\cos\varphi_s + \sqrt{1-\alpha_0^2}\sin\varphi_s;$$
$$D_{sq}(\alpha) = \alpha\cos(\varphi_{qs} - \varphi_q) - \sqrt{1-\alpha^2}\sin(\varphi_{qs} - \varphi_q);$$
$$D_s(\alpha) = \alpha\cos\varphi_{os} - \sqrt{1-\alpha^2}\sin\varphi_{os};$$

Here, $\eta_s\left(\equiv \frac{x_s}{a_s}\right)$ and $\zeta\left(\equiv \frac{y_s}{a_s}\right)$ are dimensionless (normalized) coordinates; l_{sq}, r_{0s}, and φ_{sq} are defined in Fig.1a.

By using these field representations and noting that the function $\mu(x_s)$ vanishes outside the interval $[-a_s, a_s]$, the equation (7) for the unknown functions $\{h_s(\alpha)\}_{s=1}^N$ is reduced to the following system of dual integral equations (DIE) with the kernel in trigonometric functions:

$$\begin{cases} \displaystyle\int_{-\infty}^{\infty} h_s(\alpha)\sqrt{1-\alpha^2}e^{i\varepsilon_s\alpha\eta_s}d\alpha = \frac{4\pi}{\varepsilon_s}A_s(\alpha_0)e^{i\varepsilon_s B_s(\alpha_0)\eta_s+ikr_{0s}D_s(\alpha_0)} \\ \displaystyle\quad + \sum_{q=1,q\neq s}^{N}\frac{\varepsilon_q}{\varepsilon_s}\int_{-\infty}^{\infty} h_q(\alpha)A_{sq}(\alpha)e^{i\varepsilon B_{sq}(\alpha)\eta_s+ikl_{sq}D_{sq}(\alpha)}d\alpha \qquad |\eta_s| < 1, \\ \displaystyle\int_{-\infty}^{\infty} h_s(\alpha)e^{i\varepsilon_s\alpha\eta_s}d\alpha = 0, \qquad |\eta_s| > 1 \end{cases} \tag{8}$$

In addition to the system of equations (8), it follows from the finite energy condition for the scattered fields in any bounded region of space that

$$\int_{-\infty}^{\infty}(|\alpha|+1)|h_s(\alpha)|^2 d\alpha < \infty, \qquad s = 1, 2, \ldots, N \tag{9}$$

must hold for the functions $\{h_s(\alpha)\}_{s=1}^N$.

Thus the scattering problem has been reduced to the solution of N coupled DIE with a trigonometric kernel for the unknown functions $\{h_s(\alpha)\}_{s=1}^N$. For computations, it is convenient to transform these equations using the edge condition, i.e., the specified behavior of the unknown current functions $\{\mu_s(\eta_s)\}_{s=1}^N$ near the points $\eta_s = \pm 1$ (at the PC corners).

In order to state this condition in terms of the current functions associated with the PC surface, we now consider the problems of E- and H-polarized electromagnetic wave scattering by a perfectly conducting wedge (see Fig.1d) (i.e., the boundary-value problems under Dirichlet and Neumann boundary conditions, respectively). Upon examination of the exact solution to this problem [35], the following behavior of the corresponding current functions $\rho(r,\varphi)$ and $\mu(r,\varphi)$ near the edge can be found:

$$\rho(r,\varphi) = \left\{ \begin{array}{c} \rho(r,0) \\ \rho(r,\beta) \end{array} \right\} \underset{r\to 0}{\sim} r^{\nu-1}, \qquad \frac{1}{2} \le \nu < 1$$
$$\mu(r,\varphi) = \left\{ \begin{array}{c} \mu(r,0) \\ \mu(r,\beta) \end{array} \right\} \underset{r\to 0}{\sim} \left\{ \begin{array}{c} \text{const} + r^{\nu} \\ \text{const} - r^{\nu} \end{array} \right\}, \qquad \frac{1}{2} \le \nu < 1 \tag{10}$$

where $\nu = \frac{\pi}{\beta}$; the angle β is defined in Fig.1d. The expression (10) states that when approaching the edge along $\varphi = 0$ and $\varphi = \beta$ directions, the function $\rho(r,\varphi)$ becomes singular, and the functions $\mu(r,0)$ and $\mu(r,\beta)$ tend to some constants. Physically, it means that in case of the H-polarized wave scattering, the surface current flows onto the next facet across the edge.

Proceeding from the above remark, the edge condition for the current functions ρ and μ on the PC can be expressed as

$$\rho_j(\eta) \underset{\eta\to\pm 1}{\sim} (1-\eta^2)^{\nu-1}, \quad j = 1,2,\ldots,N;$$
$$\mu_j(\eta) \underset{\eta\to\pm 1}{\sim} C_j(1-\eta) + C_{j+1}(1+\eta) + (1-\eta^2)^{\nu}, \quad j = 1,2,\ldots,N \tag{11}$$

Here, the constants $\{C_j\}_{j=1}^{N}$ depend on the frequency, amplitude and incident angle, and the functions $\{\mu_j(\eta)\}_{s=1}^{N}$ are to satisfy the continuity condition, i.e.,

$$\mu_j(+1) = \mu_j(-1) = 2C_{j+1}\,; \ldots ;\ \mu_{N+1}(+1) = \mu_1(-1) = 2C_1, \tag{12}$$

where $C_{N+1} = C_1$. If one considers a structure of separated flat strips, then the edge condition can be expressed as follows:

$$\rho_j(\eta) \underset{\eta\to\pm 1}{\sim} [1-\eta^2]^{-1/2}; \qquad \mu_j(\eta) \underset{\eta\to\pm 1}{\sim} [1-\eta^2]^{1/2} \tag{13}$$

The validity of the expressions in (13) follows from the statement that for infinitesimally thin screens, the current density function is a jump of the H_z component of the total field. Then it is clear that $\nu = 1/2$ and $\{C_j\}_{j=1}^{N} = 0$.

The technique of reducing the DIE (8) to the infinite Fredholm system of the second kind is presented below. It is the basis of a new method for solving both the DIE (8) and the scattering problem. To clarify the essentials of the proposed approach, an auxiliary problem will be considered in the following subsection.

2.2 Solution of a class of dual integral equations with the kernel in trigonometric functions

In this subsection, we consider a more simplified DIE system. Let the unknown function $h(\alpha)$ satisfy the DIE of the type:

$$\begin{cases} \int\limits_{-\infty}^{\infty} h(\alpha)\sqrt{1-\alpha^2} e^{i\varepsilon\alpha\eta} d\alpha = f(\eta), & |\eta| < 1, \\ \int\limits_{-\infty}^{\infty} h(\alpha) e^{i\varepsilon\alpha\eta} d\alpha = 0, & |\eta| > 1 \end{cases} \tag{14}$$

The solution of this system is constructed under the following assumptions:

a) The unknown function $h(\alpha)$ is the Fourier transform of a continuous function $\mu(\eta)$ ($\eta \in [-1, 1]$) satisfying

$$\begin{cases} \mu(\eta) \underset{\eta\to\pm 1}{\sim} (1-\eta^2)^\nu, & \frac{1}{2} \le \nu < 1 \\ \mu(\eta) = 0, & |\eta| > 1 \end{cases} \tag{15}$$

Besides, assume that

$$\frac{\mu(\eta)}{(1-\eta^2)^\nu} = \varphi(\eta) \in L_2[-1, 1; (1-\eta^2)^\nu] \tag{16}$$

where $L_2[-1, 1; (1-\eta^2)^\nu]$ is the Hilbert space of functions in which a scalar product is defined with the factor $(1-\eta^2)^\nu$ [36].

b) The function $h(\alpha)$ belongs to the class of functions satisfying (9). We call this class of functions $\tilde{L}_2(-\infty, \infty)$.

c) The given function $f(\eta)$ is continuous and defined in $L_2[-1, 1; (1-\eta^2)^\nu]$.

We can easily show that the unique solution of the DIE system (14) exists under the above assumptions. Indeed, application of the Parceval equality [37, 38] to the homogeneous counterpart of (14) yields

$$\int\limits_{-\infty}^{\infty} |\tilde{h}(\alpha)|^2 \sqrt{1-\alpha^2}\, d\alpha = 0 \tag{17}$$

with $\tilde{h}(\alpha)$ being the solution. It follows that $\tilde{h}(\alpha) = 0$, which gives the stated result.

For constructing the solution of the DIE system (14), an approach similar to the method of moments (MM) [14] is proposed, but the suggested scheme is different from the classical one because the procedure of semi-inversion of equation operators is incorporated.

The choice of a correct set of expansion functions for the unknown function $h(\alpha)$ is determined by the fact that $h(\alpha)$ is the Fourier transform of the function $\mu(\eta)$ satisfying the relationship (15). Let us now represent the continuous function $\varphi(\eta)$ as a uniformly convergent series using the Gegenbauer polynomials $\{C_n^{\nu+1/2}(\eta)\}_{n=0}^{\infty}$ in the space $L_2[-1, 1; (1-\eta^2)^\nu]$, where these polynomials form a basis. Then the function $\mu(\eta)$ may be represented as a series

$$\mu(\eta) = (1-\eta^2)^\nu \sum_{n=0}^{\infty} x_n C_n^{\nu+\frac{1}{2}}(\eta), \qquad \frac{1}{2} \le \nu < 1 \tag{18}$$

where $\{x_n\}_{n=0}^{\infty}$ are unknown coefficients. The Fourier transform of the function $\mu(\eta)$ is found to be

$$h(\alpha) = \int_{-1}^{1} \mu(\eta) e^{-i\varepsilon\alpha\eta} d\eta$$
$$= \frac{2\pi}{\Gamma(\nu+\frac{1}{2})} \sum_{m=0}^{\infty} (-i)^m x_m \beta_m \frac{J_{m+\nu+\frac{1}{2}}(\varepsilon\alpha)}{(2\varepsilon\alpha)^{\nu+\frac{1}{2}}}, \qquad \frac{1}{2} \le \nu < 1 \tag{19}$$

where $J_{m+\frac{1}{2}}(\varepsilon\alpha)$ and $\Gamma(\nu + \frac{1}{2})$ are the Bessel function and the gamma function, respectively, and

$$\beta_m = \Gamma(m + 2\nu + 1)/\Gamma(m + 1)$$

Thus the infinite set of expansion functions is given by $\{J_{m+\nu+\frac{1}{2}}(\varepsilon\alpha)/\alpha^{\nu+1/2}\}_{m=0}^{\infty}$. The series in (19) converges uniformly because it has been obtained by integration of uniformly convergent series (18). Then by using the asymptotic expansion for the Bessel functions for large argument, the estimate of the Fourier transform $h(\alpha)$ for large $|\alpha|$ is obtained as follows:

$$h(\alpha) \underset{|\alpha|\to\infty}{\sim} O\left(\frac{1}{\alpha^{1+\nu}}\right), \qquad \frac{1}{2} \le \nu < 1 \tag{20}$$

Hence there is a relation between the order of decay of the Fourier transform of the function $\mu(\eta)$ and its behavior near the endpoints of the interval in its definition (at the edges).

If seeking the solution of the system of dual equations (14) in the form (19), then the second equation of the system is satisfied identically for any $|\eta| > 1$. It is easily proved using the property of discontinuous Weber-Schafheitlin integrals [39, 40].

From (19), the problem has now been reduced to finding the unknowns $\{x_m\}_{m=0}^{\infty}$. To realize the semi-inversion procedure, the principal and completely continuous parts of the integral operator of the system (14) are to be separated. This procedure can be realized by introducing the function $\gamma(\alpha)$

$$\sqrt{1-\alpha^2} = i|\alpha|[1-\gamma(\alpha)]; \qquad \gamma(\alpha) \underset{|\alpha|\to\infty}{\sim} O(\alpha^{-2}) \tag{21}$$

Let us substitute (19) and (21) into the DIE (14) and expand the continuous functions $e^{i\varepsilon\alpha\eta}$ and $f(\eta)$ in terms of the Gegenbauer polynomials as follows:

$$e^{i\varepsilon\alpha\eta} = \left(\frac{2}{\varepsilon\alpha}\right)^{\nu+\frac{1}{2}} \Gamma\left(\nu+\frac{1}{2}\right) \sum_{k=0}^{\infty} i^k \left(k+\nu+\frac{1}{2}\right) J_{k+\nu+\frac{1}{2}}(\varepsilon\alpha) C_k^{\nu+\frac{1}{2}}(\eta),$$
$$f(\eta) = \sum_{k=0}^{\infty} f_k C_k^{\nu+\frac{1}{2}}(\eta); \qquad f(\eta) \in L_2[-1,1;(1-\eta^2)^{\nu}] \tag{22}$$

Then after interchanging the order of summation and integration in the resultant equations (the validity of this procedure follows from the uniform convergence of the series in (19) and (22)) and using the discontinuous Weber-Schafheitlin integrals [39, 40], we may conclude that, first, the homogeneous equation in the DIE system (14) is satisfied identically and, second,by the definition of the unknowns $\{x_n\}_{n=0}^{\infty}$, the infinite SLAE is derived as follows:

$$i\sum_{n=0}^{\infty}[1+(-1)^{k+n}]\left(C_{kn}^{(\nu+\frac{1}{2})}-d_{kn}^{(\nu+\frac{1}{2})}\right)(-i)^n x_n\beta_n=\Gamma_k,\qquad k=0,1,2,\ldots;\ \frac{1}{2}<\nu<1 \tag{23}$$

$$\begin{cases}(-1)^k x_{2k}-\sum_{n=0}^{\infty}(-1)^n(2n+1)d_{2k\,2n}^{(1)}x_{2n}=\frac{1}{2i}\Gamma_{2k}, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \nu=\frac{1}{2}\\ (-1)^k x_{2k+1}-\sum_{n=0}^{\infty}(-1)^n(2n+2)d_{2k+1\,2n+1}^{(1)}x_{2n+1}=\frac{1}{2}\Gamma_{2k+1},\end{cases} \tag{24}$$

In (23) and (24), the following notation has been employed:

$$C_{km}^{(\nu+\frac{1}{2})}=K_{\nu+\frac{1}{2}}=K_{\nu+\frac{1}{2}}(\varepsilon)\int_0^{\infty}J_{k+\nu+\frac{1}{2}}(\varepsilon\alpha)J_{m+\nu+\frac{1}{2}}(\varepsilon\alpha)\frac{d\alpha}{\alpha^{2\nu}}$$

$$V_{k+\nu+\frac{1}{2}}^{m+\nu+\frac{1}{2}}(2\nu)=\frac{\Gamma^2\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{m+k}{2}+1\right)}{\Gamma\left(\nu+\frac{1}{2}+\frac{k-m}{2}\right)\Gamma\left(\nu+\frac{1}{2}+\frac{m-k}{2}\right)\Gamma\left(\frac{k+m}{2}+2\nu+1\right)}; \tag{25}$$

$$d_{km}^{(\nu+\frac{1}{2})}=K_{\nu+\frac{1}{2}}(\varepsilon)\int_0^{\infty}\gamma(\alpha)J_{k+\nu+\frac{1}{2}}(\varepsilon\alpha)J_{m+\nu+\frac{1}{2}}(\varepsilon\alpha)\frac{d\alpha}{\alpha^{2\nu}}; \tag{26}$$

$$d_{km}^{(\nu+\frac{1}{2})}\Big|_{\nu=\frac{1}{2}}=2\int_0^{\infty}\gamma(\alpha)J_{k+1}(\varepsilon\alpha)J_{m+1}(\varepsilon\alpha)\frac{d\alpha}{\alpha^{2\nu}}=d_{km}^{1} \tag{27}$$

$$\Gamma_k=K_{(\nu+\frac{1}{2})}(\varepsilon)\frac{\varepsilon^{2\nu+1}}{2\pi}\frac{f_k}{i^k\left(k+\nu+\frac{1}{2}\right)};\qquad K_{\nu+\frac{1}{2}}(\varepsilon)=\frac{2\sqrt{\pi}\Gamma\left(\nu+\frac{1}{2}\right)}{\varepsilon^{2\nu-1}\Gamma(\nu)} \tag{28}$$

The normalization factor $K_{(\nu+\frac{1}{2})}(\varepsilon)$ has been introduced for elements of the matrix $\left\{C_{km}^{(\nu+\frac{1}{2})}\right\}_{k,m=0}^{\infty}$ so that each element does not depend explicitly on the parameter ε.

The problem has now been reduced to solving the infinite SLAE (23) and (24) for the unknowns $\{x_m\}_{m=0}^{\infty}$. According to (9), the set of coefficients $\{x_n\}_{n=0}^{\infty}$ belongs to the class of numerical sequences

$$l_2\left(\nu+\frac{1}{2}\right)=\left\{x_n:\sum_{n=0}^{\infty}|x_n|^2\beta_n<\infty\right\} \tag{29}$$

where $\beta_n=\dfrac{\Gamma(n+2\nu+1)}{\Gamma(n+1)}\underset{n\to\infty}{\sim}O(n^{2\nu})$ (if $\nu=\frac{1}{2}$, then $l_2(1)=\{x_n:\sum_{n=0}^{\infty}|x_n|^2(n+1)<\infty\}$). Let us prove that the solution of these equations exists and is unique, i.e., the Fredholm alternative [40] is valid for them. The SLAE (23) is examined first. Introducing the new coefficients as $z_k^{(\nu+1/2)}=(-i)^k x_k\sqrt{\beta_k}$, the sequence $\left\{z_k^{(\nu+1/2)}\right\}_{k=0}^{\infty}$ belongs to the space $l_2=\left\{z_k^{(\nu+1/2)}:\sum_{k=0}^{\infty}|z_k^{(\nu+1/2)}|^2<\infty\right\}$ according to (29). Then in view of (23), l_2 is the solution space for the following infinite SLAE satisfied by the unknowns $z_k^{(\nu+1/2)}$

$$\sum_{m=0}^{\infty}[A_{km}^{(\nu+1/2)}-Q_{km}^{(\nu+1/2)}]z_m^{(\nu+1/2)}=f_k^{(\nu+1/2)},\quad k=0,1,\ldots \tag{30}$$

where

$$A_{km}^{(\nu+1/2)} = \frac{1}{2}[1+(-1)^{k+m}]\sqrt{\beta_k\beta_m}C_{km}^{(\nu+1/2)}; \qquad f_k^{(\nu+1/2)} = \frac{1}{2}\sqrt{\beta_k}\Gamma_k;$$

$$Q_{km}^{(\nu+1/2)} = \frac{1}{2}[1+(-1)^{k+m}]\sqrt{\beta_k\beta_m}d_{km}^{(\nu+1/2)}$$

In the space l_2, the infinite SLAE (30) may be represented in operator form [40]:

$$(A^{(\nu+1/2)} - Q^{(\nu+1/2)})z^{(\nu+1/2)} = f^{(\nu+1/2)} \tag{31}$$

Here $z^{(\nu+\frac{1}{2})}$ and $f^{(\nu+\frac{1}{2})}$ are column vectors given by the unknowns $\{z_k^{(\nu+\frac{1}{2})}\}_{k=0}^{\infty}$ and the free terms $\{f_k^{(\nu+\frac{1}{2})}\}_{k=0}^{\infty}$ of (30), respectively. Operators $A^{(\nu+\frac{1}{2})}$ and $Q^{(\nu+\frac{1}{2})}$ have been given by the matrices $\{A_{km}^{(\nu+\frac{1}{2})}\}_{k,m=0}^{\infty}$ and $\{Q_{km}^{(\nu+\frac{1}{2})}\}_{k,m=0}^{\infty}$. Let us verify that $f^{(\nu+\frac{1}{2})} \in l_2$. From the definition of $f^{(\nu+\frac{1}{2})}$, we find the inequality:

$$\sum_{k=0}^{\infty}\beta_k|\Gamma_k|^2 \le C\sum_{k=0}^{\infty}\beta_k\left|\frac{f_k}{k+\nu+\frac{1}{2}}\right|^2,$$

where C is a constant. Since $\{f_k\}_{k=0}^{\infty}$ are coefficients of the expansion in the Gegenbauer polynomials for the function $f(\eta)$ of the space $L_2[-1,1;(1-\eta^2)^\nu]$, the inequality $\sum_{k=0}^{\infty}|f_k|^2\beta_k/\left(k+\nu+\frac{1}{2}\right) < \infty$ holds for $\{f_k\}_{k=0}^{\infty}$. Then it is evident that $\sum_{k=0}^{\infty}|f_k^{(\nu+1/2)}|^2 < \infty$, which gives the stated result.

It is easy to show that the operator $A^{(\nu+\frac{1}{2})}$ is positive definite, i.e., it has a double-side continuous inversion operator.

Let us show that the $Q^{(\nu+\frac{1}{2})}$ operator in (31) is completely continuous. For this purpose, it is necessary to prove that the operator norm $\|Q^{(\nu+\frac{1}{2})}\|$ is bounded. Using (26) and (30) together with the Cauchy-Bunjakovskii inequality, the following is obtained:

$$\begin{aligned}\|Q^{(\nu+\frac{1}{2})}\| &\le \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\left|Q_{km}^{(\nu+\frac{1}{2})}\right|^2 \\ &\le K_{\nu+\frac{1}{2}}^2\sum_{k=0}^{\infty}\beta_k\int_0^{\infty}\frac{J_{k+\nu+\frac{1}{2}}^2(\varepsilon\alpha)}{\alpha^{2\nu+1+s}}d\alpha\sum_{m=0}^{\infty}\beta_m\int_0^{\infty}|\gamma_1(\alpha)|^2\frac{J_{m+\nu+\frac{1}{2}}^2(\varepsilon\alpha)}{\alpha^{2\nu+1-s}}d\alpha\end{aligned} \tag{32}$$

Here s is some parameter such that $0 < s < 1$, and

$$|\gamma_1(\alpha)| = |\alpha\gamma(\alpha)| = |\alpha + i\sqrt{1-\alpha^2}| = \begin{cases}1, & \alpha < 1\\ \alpha - \sqrt{1-\alpha^2}, & \alpha \ge 1\end{cases}$$

For estimation of the function $|\gamma_1(\alpha)|$ for every α, let us use the following statement in [41]:

$$\alpha - \sqrt{\alpha^2-1} = \sum_{k=1}^{\infty}(-1)^{k+1}C_{1/2}^k\alpha^{-2k+1} \le \frac{1}{\alpha}\sum_{k=1}^{\infty}|C_{1/2}^k| = \frac{\sqrt{2}}{\alpha}; \quad \alpha > 1,$$

$$C_{1/2}^k = \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\ldots\left(\frac{1}{2}-(k-1)\right)}{\Gamma(k+1)} \tag{33}$$

Since there exists a parameter $0 < s < 1$ such that $\frac{\sqrt{2}}{\alpha} < \frac{1}{\alpha^s}$, we find that $|\gamma_1(\alpha)| < \frac{1}{\alpha^s}$. If one substitutes this estimation into (32) and evaluates the integrals, then

$$\|Q^{(\nu-\frac{1}{2})}| \leq K_{\nu+\frac{1}{2}}(\varepsilon) \sum_{k=0}^{\infty} V_{k+\nu+\frac{1}{2}}^{k+\nu+\frac{1}{2}}(2\nu+s+1)\beta_k \tag{34}$$

The series in (34) converges because it is seen using (25) that

$$V_{k+\nu+\frac{1}{2}}^{k+\nu+\frac{1}{2}}(2\nu+s+1) \underset{k\to\infty}{\sim} O(k^{-2\nu-s-1}); \qquad \beta_k \underset{k\to\infty}{\sim} O(k^{2\nu})$$

Hence, $Q^{(\nu+\frac{1}{2})}$ is a completely continuous operator.

Thus it has been verified that (31) belongs to the class of operator equations [40] such that the Fredholm alternative is valid. The approximate solution for the infinite SLAE (30) can be obtained by the truncation method. Indeed, as the positively determined operator $A^{(\nu+\frac{1}{2})}$ may be represented in the form $A^{(\nu+\frac{1}{2})} = T + cI$ (see [41]) with T being a positive operator, I is the identity operator, and c is a real number; so in view of the complete continuity of the operator $Q^{(\nu+\frac{1}{2})}$, the conclusion of applicability of the truncation method to the infinite SLAE (30) follows (see [42]).

Now let us consider the infinite SLAE (29) after turning to the new unknowns $z_k^{(1)} = (-i)^k\sqrt{k+1}x_k$ in the space l_2:

$$z_k^{(1)} - \sum_{m=0}^{\infty} Q_{km}^{(1)} z_m^{(1)} = f_k^{(1)} \tag{35}$$

It is obvious that $\{f_k^{(1)}\}_{k=0}^{\infty} \in l_2$. According to the estimation of the operator norm $\|Q^{(1)}\|$ (see expression (34) for $\nu = 1/2$), the matrix $\{Q_{km}^{(1)}\}_{k,m=0}^{\infty}$ gives a completely continuous operator in the space l_2. Then the infinite SLAE (35) is the Fredholm equation of the second kind. Hence, these equations are also solvable with the truncation method.

Let us prove that the infinite SLAE (24) is the Fredholm equation of the second kind in the space l_2. For this purpose, let us estimate the norm of the matrix $\{(m+1)d_{km}^{(1)}\}_{k,m=0}^{\infty}$ by taking into account (27) and the Cauchy-Bunjakovskii inequality. This estimation is:

$$\|d^{(1)}\|^2 \leq \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(m+1)^2|d_{km}^{(1)}|^2 \leq 4\sum_{k=0}^{\infty}\int_0^{\infty} J_{k+1}^2(\varepsilon\alpha)\frac{d\alpha}{\alpha^2}\sum_{m=0}^{\infty}(m+1)^2\int_0^{\infty} J_{m+1}^2(\varepsilon\alpha)|\gamma_1(\alpha)|^2\frac{d\alpha}{\alpha^2}$$

According to (33), it is seen that $|\gamma_1(\alpha)| \leq 1$ holds for $\alpha < 1$ and $|\gamma_1(\alpha)| \leq \frac{\sqrt{2}}{\alpha}$ for $\alpha \geq 1$. Therefore we derive

$$\|d^{(1)}\|^2 \leq \sum_{k=0}^{\infty}\frac{\Gamma\left(k+\frac{1}{2}\right)}{\Gamma\left(k+\frac{5}{2}\right)}\sum_{m=0}^{\infty}(m+1)^2\left\{\int_0^1 J_{m+1}^2(\varepsilon\alpha)\frac{d\alpha}{\alpha^2} + 2\int_1^{\infty} J_{m+1}^2(\varepsilon\alpha)\frac{d\alpha}{\alpha^4}\right\}$$

Since $\sum_{m=0}^{\infty}(m+1)^2 J_{m+1}^2(\varepsilon\alpha) = \frac{\varepsilon^2\alpha^2}{4}$ [39], the following estimate is obtained after evaluating the corresponding integrals and sums:

$$\|d^{(1)}\|^2 \leq 3\varepsilon^3/4\pi < \infty \tag{36}$$

Thus the matrix $\{(m+1)d_{km}^{(1)}\}_{k,m=0}^{\infty}$ gives a completely continuous operator. Since the free terms $\{\Gamma_k\}_{k=0}^{\infty}$ of (24) also belong to the space $l_2(1)$, it is clear that the infinite SLAE (24) is the Fredholm equation of the second kind, which gives the stated result.

From the existence of the unique solution for the infinite SLAE (30), the existence of the unique solution for the DIE system in space $\tilde{L}_2(-\infty,\infty)$ follows in accordance with the formula (19).

Various interior and exterior problems in electromagnetics may be reduced to the operator equations as shown in (31). For example, many interior problems of electromagnetics [25–27, 31] have been reduced to the infinite SLAE of the first kind by means of the partial domain method [26, 33] upon consideration of the edge condition but without investigation of the solvability. However, if the procedure of the convergence acceleration is carried out [25–27], then the derived SLAE of the first kind can be reduced to the class of operator equations of the (31) type (see [32]).

2.3 Reduction of the problem of H-polarized wave scattering by polygonal cylinders to the SLAE

The procedure for the reduction of the DIE (8) to the system of N coupled linear algebraic equations is similar to that described previously in Section 2.2, but it requires somewhat more careful considerations. Since there are non-zero currents along the edges of the PC, the functions $\{\mu_s(\eta)\}_{s=1}^{N}$ appearing in this diffraction problem cannot be represented in the form (18). At both endpoints of the interval $[-1,1]$, the functions $\{\mu_s(\eta)\}_{s=1}^{N}$ tend to some constants $\{C_s\}_{s=1}^{N}$ as has been pointed out previously (see (11)). Therefore these functions are to be in the form of

$$\mu_s(\eta_s) = C^s(1-\eta_s) + C^{s+1}(1+\eta_s) + (1-\eta_s^2)^{\nu}\sum_{n=0}^{\infty}\mu_n^s C_n^{\nu+\frac{1}{2}}(\eta_s) \tag{37}$$

Thus an additional problem of determining the constants $\{C_s\}_{s=1}^{N}$ arises. The Fourier transform of the function $\mu_s(\eta)$ is:

$$h_s(\alpha) = C^s(-2i)K_s^+(\alpha) + C^{s+1}2iK_s^-(\alpha) + \frac{\pi}{2^{2\nu}\Gamma(\nu+\frac{1}{2})}\sum_{n=0}^{\infty}\hat{\mu}_n^s\beta_n j_{n+\nu+\frac{1}{2}}(\varepsilon_s\alpha) \tag{38}$$

where $\{j_{n+\nu+\frac{1}{2}}(z)\}_{n=0}^{\infty}$ are functions connected with the Bessel functions through the simple relationship $j_{n+\nu+\frac{1}{2}}(z) = (z/2)^{-\nu-1/2}J_{n+\nu+\frac{1}{2}}(z)$, and

$$K_s^{\pm}(\alpha) = \frac{e^{\pm i\varepsilon_s\alpha}}{\varepsilon_s\alpha} - \frac{\sin\varepsilon_s\alpha}{(\varepsilon_s\alpha)^2}; \qquad \hat{\mu}_{2k} = (-1)^k\mu_{2k}; \qquad \hat{\mu}_{2k+1} = i(-1)^{k+1}\mu_{2k+1}$$

Although the expression (38) for the unknown function $h_s(\alpha)$ is somewhat complicated, we can apply a method similar to that developed in Section 2.2 for the reduction of the problem to the SLAE.

As before let us substitute the representation (38) for the functions $\{h_s(\alpha)\}_{s=1}^{N}$ to the DIE system (8) and separate the principal and completely continuous parts of the integral operators of these equations using (21). Then assuming that the right-hand sides of the equations in (8) belong to the function class $L_2[-1,1;(1-\eta^2)^{\nu}]$ and using the orthogonality relation of the Gegenbauer polynomials for determining the unknowns $z_n^s = (-i)^n\sqrt{\beta_n}\mu_n^s$, the following coupled infinite SLAE is obtained:

$$\sum_{q=1}^{N}\tilde{A}_{m0}^{sq}C_q + i\sum_{n=0}^{\infty}(A_{mn}^s - \tilde{Q}_{mn}^s)z_n^s + \sum_{q=1}^{N}\sum_{n=0}^{\infty} z_n^q P_{mn}^{sq} = \Pi_m^s \tag{39}$$

where

$$\tilde{A}_{m0}^{ss} = \left\{-\frac{\gamma}{4}\varepsilon_s^2\int_{-\infty}^{\infty} K_s^+(\alpha)\sqrt{1-\alpha^2}j_{m+\nu+\frac{1}{2}}(\varepsilon_s\alpha)\,d\alpha \right.$$
$$\left. -\frac{\gamma}{4}\varepsilon_{s-1}\varepsilon_s\int_{-\infty}^{\infty} K_{s-1}^-(\alpha)e^{ikr_{s-1,s}D_{s,s-1}}A_{s,s-1}j_{m+\nu+\frac{1}{2}}(\varepsilon_s B_{s,s-1})\,d\alpha\right\}\sqrt{\beta_m} \tag{40}$$

$$\varepsilon_0 = \varepsilon_N; \qquad K_0^\pm(\alpha) = K_N^\pm(\alpha); \qquad D_{s0} = D_{sN}; \qquad A_{s0} = A_{sN};$$
$$B_{s0} = B_{sN}; \qquad \gamma = 2i\frac{2^{2\nu}\Gamma(\nu+\frac{1}{2})}{\pi};$$

$$\tilde{A}_{m0}^{sq} = \sqrt{\beta_m}\gamma\frac{\varepsilon_q\varepsilon_s}{4}\int_{-\infty}^{\infty} K_q^+(\alpha)e^{ikr_{qs}D_{sq}}A_{sq}j_{m+\nu+\frac{1}{2}}(\varepsilon_s B_{sq})\,d\alpha$$
$$+\sqrt{\beta_m}\begin{cases}\gamma\frac{\varepsilon_s^2}{4}\int_{-\infty}^{\infty} K_s^-(\alpha)\sqrt{1-\alpha^2}j_{m+\nu+\frac{1}{2}}(\varepsilon_s\alpha)\,d\alpha, & q=s+1;\\ -\gamma\frac{\varepsilon_{q-1}\varepsilon_s}{4}\int_{-\infty}^{\infty} K_{q-1}^-(\alpha)e^{ikr_{q-1,s}D_{s,q-1}}A_{s,q-1}j_{m+\nu+\frac{1}{2}}(\varepsilon_s B_{s,q-1})\,d\alpha, & q\neq s,s+1\end{cases}$$

$$A_{mn}^s - \tilde{Q}_{mn}^s = (-i)\frac{\varepsilon_s^2}{4}\sqrt{\beta_n\beta_m}\int_{-\infty}^{\infty} j_{n+\nu+\frac{1}{2}}(\varepsilon_s\alpha)j_{m+\nu+\frac{1}{2}}(\varepsilon_s\alpha)\sqrt{1-\alpha^2}\,d\alpha$$
$$= \frac{\varepsilon_s^{2\nu-1}2^{2\nu}}{K_{\nu+\frac{1}{2}}(\varepsilon_s)}[A_{mn}^{(\nu+\frac{1}{2})} - Q_{mn}^{(\nu+\frac{1}{2})}];$$

$$P_{mn}^{sq} = -\frac{\varepsilon_s\varepsilon_q}{4}\sqrt{\beta_n\beta_m}\int_{-\infty}^{\infty} j_{m+\nu+\frac{1}{2}}(\varepsilon_q\alpha)j_{n+\nu+\frac{1}{2}}(\varepsilon_s B_{sq})e^{ikr_{qs}D_{sq}\alpha_0}\,d\alpha A_{sq}^{(\alpha)}e^{ikr_{qs}D_{sq}} \tag{41}$$

$$\Pi_m^s = \varepsilon_s\cdot 2^{2\nu}\Gamma\left(\nu+\frac{1}{2}\right)A_s e^{ikr_{0s}D_s(\alpha_0)}j_{m+\nu+\frac{1}{2}}(\varepsilon_s B_s(\alpha_0))\sqrt{\beta_m} \tag{42}$$

The matrix $\tilde{A}_{m0}^{sq}$ is generated by the edge constants $\{C_q\}_{q=1}^N$. Besides, the system (39) is to be completed with N equations for the unknowns $\{C^q\}_{q=1}^N$ and $\{\hat{\mu}_m^s\}_{m=1}^\infty$.

From the definition of the current function on the PC (see (3)) and the edge condition (11), it follows that the total field near $(s+1)$ edges tends to some constant:

$$H_z^n\Big|_{\substack{x_s=a_s\\ y_s=0}} = \left(H_z^i + \sum_{q=1}^{N} H_z^q(x_s,y_s)\right)\Big|_{\substack{x_s=a_s\\ y_s=0}} = -\mu_s(+1) = -2C^{s+1}$$

As has been stated above, $H_z^s(x_s,0) = -\frac{1}{2}\mu_s(x_s)$ (see (6)). Then $H_z^s(a_s,0) = -C^{s+1}$, $H_z^{s+1}(a_{s+1},0) = -C^{s+1}$. Therefore the relation

$$\left\{H_z^i(x_s,y_s) + \sum_{q=1,q\neq s,s+1} H_z^q(x,y_s)\right\}\Big|_{\substack{x_s=a_s\\ y_s=0}} = 0 \tag{43}$$

may be obtained. If one substitutes the representations (8) and (38) into the relation (43), then the following expression is obtained:

$$e^{i\varepsilon_s B_s(\alpha_0)+i\varepsilon_s D_s(\alpha_0)} + \sum_{q=1,q\neq s,s+1}^{N}\left(\frac{\varepsilon_q}{4\pi}\int_{-\infty}^{\infty}(C^q(-2i)K_q^+(\alpha)+C^q(-2i)K_q^-(\alpha)\right.$$

$$\left.+\frac{\pi}{2^{2\nu}\Gamma(\nu+\frac{1}{2})}\sum_{n=0}^{\infty}\hat{\mu}_n^q\beta_n j_{n+\nu+1/2}(\varepsilon_q\alpha)\right)e^{i\varepsilon_s B_{sq}+ik\vec{r}_{qs}D_{sq}}d\alpha = 0$$

After its transformation, N additional equations required for the SLAE (39) are derived as follows:

$$\sum_{q=1}^{N} C^q BD^{sq} + \sum_{q=1,q\neq s,s+1}^{N}\sum_{n=0}^{\infty} z_n^q B_{0,n}^{sq} = P^s, \qquad s=1,2\ldots,n, \tag{44}$$

where

$$BD^{sq} = \begin{cases} \Omega_{s,s-1}^-, & s=q\,, \\ 0, & q=s-1\,, \qquad \Omega_{sq}^{\pm} = \Gamma\varepsilon_q\displaystyle\int_{-\infty}^{\infty} K_q^{\pm}(\alpha)e^{i\varepsilon_s B_{sq}+ikr_{qs}D_{sq}}d\alpha \\ -\Omega_{s,s+2}^+, \\ \Omega_{s,q-1}^- - \Omega_{s,q}^+, \end{cases} \tag{45}$$

$$B_{on}^{sq} = \varepsilon_q\sqrt{\beta_n}\int_{-\infty}^{\infty} j_{n+\nu+\frac{1}{2}}(\varepsilon_q\alpha)e^{i\varepsilon_s B_{sq}+ik\vec{r}_{qs}D_{sq}}d\alpha$$

$$P^s = -42^\nu\Gamma(\nu+\frac{1}{2})e^{i\varepsilon_s B_s(\alpha_0)+ikr_{qs}D_s(\alpha_0)} \tag{46}$$

On the basis of (39) and (44), the original infinite SLAE is transformed into the following form:

$$\begin{cases} \displaystyle\sum_{q=1}^{N} A_{mo}^{sq}C^q + i\sum_{n=0}^{\infty}(A_{mn}^s - \hat{Q}_{mn}^s)z_n^s + \sum_{q=1}^{N}\sum_{n=0}^{\infty} z_n^q P_{mn}^{sq} = \Pi_m^s \\ \displaystyle\sum_{q=1}^{N} C^q BD^{sq} + \sum_{q=1q\neq s,s+1}^{N}\sum_{n=0}^{\infty} z_n^2 B_{on}^{sq} = P^s \end{cases} \tag{47}$$

The matrices $\{P_{mn}^{sq}\}_{m,n=0}^{\infty}$ $(s \neq q)$ are shown to generate completely continuous operators $\{P_{jq}\}_{j\neq q=1}^{N}$ in the space l_2 (in Section 4, the explicit estimation of these operators is given). If $\{\hat{A}_{mo}^{sq}\}_{m=0}^{\infty}$, $\{B_{on}^{sq}\}_{n=0}^{\infty}$, and $\{\Pi_{m}^{s}\}_{m=0}^{\infty}$ form convergent sequences in the space l_2, then it can be concluded that the infinite SLAE (47) belongs to the class of matrix equations such as (30), whose solvability has been investigated in detail (see Section 2.2).

Note that the elements of matrices $\{P_{mk}^{jq}\}_{m,k=0}^{\infty}$ describing the electromagnetic interaction of the PC facets (as one can see from (41)) are represented by improper integrals of the Bessel functions. The asymptotic behavior of the integrands $N_{mk}^{jq}(\alpha)$ depends on the values of the indices j and q .

For example, when $q = j+1$ or $q \neq j+1$, i.e., when the operators $\{P_{jq}\}_{j,q=1}^{N}$ describe the interaction of adjoining PC facets or non-adjoining ones respectively, the rates of decay of the functions $\{N_{mk}^{jq}\}_{m,k=0}^{\infty}$ for $|\alpha| \to \infty$ differ basically from each other. In fact, we find that

$$N_{mk}^{j,j+1}(\alpha) \underset{|\alpha|\to\infty}{\sim} O\left(\frac{1}{\alpha^{2\nu+1}}\right); \qquad N_{mk}^{jq}(\alpha) \underset{|\alpha|\to\infty}{\sim} O\left(\frac{e^{-\alpha k l_{jq}}}{\alpha^{2\nu+1}}\right) \tag{48}$$

This point needs further consideration. The values $A_{jq}(\alpha), B_{jq}(\alpha)$, and $D_{jq}(\alpha)$ involved in the integrand of $N_{mk}^{jq}(\alpha)$ become complex values for $|\alpha| \to \infty$ as one can see from the formula (9). As the complex value $B_{jq}(\alpha) = \alpha\cos(\varphi_j - \varphi_q) + \sqrt{1-\alpha^2}\sin(\varphi_j - \varphi_q)$ is involved in the Bessel function argument, the functions $N_{mk}^{jq}(\alpha)$ have an exponentially increasing factor $\exp\left(\varepsilon_j\sqrt{\alpha^2-1}\sin(\varphi_j-\varphi_q)\right)$. On the other hand, however, the functions $N_{mk}^{jq}(\alpha)$ have an exponentially decaying factor $\exp\left(-kr_{jq}\sqrt{\alpha^2-1}\sin(\varphi_j-\varphi_{jq})\right)$. Then using the asymptotic expansion for the Bessel functions, the functions $N_{mk}^{jq}(\alpha)$ behave like

$$N_{mk}^{jq}(\alpha) \underset{|\alpha|\to\infty}{\sim} \frac{1}{\alpha^{2\nu+1}} e^{-\sqrt{\alpha^2-1}[kr_{jq}\sin(\varphi_j-\varphi_{jq})-\varepsilon_j\sin(\varphi_j-\varphi_q)]}$$

For $q = j+1$, the parameters $r_{jq} = l_{jq}$ and a_j are connected through a simple relation $l_{jq} = 2a_j\cos(\varphi_j - \varphi_{jq})$ (see Fig.1d). Then

$$N_{mk}^{j,j+1}(\alpha) \underset{|\alpha|\to\infty}{\sim} \alpha^{-2\nu-1}\exp\left(-\sqrt{\alpha^2-1}\varepsilon_j\sin(\varphi_j+\varphi_q-2\varphi_{jq})\right)$$

As the identity $\varphi_j + \varphi_q - 2\varphi_{jq} = 0$ or 2π is valid for the j-th and $(j+1)$-th facets of a regular polygon, $N_{mk}^{j,j+1}(\alpha) \underset{|\alpha|\to\infty}{\sim} O(\alpha^{-2\nu-1})$. For $q \neq j+1$ (non-adjoining facets), it is obvious that $l_{jq}\sin(\varphi_j - \varphi_{jq}) > a_j\sin(\varphi_j - \varphi_{jq})$. The asymptotic representation

$$N_{mk}^{jq}(\alpha) \underset{|\alpha|\to\infty}{\sim} \frac{1}{\alpha^{2\nu+1}} e^{-\alpha k[r_{jq}\sin(\varphi_j-\varphi_{jq})-a_j\sin(\varphi_j-\varphi_q)]}$$

is now derived. The adjoining facets are characterized by a slow decrease of the functions $N_{mk}^{jq}(\alpha)$ as can be seen from (48). Therefore it is necessary to use the procedure of convergence acceleration for computation of the improper integrals involved in the matrix elements $P_{km}^{j,j+1}$.

Note that the recurrence formula

$$L_{mn} = \frac{m-1}{n}(L_{m-1,n-1} + L_{m-1,n+1}) - L_{m-2,n}$$

is valid for the block matrix elements, which can be derived from the recurrence relationship for the Bessel functions. Here L_{mn} represents A^s_{mn} or Q^s_{mn}.

Now it is desirable to give formulas for the main electromagnetic characteristics of the scatterer. The solution of the problem of wave scattering by a bounded body results in characteristics of primary interest such as the radiation pattern (RP), the total scattering cross-section σ^H_s , and the back scattering cross-section. The current distribution on the PC facets is described by the functions $\{\mu_j(\eta)\}_{j=1}^N$ expanded in terms of the series (37) using the Gegenbauer polynomials. The unknown coefficients $\{\mu^s_m\}_{m=0}^{\infty}$ are determined by solving the infinite SLAE (47). For this purpose, the fields $\{H^j_z\}_{j=0}^N$ in far zone should be investigated. Introduce a cylindrical coordinate system (r,φ,z) which is coaxial to the general coordinate system $(xoy)(x = r\cos\varphi, y = r\sin\varphi)$. Noting that

$$x^1_j = x\cos\varphi_j + y\sin\varphi_j - r_{oj}\cos(\varphi_j - \varphi_{oj})$$
$$y^1_j = -x\sin\varphi_j + y\cos\varphi_j - r_{oj}\sin(\varphi_j - \varphi_{oj})$$

and substituting the asymptotic expansion of the Hankel function $H^{(1)}_0(r)$ for $|r| \to \infty$ into the formula (38), the far field representation in the cylindrical coordinate system (r,φ,z) is deduced as follows:

$$H^j_z = A(r)\Phi_j(\varphi)$$

where

$$A(r) = \sqrt{\frac{2}{\pi kri}}e^{ikr},$$

and

$$\Phi_j(\varphi) = -\frac{\varepsilon_j}{2}\sin(\varphi-\varphi_j)e^{ikr_{oj}\cos(\varphi_j-\varphi_{oj})}\int_{-1}^{1}\mu_j(\eta_j)e^{-i\varepsilon_j\cos(\varphi-\varphi_j)}d\eta$$

gives the radiation pattern for each element. On taking account of

$$\int_{-1}^{1}\mu_j(\eta_j)e^{-i\varepsilon_j\cos(\varphi-\varphi_j)}d\eta = h_j(\cos(\varphi-\varphi_j))$$

with the function $h_j(\cos(\varphi-\varphi_j))$ being defined by the expression (38), the radiation pattern $\Phi(\varphi)$ for the total scattered field is derived as

$$\Phi(\varphi) = \sum_{j=1}^{N}\Phi_j(\varphi) = -\frac{1}{2}\sum_{j=1}^{N}\varepsilon_j\sin(\varphi-\varphi_j)e^{-ikr_{oj}\cos(\varphi-\varphi_j)}$$
$$\times\left\{[C_{j+1}K^-_j(\cos(\varphi-\varphi_j)) - C_jK^+_j(\cos(\varphi-\varphi_j))]\right.$$
$$\left.+\frac{\pi}{\Gamma(\nu+\frac{1}{2})}\sum_{m=0}^{\infty}\sqrt{\beta_m}z^j_m\frac{J_{m+\nu+1/2}(\varepsilon_j\cos(\varphi-\varphi_j))}{(2\varepsilon_j\cos(\varphi-\varphi_j))^{\nu+1/2}}\right\} \tag{49a}$$

According to the optical theorem [44], the total scattering cross-section is given by

$$\sigma^H_S = -\frac{4}{k}Re\Phi(\theta_0) \tag{49b}$$

3. *H*-POLARIZED PLANE WAVE SCATTERING BY A FINITE NUMBER OF FLAT STRIPS

As a special case, the method developed in the previous section for solving the wave scattering by a PC provides a rigorous solution for the scattering by infinitesimally thin flat strips (see Fig.1b). In order to obtain the solution of this problem, it is necessary that:

a) the PC facets are disengaged, i.e., the linear dimensions $\{2a_j\}_{j=1}^{N}$ of flat strips are such that they cannot touch one another.

b) the parameter specifying the current function singularity at the edge is taken as $\nu = 1/2$.

Under the above requirement, the current density functions on the strips may be represented as

$$\mu_j(\eta) = (1-\eta^2)^{\frac{1}{2}} \sum_{m=0}^{\infty} x_m^j U_m(\eta) \tag{50}$$

by taking account of (18), where $\{U_m(\eta)\}_{m=0}^{\infty}$ are the Chebyshev polynomials of the second kind ($U_m(\eta) = C_m^1(\eta)$). To find the unknowns $\{x_m^j\}_{m=0}^{\infty}$ according to the scheme stated in Section 2.2, the infinite coupled SLAE of the following form is derived:

$$\left\{\begin{aligned}
&(-1)^k x_{2k}^j - \sum_{m=0}^{\infty}(-1)^m(2m+1)x_{2m}^j d_{2k2m}^{(1)} \\
&= \frac{1}{2i}\Gamma_{2k}^{(j)} + \frac{1}{2i}\sum_{q=1,q\neq j}^{N}\sum_{m=0}^{\infty}(-i)^m(m+1)x_{2m}^q P_{m2k}^{jq} \\
&(-1)^k x_{2k}^j - \sum_{m=0}^{\infty}(-1)^m(2m+2)x_{2m+1}^j d_{2k+1,2m+1}^{(1)} \\
&= \frac{1}{2i}\Gamma_{2k}^{(j)} + \frac{1}{2i}\sum_{q=1,q\neq j}^{N}\sum_{m=0}^{\infty}(-i)^m(m+1)x_m^q P_{m,2k+1}^{jq}
\end{aligned}\right. \tag{51}$$

where

$$\begin{aligned}
P_{km}^{jq} &= 2\int_{-\infty}^{\infty} A_{jq}(\alpha)J_{m+1}(\varepsilon_q\alpha)J_{k+1}(\varepsilon_j B_{jq}(\alpha))\frac{e^{ikl_{jq}D_{jq}(\alpha)}}{\alpha B_{jq}(\alpha)}d\alpha \\
\Gamma_k^{(1)} &= \frac{4e^{ikr_{0j}D_j(\alpha_0)}A_j(\alpha_0)J_{k+1}(\varepsilon_j B_j(\alpha_0))}{\varepsilon_j B_j(\alpha_0)}
\end{aligned} \tag{52}$$

and the elements of the matrix $\{d_{km}^{(1)}\}_{k,m=0}^{\infty}$ are determined by the relationship (27).

The infinite SLAE (51) belongs to the class of matrix equations such as (24) and (35); in other words, this is a system of the Fredholm equations of the second kind. To verify this statement, it is sufficient to prove the inequalities:

$$\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}(m+1)^2|P_{km}^{jq}|^2 < \infty\,; \qquad \sum_{k=0}^{\infty}|\Gamma_k^{(j)}| < \infty \tag{53}$$

This means that the matrices $\{P_{km}^{jq}\}_{k,m=0}^{\infty}(j \neq q = 1,2,\ldots,N)$ corresponding to the interaction between strips generate completely continuous operators $\{P_{km}^{jq}\}_{k,m=0}^{\infty}(j \neq q)$ in the space l_2, and the free terms of the equations (51) also belong to the space l_2. Recall that the complete continuity of the operators corresponding to the matrices $\{d_{km}^{(1)}\}_{k,m=0}^{\infty}$ was proved in Section 2.2. For clarity, the validity of the relationship (53) can be proved for the case of wave scattering by the structure of two identical strips ($a_1 = a_2$) lying in the same plane (Fig.1e). Using the solution of the problem as a model, the conclusion that it is impossible to join the infinitesimally thin and perfectly conducting strips without changing the edge condition $\nu = 1/2$ at the joint, will be verified below. The parameters of a two-strip system (Fig.1e) correspond to:

$$\varphi_1 = 0; \varphi = 2\pi; \varphi_{12} = 0; \varphi_{21} = \pi; \varphi_{02} = 0; l_{12} = l_{21}; B_{12} = B_{21} = \alpha;$$
$$A_{12} = A_{21} = -\sqrt{1-\alpha^2}; D_{12} = -\alpha; D_{21} = \alpha; D_2(\alpha_0) = \alpha_0 = \cos\theta_0; r_{01} = r_{02} = \frac{l_{21}}{2};$$
$$B_1(\alpha_0) = B_2(\alpha_0) = \alpha_0; A_1(\alpha_0) = A_2(\alpha_0) = \sqrt{1-\alpha_0^2}; D_1(\alpha_0) = -\alpha_0 \tag{54}$$

Then the following expression can be obtained :

$$\sum_{k=0}^{\infty} |\Gamma_k^{(j)}|^2 = 16\frac{1-\alpha_0^2}{\alpha_0^2\varepsilon_0^2}\sum_{k=0}^{\infty} J_{k+1}^2(\varepsilon_j\alpha_0) = 8\frac{1-\alpha_0^2}{\alpha_0^2\varepsilon_0^2}\left[1 - J_0^2(\varepsilon_j\alpha_0)\right] < \infty$$

To check the second inequality in (53), the integrals included in the matrix elements P_{mk}^{jq} should be investigated more carefully. Then under (54), these matrix elements are transformed into:

$$\left\{\begin{matrix} P_{mk}^{12} \\ P_{mk}^{21} \end{matrix}\right\} = -2\int_{-\infty}^{\infty} \frac{\sqrt{1-\alpha_2}}{\alpha^2} J_{m+1}(\varepsilon_2\alpha)J_{k+1}(\varepsilon_2\alpha)\left\{\begin{matrix} e^{-ikl_{12}\alpha} \\ e^{ikl_{12}\alpha} \end{matrix}\right\} d\alpha \tag{55}$$

If one tries to estimate these integrals according to the above-stated scheme (on the basis of the Cuachy-Bunjakovskii inequality), the validity of (53) cannot be established because when a function is estimated in its absolute value, the oscillating factor $e^{\pm ikl_{12}\alpha}$ improving the convergence rate of the integrals in (55) disappears. In order to obtain the desired result, the principal part of the integrand is separated in a familiar manner. As before, the function $\gamma(\alpha)$ is introduced by means of the formula $\sqrt{1-\alpha^2} = i|\alpha|[1-\gamma(\alpha)]$. The result is :

$$P_{mk}^{12} = -2i\int_0^{\infty} J_{m+1}(\varepsilon_1\alpha)J_{k+1}(\varepsilon_1\alpha)\left[e^{-ikl_{12}\alpha} + (-1)^{k+m}e^{ikl_{12}\alpha}\right]\frac{d\alpha}{\alpha}$$
$$+ 2i\int_0^{\infty} J_{m+1}(\varepsilon_1\alpha)J_{k+1}(\varepsilon_1\alpha)\gamma(\alpha)\left[e^{-ikl_{12}\alpha} + (-1)^{k+m}e^{ikl_{12}\alpha}\right]\frac{d\alpha}{\alpha} (\equiv \tilde{P}_{mk}^{12} + \hat{P}_{mk}^{12})$$

Then it is obvious that $P_{mk}^{21} = (-1)^{m+k}P_{mk}^{12}$. From the definition of $\hat{P}_{mk}^{12}$, it follows that the estimate of this matrix norm is identical to (36). As for $\tilde{P}_{mk}^{12}$, the corresponding integrals are tabulated (see [39]) and then

$$\tilde{P}_{mk}^{12} = \left[1 - i^{m+k}\right]\left(\frac{2a_1}{l_{12}}\right)^{m+k+2}$$
$$\times \sum_{p=0}^{\infty} \frac{\Gamma^2(p+\frac{m+k+3}{2})(\Gamma(p+\frac{m+k}{2}+2)(\Gamma(p+\frac{m+k}{2}+1)}{\Gamma(p+m+k+3)\Gamma(p+m+2)\Gamma(p+k+2)\Gamma(p+1)}\left(\frac{2a_1}{l_{12}}\right)^{2p} \tag{56}$$

Note that the term of this series decreases as $O\left[\frac{1}{p^2}\left(\frac{2a_1}{l_{12}}\right)^{2p}\right]$ for $p \to \infty$, i.e., the series converges for all $l_{12} \geq 2a_1$. The desired result follows from the representation (56). Namely, the norm of matrix $\{P^{12}_{mk}(m+1)\}^{\infty}_{m,k=0}$ is:

$$||\tilde{P}_{12}||^2 = \sum_{k=0}^{\infty}\sum_{m=0}^{\infty} |P_{mk}|^2(m+1)^2 < \infty$$

and is bounded only under the strict requirement $l_{12} > 2a_1$. Hence the interaction matrices $\{P^{12}_{mk}(m+1)\}$ generate the completely continuous operators only if the strips do not touch each other $l_{12} > 2a_1$.

The verification reveals the fact that the interaction operators P_{jq} are always completely continuous if the strip contours and their prolongations do not cross each other. Thus, it has been shown that the infinite SLAE (51) is the Fredholm equation of the second kind, and its approximate solution may be obtained by the truncation method with any preassigned accuracy.

On the basis of elaborated numerical algorithms, the electromagnetic characteristics have been calculated for various structures of practical interest such as 2- and 4- mirror strip open resonators (OR) (Figs.2-5). Figures 2 and 3 illustrate the frequency dependence of the total scattering cross-section $\frac{\sigma_s^H}{4a}$ for the 2-mirror strip OR at different values of the parameter $\delta = h/a$ and the incidence angle θ_0. The current distribution on the strips and the field pattern are also shown in Fig.2. These results (Fig.2, curve 1) completely agree with those obtained on the basis of mathematically rigorous method [17]. As is seen from Figs.2 and 3, the dependence $\frac{\sigma_s^H}{4a}$ versus $\kappa = ka$ has sharp resonances due to the excitation of quasi-eigen oscillations inside the resonator. It is known [17] that the values κ corresponding to the resonances for $\delta \leq 1$ are always less than $\frac{m\pi}{2}$ (for the normal wave incidence, $\kappa < (m - \frac{1}{2})\pi$, $m = 1, 2, \ldots$). In this case, the resonances due to the excitation of "piston-like" natural mode prevail. If $\delta \geq 1$, the resonances occur mainly due to the higher order modes for κ values close to $n\pi/(2\delta)(n = 1, 2, \ldots)$ as shown in Fig.4.

The frequency dependences of $\frac{\sigma_s^H}{4a}$ for the 4-mirror strip OR (of identical strips) are presented in Figs.5 and 6. These curves have sharp resonances which occur when the frequency parameter is close to $\pi\sqrt{m^2+n^2}$ where $m, n = 0, 1, 2 \ldots$; in other words, at frequencies close to the natural frequencies of the interior domain of resonator. Besides, it is natural to assume the presence of natural frequencies of the 2-mirror OR, because the 4-mirror OR may be considered as a system of two coupled 2-mirror strip OR. The frequency dependences for the 4-mirror strip OR shown in Fig.6 confirm this consideration. The resonances of $\frac{\sigma_s^H}{4a}$ in low-frequency range (they may be named principal resonances) are just observed at frequencies shifted to lower ones with respect to those of the 2-mirror strip OR (compare the curves in Fig.2 ($\delta = 1$) and Fig.6). Figure 7 shows the surface current distribution on the mirrors of the 4-mirror OR for different values of its parameters.

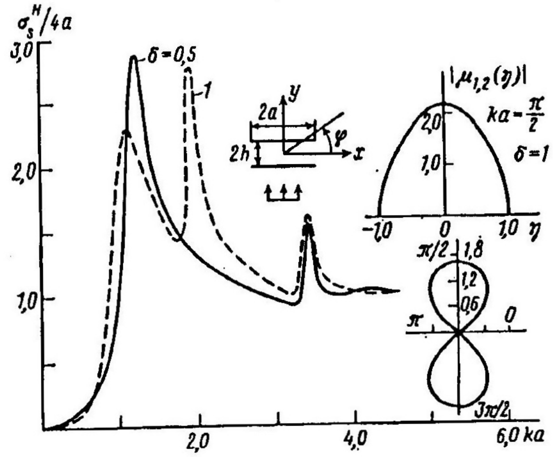

Fig. 2. Frequency dependence of the total scattering cross-section (SCS) $\sigma_s^H/4a$ for a 2-mirror strip open resonator (OR) with $\delta\,(\equiv h/a) = 0.5$ (solid line) and $\delta = 1.0$ (dashed line). Current distribution function $\mu(\eta)$ on the strips and radiation pattern (RP) for $ka = \pi/2$ and $\delta = 1.0$.

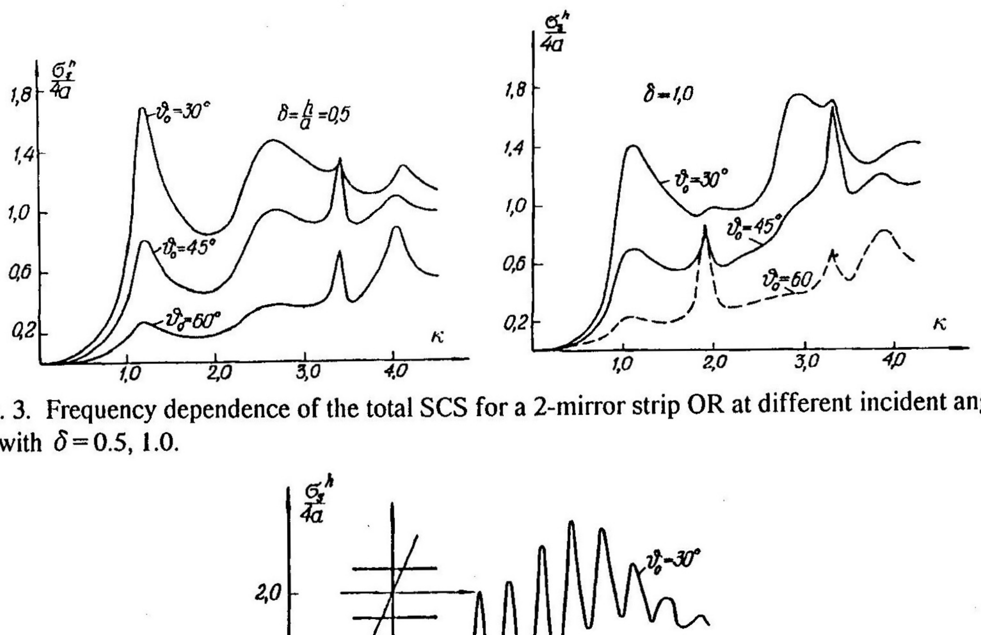

Fig. 3. Frequency dependence of the total SCS for a 2-mirror strip OR at different incident angles θ_0 with $\delta = 0.5$, 1.0.

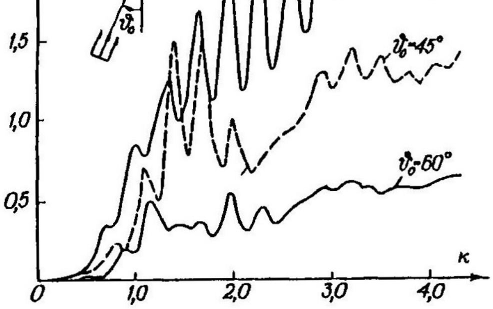

Fig. 4. Frequency dependence of the total SCS for a 2-mirror strip OR at different incident angles θ_0 with $\delta = 5.0$.

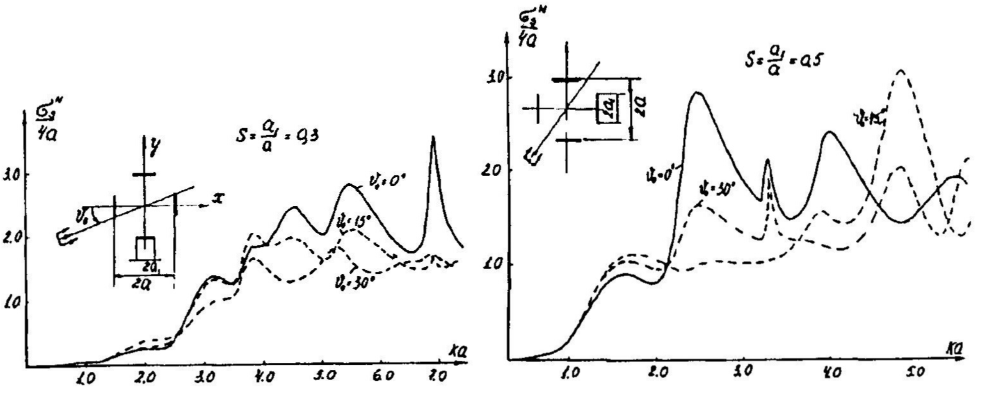

Fig. 5. Frequency dependence of the total SCS for a 4-mirror strip OR at different incident angles θ_0 with $S(\equiv a_1/a) = 0.3, 0.5$.

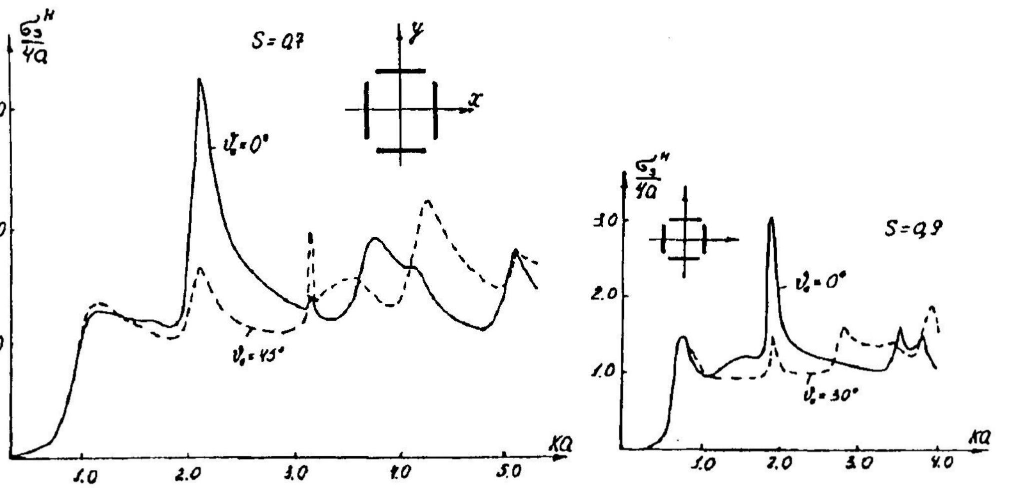

Fig. 6. Frequency dependence of the total SCS for a 4-mirror strip OR at different incident angles θ_0 with $S = 0.7$.

4. WAVE SCATTERING BY A FLAT STRIP

In this section, we consider the H-polarized plane electromagnetic wave scattering by a single flat strip as a special case. This problem is formulated in terms of the infinite SLAE of the second kind (51) with $j = 1$. Then all the linear dimensions of the strips $\{a_q\}_{q\neq1}^{N} = 0$ besides one of them ($j = 1$), and all the mutual interaction matrices $\{P_{mk}^{jq}\}_{m,k=o}^{\infty} = 0$. This is a classical problem in diffraction theory, and a considerable number of investigations have been devoted to seeking its rigorous and approximate solutions. The most complete bibliography on this matter can be found in monographs [42–45]. It should be noted [21, 46] that the solution methods developed in recent publications are a special case of the present

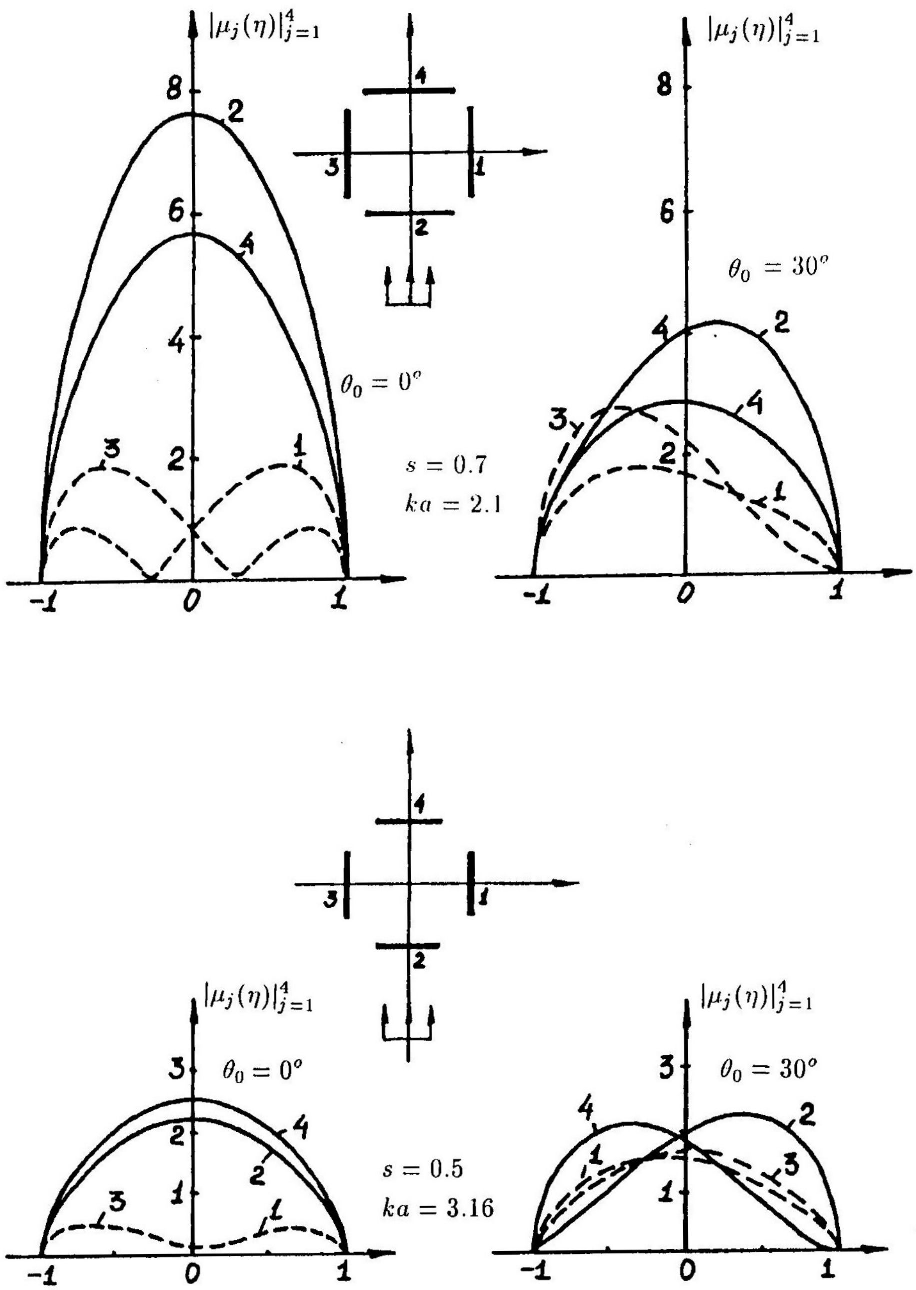

Fig. 7. Current distribution function on the strips of a 4-mirror strip OR at different values of S, θ_0 and ka.

approach. By means of the MM, the original DIE was reduced to the infinite SLAE of the first kind with slowly decaying matrix elements [21] and to the SLAE of the second kind in [46]. However, the solvability of these equations was not thoroughly investigated.

For the problem considered here, the infinite SLAE of the second kind takes the form :

$$\begin{cases} (-1)^k x_{2k}^{(1)} - \sum_{m=0}^{\infty}(-1)^m(2m+1)d_{2k,2m}^{(1)}x_{2m}^{(1)} = -2i\frac{\sqrt{1-\alpha_0^2}}{\alpha_0}\frac{J_{2k+1}(\varepsilon\alpha_0)}{\alpha_0} \\ (-1)^k x_{2k+1}^{(1)} - \sum_{m=0}^{\infty}(-1)^m(2m+2)d_{2k+1,2m+1}^{(1)}x_{2m+1}^{(1)} = 2\frac{\sqrt{1-\alpha_0^2}}{\alpha_0^2}J_{2k+1}(\varepsilon\alpha_0) \end{cases} \quad (57)$$

It should be noted that on the basis of the SLAE (57), the integral equations of the second kind for the unknown Fourier transforms can also be obtained. For verifying this statement, let us represent the Fourier transform function $h_1(\pm\alpha)$ as a sum of the even and odd parts, i.e., $h_1(\pm\alpha)$ $(= h_1^+(\alpha) \pm h_1^-(\alpha)/2)$. In view of (19) (under $\nu = 1/2$), we obtain

$$h_1^+(\alpha) = 2\pi\sum_{m=0}^{\infty}(-1)^m x_{2m}^{(1)}(2m+1)\frac{J_{2k+1}(\varepsilon_1\alpha)}{\varepsilon_1\alpha}$$
$$h_1^-(\alpha) = -2\pi i\sum_{m=0}^{\infty}(-1)^m x_{2m+1}^{(1)}(2m+2)\frac{J_{2k+2}(\varepsilon_1\alpha)}{\varepsilon_1\alpha} \quad (58)$$

If one substitutes $\{x_m^{(1)}\}_{m=0}^{\infty}$, the solution of the SLAE (57), into the series (58) and recalls that the quantities $d_{km}^{(1)}$ are defined by the formula (27), then

$$h_1^+(\alpha) = -2i\frac{2\pi}{\varepsilon_1}\frac{\sqrt{1-\alpha_0^2}}{\alpha\alpha_0}\sum_{k=0}^{\infty}(2k+1)J_{2k+1}(\varepsilon_1\alpha)J_{2k+1}(\varepsilon_1\alpha_0)$$
$$+\frac{4\pi}{\varepsilon_1}\sum_{k=0}^{\infty}(2k+1)\frac{J_{2k+1}(\varepsilon_1\alpha)}{\alpha}\sum_{k=0}^{\infty}(-1)^m(2k+1)x_{2m+1}^{(1)}\int_0^{\infty}\gamma(\beta)J_{2k+1}(\varepsilon_1\beta)J_{2m+1}(\varepsilon_1\beta)\frac{d\beta}{\beta}$$

Through interchanging the order of integration and summation (this procedure is possible due to the uniform convergence of the series and the absolute integrability of the integrands) and introducing the notation

$$K^+(\alpha,\beta) = \frac{1}{\alpha\beta}\sum_{k=0}^{\infty}(2k+1)J_{2k+1}(\varepsilon_1\alpha)J_{2k+1}(\varepsilon_1\beta) \quad (59)$$

the following integral equation is obtained:

$$h^+(\alpha) = -i\frac{4\pi}{\varepsilon_1}\sqrt{1-\alpha_0^2}K^+(\alpha,\alpha_0) + 2\int_0^{\infty}\beta\gamma(\beta)K^+(\alpha,\beta)h_1^+(\beta)d\beta \quad (60)$$

The expression for $h^-(\alpha)$ can be obtained in a similar fashion:

$$h^-(\alpha) = \frac{4\pi}{\varepsilon_1}\sqrt{1-\alpha_0^2}K^-(\alpha,\alpha_0) + 2\int_0^{\infty}\beta\gamma(\beta)K^-(\alpha,\beta)h_1^-(\beta)d\beta \quad (61)$$

where

$$K^-(\alpha,\beta) = \frac{1}{\alpha\beta}\sum_{k=0}^{\infty}(2k+2)J_{2k+2}(\varepsilon_1\alpha)J_{2k+2}(\varepsilon_1\beta) \tag{62}$$

Equations (60) and (61) are the Fredholm integral equations of the second kind. The kernel representation $K^{\pm}(\alpha,\beta)$ may be simplified to

$$K^+(\alpha,\beta) = \frac{\varepsilon_1}{2(\alpha^2-\beta^2)}\left[\alpha J_1(\varepsilon_1\alpha)J_0(\varepsilon_1\beta) - \beta J_0(\varepsilon_1\alpha)J_1(\varepsilon_1\beta)\right]$$
$$K^-(\alpha,\beta) = \frac{\varepsilon_1}{2(\alpha^2-\beta^2)}\left[\beta J_1(\varepsilon_1\alpha)J_0(\varepsilon_1\beta) - \alpha J_0(\varepsilon_1\alpha)J_1(\varepsilon_1\beta)\right] \tag{63}$$

by means of summation of the series in (59) and (62). Note that the Fredholm equations of the second kind (60) and (61) with the kernels (59) and (63) are identical to the analogous equations obtained by the Riemann-Hilbert problem method [38,43] and by the method of the Abel integral transform [49].

Determining the coefficients $\{x_n^{(1)}\}_{n=0}^{\infty}$ from the SLAE (57), the surface current density function and the radiation pattern (RP) can be evaluated by (49) and (50), respectively. In particular, the RP is found to be:

$$\Phi(\varphi) = -\frac{-\pi\sin\varphi}{2\varepsilon_1\cos\varphi}\sum_{n=0}^{\infty}(-i)^n(n+1)x_n^{(1)}J_{n+1}(\varepsilon_1\cos\varphi)$$

from which the total scattering cross-section (SCS) can be found using the formula (49b) as

$$\frac{\sigma_S^H}{4a_1} = \frac{\pi}{\varepsilon_1^2}\frac{\sin\theta_0}{\cos\theta_0}\sum_{n=0}^{\infty}(n+1)J_{n+1}(\varepsilon_1\cos\theta_0)Re\left\{(-i)^n x_n^{(1)}\right\}$$

The well-conditioned algorithms for computation of these characteristics have been developed to check the efficiency of the obtained solution. The infinite SLAE (57) is solved by the truncation method for arbitrary physical parameters using the Crout algorithm [53]. However, as follows from the estimation of the norm of the matrix $\{d_{kn}^{(1)}\}_{k,n=0}^{\infty}$ (see (36)) for the frequency parameter $\varepsilon_1 = ka_1 < 1$, i.e., in low-frequency range, this system of equations can be solved by simple iterations.

Improper integrals involved in the expressions for matrix elements d_{kn}^1 are computed by the Simpson method (see the details in Section 4). In order to keep the error less than 10^{-5}, the infinite integration interval may be replaced by the finite one (0,25).

Tables 1 and 2 present the correlation between the truncation order N_1 of the infinite SLAE (57) and the accuracy of the field $H_z^{(1)}$ and the total scattering cross-section σ_s^h in the far zone. This is to demonstrate the good convergence of the sequence of solutions when the truncation order N_1 increases. Similar results have also been obtained for other values of the parameters ε_1 and θ_0. Numerous calculations show that for the reasonable accuracy, the truncation order N_1 should be chosen as $entier(\varepsilon_1)$.

The current density functions on the strip at various incidence angles θ_0 and normalized widths $\varepsilon_1 = ka_1$ of a strip are shown in Fig.8. These results agree well with those given in the monograph [47]. Frequency dependences of the total SCS and the RP at different incident angles θ_0 are shown in Fig.9. It should only be pointed out that if $\varepsilon_1 \to \infty$, then the total scattering cross-section tends to the geometric-optical limit $\sigma_S^H/4a_1 \underset{\varepsilon_1\to\infty}{\sim} \sin\theta_0$.

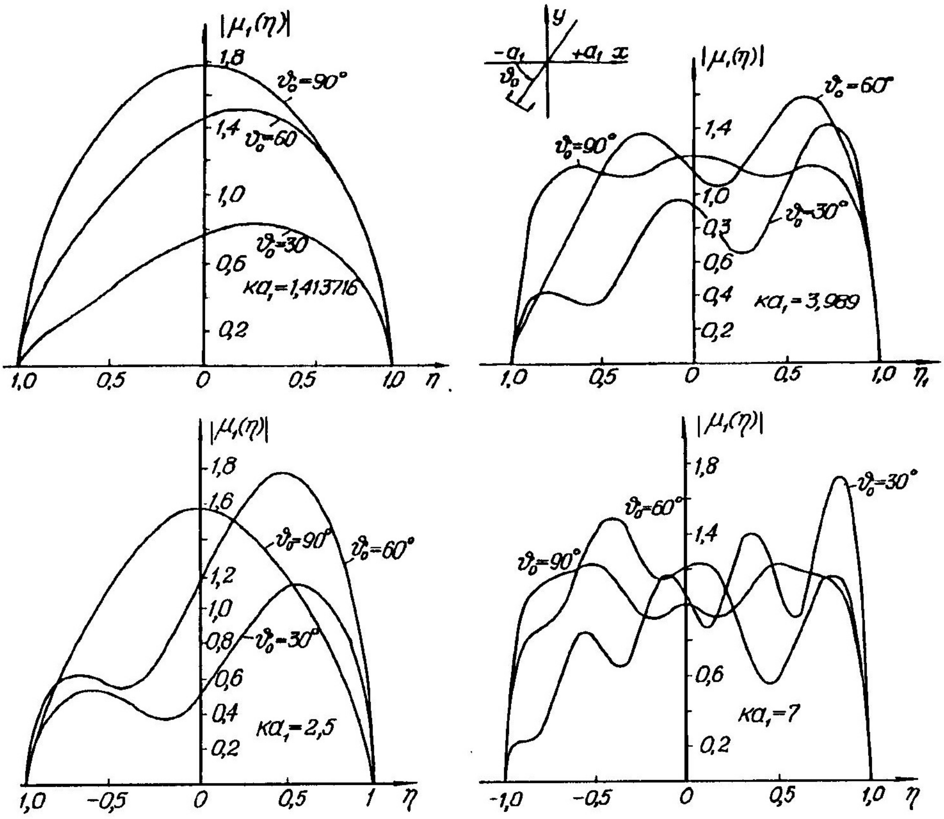

Fig. 8. Current distribution function on the strip at different values of θ_0 and $\varepsilon_1 (\equiv ka_1)$.

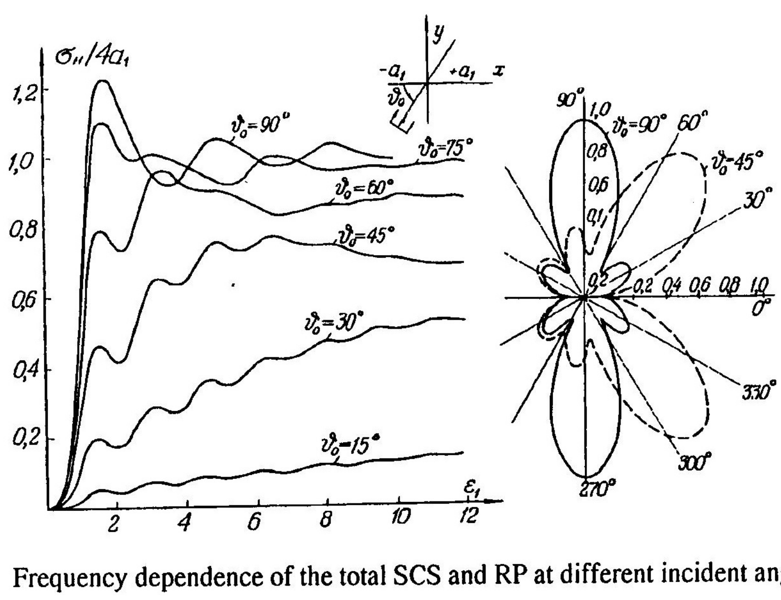

Fig. 9. Frequency dependence of the total SCS and RP at different incident angles θ_0.

Table 1. Computational accuracy of RP for various values of the truncation order N_1 of the infinite SLAE.

$\theta_0 = 45^0 \quad \varepsilon_1 = ka_1 = \sqrt{28}$						
$Re\Phi(\varphi)$						
φ \ N_1	1	2	3	4	5	10
2.5^o	0.02079	0.03344	0.03433	0.03434	0.03434	0.03434
45^o	0.63133	0.73543	0.73765	0.73766	0.73766	0.73766
90^o	-0.1562	-0.1864	-0.1857	-0.1857	-0.1857	-0.1857
135^o	0.12825	0.188556	0.18950	0.18950	0.18950	0.18950
177.5^o	0.00582	0.01206	0.01267	0.01269	0.01269	0.01269
$Im\Phi(\varphi)$						
2.5^o	-0.01632	-0.016565	-0.0164	-0.01641	-0.01641	-0.01641
45^o	-0.26932	-0.18202	-0.17636	-0.17628	-0.17628	-0.17628
90^o	0.15452	0.08510	0.08096	0.08091	0.08091	0.08091
135^o	-0.11856	-0.02106	-0.01527	-0.01519	-0.01519	-0.01519
177.5^o	0.0081	0.00599	0.00630	0.00630	0.00630	0.00630

Table 2. Computational accuracy of the total SCS $\sigma_s^H/4a_1$ for various values of N_1.

	$\varepsilon_1 = ka_1 = \sqrt{28}$	
	$\theta_0 = 45^o$	$\theta_0 = 90^o$
N_1	$\sigma_S^H/4a_1$	$\sigma_S^H/4a_1$
1	0.63133	1.037816
2	0.73543	1.042125
3	0.737654	1.04048
4	0.737655	1.04045
5	0.73765	1.040451
10	0.737655	1.040451

5. H-POLARIZED PLANE WAVE SCATTERING BY A RECTANGULAR CYLINDER

Among a number of cylindrical structures treatable by the proposed approach, a cylinder of rectangular cross section is typical. This is a sole structure formed by strips of different widths and joined at the same (right) angles (the fact of identical corner geometries is important for application of the method in the present version). In this case, the proper behavior of the current at the cylinder's edge point can be described by setting $\nu = 2/3$ in (10). It is obvious that the symmetry of the structure can simplify the computational algorithm. The scatterer symmetry leads to the existence of a certain relationship between the elements of separate matrix blocks, which simplifies calculation of the whole SLAE matrix. When there is a higher degree of symmetry, more relationships hold. Thus blocks of the SLAE matrix have simpler relationships in case of a square prism than of a rectangular one, in other words, the elements of one block are expressed more easily via elements of others for the square prism. This is true for any regular polygonal cylinders.

For the rectangular prism ($N = 4, \nu = 2/3$), the quantities A_{sq}, B_{sq},and C_{sq} in the expressions of matrix elements of the SLAE (47) have the form:

$$\{A_{sq}\}_{s,q=1}^{4} = \begin{bmatrix} 0 & -\alpha & \sqrt{1-\alpha^2} & \alpha \\ \alpha & 0 & -\alpha & \sqrt{1-\alpha^2} \\ \sqrt{1-\alpha^2} & \alpha & 0 & -\alpha \\ -\alpha & \sqrt{1-\alpha^2} & \alpha & 0 \end{bmatrix}$$

$$\{B_{sq}\}_{s,q=1}^{4} = \begin{bmatrix} 0 & -\sqrt{1-\alpha^2} & \alpha & \sqrt{1-\alpha^2} \\ \sqrt{1-\alpha^2} & 0 & \sqrt{1-\alpha^2} & -\alpha \\ -\alpha & \sqrt{1-\alpha^2} & 0 & -\sqrt{1-\alpha^2} \\ -\sqrt{1-\alpha^2} & -\alpha & -\sqrt{1-\alpha^2} & 0 \end{bmatrix}$$

$$\{D_{sq}\}_{s,q=1}^{4} = \begin{bmatrix} 0 & D_{12} & \sqrt{1-\alpha^2} & D_{14} \\ D_{21} & 0 & D_{23} & \sqrt{1-\alpha^2} \\ \sqrt{1-\alpha^2} & D_{32} & 0 & D_{34} \\ D_{41} & \sqrt{1-\alpha^2} & D_{43} & 0 \end{bmatrix}$$

$$D_{12}(\alpha) = D_{34}(\alpha) = D_{14}(-\alpha); D_{14}(\alpha) = D_{32}(\alpha) = \alpha\cos\varphi_{12} + \sqrt{1-\alpha^2}\sin\varphi_{12}$$
$$D_{41}(\alpha) = D_{23}(\alpha) = D_{21}(-\alpha); D_{21}(\alpha) = D_{23}(\alpha) = \alpha\sin\varphi_{12} + \sqrt{1-\alpha^2}\cos\varphi_{12}$$

The block structure of the SLAE is illustrated in Fig.10. Four diagonal cells describe the properties of four single-strip scatterers constructing the cylinder. Twelve non-diagonal cells account for the mutual interaction of cylinder facets. The numbers of lines and columns in these matrices correspond to the truncation parameters, which, in turn, depend on the electrical dimension of the scatterer. Besides there are nine extra matrices caused by the constants C^q: four extra matrices have four columns and the row number is defined by the truncation parameter, four other matrices have four rows but their column number is defined by the truncation parameter and the ninth matrix has the size 4×4.

First of all, let us show that the matrices $\{P_{mn}^{sq}\}_{m,n=0}^{\infty}$ give completely continuous operators $P_{cq}(s \neq q = 1,2,3,4)$ (see Section 2). Two kinds of operators may be distinguished: the operators describing the parallel-facet interaction and the operators of perpendicular-facet interaction. The norms of the interaction operators $P_{24}, P_{13}, P_{12}, P_{31}$ for non-adjacent facets are bounded in l_2. It is sufficient to estimate the norm of one of them because the rest norms of operators are similar. Now we obtain

$$P_{mn}^{24} = \sqrt{\beta_m\beta_n}\varepsilon_2^2 2^{2\nu-1}\int_{-\infty}^{\infty} J_{m+\nu+1/2}(\varepsilon_2\alpha)J_{n+\nu+1/2}(\varepsilon_2\alpha)(-1)^n\frac{\sqrt{1-\alpha^2}}{(\varepsilon_2\alpha)^{2\nu+1}}e^{i\varepsilon_1\sqrt{1-\alpha^2}}d\alpha$$
$$= \sqrt{\beta_m\beta_n}\frac{2^{2\nu-1}}{\varepsilon_2^{2\nu-1}}((-1)^m + (-1)^n)$$
$$\times\int_0^{\infty} J_{m+\nu+1/2}(\varepsilon_2\alpha)J_{n+\nu+1/2}(\varepsilon_2\alpha)e^{i\varepsilon\sqrt{1-\alpha^2}}\frac{\sqrt{1-\alpha^2}}{\alpha^{2\nu+1}}d\alpha \qquad (64)$$

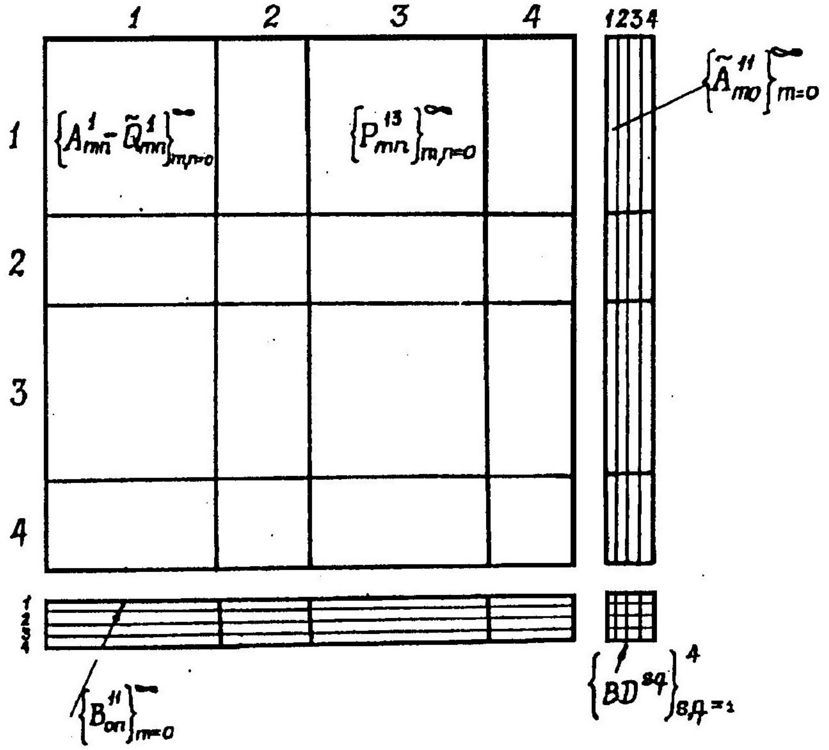

Fig. 10. Block structure of the SLAE.

It is known that the product of the Bessel functions can be expanded in terms of a uniformly convergent power series of the argument [50]:

$$J_{k+\nu+1/2}(\varepsilon_2\alpha)J_{m+\mu+1/2}(\varepsilon_2\alpha) = \frac{1}{\sqrt{\pi}}\sum_{p=0}^{\infty} Q_{k,m,p}^{(\nu,\mu)}(\varepsilon_2\alpha)^{2p+k+m+\nu+\mu+1},$$

$$Q_{k,m,p}^{(\nu,\mu)} = \frac{(-1)^p\Gamma(p+\frac{k+m+\nu+\mu}{2}+1)\Gamma(p+\frac{k+m+\nu+\mu+3}{2})}{\Gamma(p+1)\Gamma(p+k+\nu+\frac{3}{2})\Gamma(p+k+\mu+\frac{3}{2})\Gamma(p+k+\mu+\nu-1)} \tag{65}$$

Using this expression, (64) can be estimated as follows:

$$|P_{mn}^{24}| \le 2\sqrt{\beta_m\beta_n}\frac{2^{2\nu-1}}{\varepsilon_2^{2\nu-1}}\left\{|Q_{k,m,p}^{(\nu,\mu)}|\varepsilon_2^{n+m}\left[\frac{\delta_{n0}\delta_{m0}}{\varepsilon_1}|K_1(\varepsilon_1)| + \frac{e^{-\varepsilon_1}}{2}B\left(\frac{n+m+1}{2},\frac{3}{2}\right)\right] + \sum_{p=1}^{\infty}|Q_{k,m,p}^{(\nu,\mu)}|\varepsilon_2^{2p+n+m}\left\{\frac{e^{-\varepsilon_1}}{2}B\left(p+\frac{n+m+1}{2},\frac{3}{2}\right) + \left|\frac{\partial^{2p+m+n-1}}{\partial\varepsilon_1^{2p+m+n-1}}\left[\frac{1}{\varepsilon_1}K_2(\varepsilon_1)\right]\right|\right\}\right\} \tag{66}$$

where $B(\alpha,\beta)$ is the Beta function; $\{K_j(\varepsilon_1)\}$ for $j=1,2$ is the MacDonald function; δ_{k0} is the Kronecker delta. Based on this estimation, it is easy to show that:

$$\|P^{24}\| \le \left\{\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}|P_{mn}^{24}|^2\right\}^{1/2} < \infty \tag{67}$$

To estimate the norm of the interaction operators of the adjacent facets, it is sufficient to do this for the operator P_{12}:

$$P_{mn}^{12} = \sqrt{\beta_n\beta_m}(-1)^n \frac{2^{2\nu-1}}{(\varepsilon_1\varepsilon_2)^{\nu-1/2}} \int_0^\infty \frac{\alpha e^{i\varepsilon_1\sqrt{1-\alpha^2}}}{\alpha(\sqrt{1-\alpha^2})^{\nu+1/2}} J_{m+\nu+1/2}(\varepsilon_2\alpha)$$
$$\times J_{m+\nu+1/2}(\varepsilon_1\sqrt{1-\alpha^2}) \left[-e^{-i\varepsilon_2\alpha} + (-1)^m e^{i\varepsilon_2\alpha}\right] d\alpha$$

Let

$$N_{mn}^{12}(\alpha) = \frac{\alpha e^{i\varepsilon_1\sqrt{1-\alpha^2}}}{(\alpha\sqrt{1-\alpha^2})^{\nu+1/2}} J_{m+\nu+1/2}(\varepsilon_2\alpha) J_{m+\nu+1/2}(\varepsilon_1\sqrt{1-\alpha^2}) \underset{|\alpha|\to\infty}{\sim} O\left(\frac{1}{\alpha^{2\nu+1}}\right)$$

Such a slow decay of the function $N_{mn}^{12}(\alpha)$ requires convergence acceleration. To this end, let us investigate the asymptotic behavior of the function $N_{mn}^{12}(\alpha)$. We find that

$$N_{mn}^{12}(\alpha) \underset{|\alpha|\to\infty}{\sim} \frac{i^n e^{-\varepsilon_1\alpha}}{\alpha^{2\nu}} I_{m+\nu+1/2}(\varepsilon_1\alpha) J_{m+\nu+1/2}(\varepsilon_2\alpha) [\equiv \tilde{N}_{mn}^{12}(\alpha)]$$

where $I_{m+\nu+1/2}(\varepsilon_1\alpha)$ is the modified Bessel function. Then we can express P_{nm}^{12} as

$$P_{nm}^{12} = \tilde{P}_{nm}^{12} - \sqrt{\beta_n\beta_m}(-1)^n \frac{2^{2\nu-1}}{(\varepsilon_1\varepsilon_2)^{\nu-1/2}} \int_0^\infty (N_{mn}^{12} - \tilde{N}_{mn}^{12})(e^{i\varepsilon_2\alpha}(-1)^m - e^{-i\varepsilon_2\alpha}) d\alpha$$

where

$$\tilde{P}_{nm}^{12} = (-1)^n \sqrt{\beta_n\beta_m} \frac{2^{2\nu-1}}{(\varepsilon_1\varepsilon_2)^{\nu-1/2}} \int_0^\infty \tilde{N}_{mn}^{12} \left(e^{i\varepsilon_2\alpha}(-1)^m - e^{-i\varepsilon_2\alpha}\right) d\alpha$$

If $\varepsilon_1 \geq \varepsilon_2$ then

$$\tilde{P}_{nm}^{12} = (-1)^n \sqrt{\beta_n\beta_m} \frac{2^{2\nu-1}}{(\varepsilon_1\varepsilon_2)^{\nu-1/2}} \left[B_{mn}^{(+)} - (-1)^m B_{mn}^{(-)}\right],$$
$$B_{mn}^{(\pm)} = (\mp i)^{m+\nu+\frac{1}{2}} \left\{ \frac{(\mp i\frac{\varepsilon_2}{\varepsilon_1})^{m+\nu+\frac{1}{2}}}{2^{m+2}\pi} \frac{\Gamma(\nu-m-1)\Gamma(n+m+2)}{\Gamma(n-m+2\nu)\Gamma(m+\nu+\frac{3}{2})}\right.$$
$$\times {}_3F_2\left(1-2\nu+m+n, m+n+2, m+\nu+1; m-\nu+2, 2m+2\nu+2; \mp i\frac{\varepsilon_2}{\varepsilon_1}\right)$$
$$+ \sqrt{\pm i\frac{\varepsilon_1}{\varepsilon_2}} \frac{1}{\pi(\pm 2i\varepsilon_2)^{1-2nu}} \frac{\Gamma(m+1-\nu)\Gamma(2\nu)}{\Gamma(m+3\nu+1)}$$
$$\left.\times {}_3F_2\left(-n-\nu, n+\nu+1, 2\nu; m+3\nu+1; \nu-m; \mp i\frac{\varepsilon_2}{\varepsilon_1}\right)\right\} \tag{68}$$

where ${}_3F_2(\ldots,\ldots,\ldots,)$ is a generalized hypergeometric function. The corresponding hypergeometric series is absolutely convergent for $\varepsilon_1 \geq \varepsilon_2$, and the asymptotic behavior of the series term is found to be $O\left[n^{-2\nu-1}\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^n\right]$.

The asymptotic limit of the matrix elements P^{12}_{mn} for $n, m \to \infty$ is derived by the asymptotic behavior of the quantities $\tilde{P}^{12}_{mn}$. Even in the most unfavourable case due to the properties of the hypergeometric function, the following expressions for $\tilde{P}^{12}_{mn}$ can be obtained:

$$P^{12}_{mn} \underset{m,n\to\infty}{\approx} \tilde{P}^{12}_{mn} \underset{m,n\to\infty}{=} O\left[(mn)^{-3\nu}\right] (m \neq n),$$
$$P^{12}_{nn} \underset{n\to\infty}{\approx} \tilde{P}^{12}_{nn} \underset{n\to\infty}{=} O(n^{2\nu}), \quad \nu = 2/3 \tag{69}$$

Thus the operators corresponding to the matrices such as $\{P^{12}_{mn}\}^{\infty}_{n,m=0}$ are completely continuous in the space l_2. The system of linear algebraic equations (47) is solvable by the truncation method. Two following statements should be made:

First, the complete continuity of the interaction operators for adjacent facets is provided by the correct account of the singularity of current functions at the edges (at the joints);

Second, the continuous transition from a rectangular cylinder to a flat strip does not take place because, as follows from (66), the norm of the operator P_{24} increases unlimitedly if the thickness a_1 tends to zero ($\varepsilon_1 \to 0$).

The change to flat strip structures (Fig.1,b,c,g) must be realized in accordance with Section 3.

Now let us make some remarks on numerical realization of the proposed approach. Recall that the symmetry of structures provides simple relations between the interaction matrices $\{P^{jq}_{kn}\}^{\infty}_{k,m=0}$. For a rectangular cylinder, this enables one to calculate only four interaction matrices instead of twelve ones.

The principal procedure in numerical computations is the calculation of the matrices $\{Q^{(\nu+1/2)}_{kn}\}^{\infty}_{k,n=0}$, $\{P^{jq}_{kn}\}^{\infty}_{n,m=0}$ for the infinite SLAE (47). The elements of these matrices are represented by improper integrals, i.e., integrals over infinite intervals. However, when the integration parameter increases, the integrands in $Q^{(\nu+1/2)}_{kn}$ and $P^{j,j+1}_{kn}$ decrease as $O(1/\alpha^{2\nu+3})$ and $O(1/\alpha^{2\nu+1})$, respectively. Then these integrals can be calculated directly by Simpson's rule. However, they can be also reduced to series representations. A brief summary of this procedure is presented below.

Let the matrix elements be represented in the form:

$$Q^{(\nu+1/2)}_{kn} = \frac{1}{2}\sqrt{\beta^{(\nu+1/2)}_{k}\beta^{(\nu+1/2)}_{n}}\left[B^{(1)}_{kn} + B^{(2)}_{kn}\right]\left[1 + (-1)^{(k+n)}\right] \tag{70}$$

where

$$B^{(1)}_{kn} = K_{\nu+1/2}(\varepsilon_j)\int_0^1 J_{k+\nu+1/2}(\varepsilon_j\alpha)J_{n+\nu+1/2}(\varepsilon_j\alpha)\gamma(\alpha)\frac{d\alpha}{\alpha^{2\nu}}, \tag{71}$$

$$B^{(2)}_{kn} = K_{\nu+1/2}(\varepsilon_j)\int_1^\infty J_{k+\nu+1/2}(\varepsilon_j\alpha)J_{n+\nu+1/2}(\varepsilon_j\alpha)\gamma(\alpha)\frac{d\alpha}{\alpha^{2\nu}} \tag{72}$$

After substitution of (65) into (71) and in view of $\gamma(\alpha) = 1 + \dfrac{i}{\alpha}\sqrt{1-\alpha^2}$, the resultant integral (see [51]) for $B^{(1)}_{kn}$ can be evaluated in closed form and the result is:

$$B^{(1)}_{kn} = K_{\nu+1/2}(\varepsilon_j)\frac{\varepsilon_j^{2\nu+1}}{\sqrt{\pi}}\sum_{p=0}^{\infty} Q^{\nu\nu}_{k,n,p}\varepsilon_j^{2p+k+n}\left[\frac{1}{2p+k+n+2} + \frac{i\sqrt{\pi}\Gamma(p+\frac{k+n+1}{2})}{4\Gamma(p+2+\frac{k+n}{2})}\right] \tag{73}$$

The expression (33) for the function $\gamma_1(\alpha) = \alpha\gamma(\alpha)$ under $\alpha \geq 1$ and a contour integral representation [51] for products of the Bessel functions are necessary for calculation of the integral (72):

$$J_{k+\nu+1/2}(\varepsilon_j\alpha)J_{n+\nu+1/2}(\varepsilon_j\alpha) = \frac{1}{2\pi}\int\limits_{c-i\infty}^{c+i\infty} \frac{\Gamma(-S)}{\sqrt{\pi}} \frac{\Gamma(S+\frac{k+n+2\nu+1}{2})}{\Gamma(S+k+\nu+\frac{3}{2})} \times \frac{\Gamma(S+\frac{k+n+2\nu+3}{2})(\varepsilon_j\alpha)^{2s+k+n+2\nu+1}}{\Gamma(S+k+n+\nu+\frac{3}{2})\Gamma(S+k+n+2\nu+2)}ds \tag{74}$$

where

$$-Re\left(\frac{k+n}{2}+\nu+1\right) < ReS = c < O$$

Computation of the appearing Euler integrals of the first kind [51] and the contour integration on the basis of the residue theory yields

$$B_{kn}^{(2)} = \frac{K_{\nu+1/2}(\varepsilon_j)}{2\sqrt{\pi}}\left\{\sum_{p=1}^{q-1}(-1)^{p+1}C_{1/2}^{p}\varepsilon_j^{2\nu}\left[L_{knp}^{\nu}\varepsilon_j^{2p-1}+\sum_{m=0}^{\infty}Q_{k,n,m}^{(\nu,\nu)}\frac{\varepsilon_j{}^{2m+k+n+1}}{m+q-p}\right]\right.$$
$$+\sum_{p=q}^{\infty}(-1)^{p}C_{1/2}^{p}\left\{\frac{L_{knp}^{\nu}\varepsilon_j^{2p+2\nu-1}}{\Gamma(p-q+1)}\left[2ln\varepsilon_j+\psi(p+\nu)+\psi(p+\nu+\frac{1}{2})\right.\right.$$
$$-\psi(p-q+1)-\psi\left(p+\frac{k-n+1}{2}+\nu\right)-\psi\left(p+\frac{n-k+1}{2}+\nu\right)$$
$$\left.\left.\left.-\psi\left(p+\frac{n+k}{2}+2\nu+1\right)\right]+\sum_{m=0,m\neq p-q}^{\infty}\frac{Q_{k,n,m}^{(\nu,\nu)}\varepsilon_j^{2m+k+n+2\nu+1}}{m+q-p}\right\}\right\} \tag{75}$$

where $\psi(x) = [ln\Gamma(x)]'$ is the ψ-function, $q = 1 + \frac{k+n}{2}$, and L_{knp}^{ν} quantities are expressed as

$$L_{knp}^{\nu} = \frac{\Gamma(p+\nu)\Gamma\left(p+\nu+\frac{1}{2}\right)}{\Gamma\left(p+\frac{k-n+1}{2}+\nu\right)+\Gamma\left(p+\frac{n-k+1}{2}+\nu\right)\Gamma\left(p+\frac{n+k}{2}+2\nu+1\right)}$$

The series in (73) and (75) converge rapidly enough for small values of parameter $\varepsilon_j \sim (0:10)$.

Let us consider the improper integrals in the matrix elements P_{kn}^{jq} again. As has been noted in Section 4, the convergence of the integrals in the quantities P_{kn}^{jq} with $q \neq j+1$ describing the non-adjacent facet interaction, is sufficient for $\{\varepsilon_j\}_{j=1}$ not too small. As for the matrices $\{P_{kn}^{j,j+1}\}_{k.n=0}^{\infty}$, the improper integrals in their elements converge slowly. That is why the convergence acceleration procedure is necessary (see (68)).

Now the question arises: What does such convergence acceleration of improper integrals yield? The results of computation of the functions $N_{kn}^{12}(\alpha)$ (after application of the convergence acceleration procedure and without it) presented in Fig.11 are the answer to this question. They demonstrate evidently more rapid decaying of the integrand $N_{kn}^{12}(\alpha)-\tilde{N}_{kn}^{12}(\alpha)$ at some values of parameters ε and indices k, n. As a result, the computation time for the evaluation of the integrals decreases considerably, and a better accuracy is achieved.

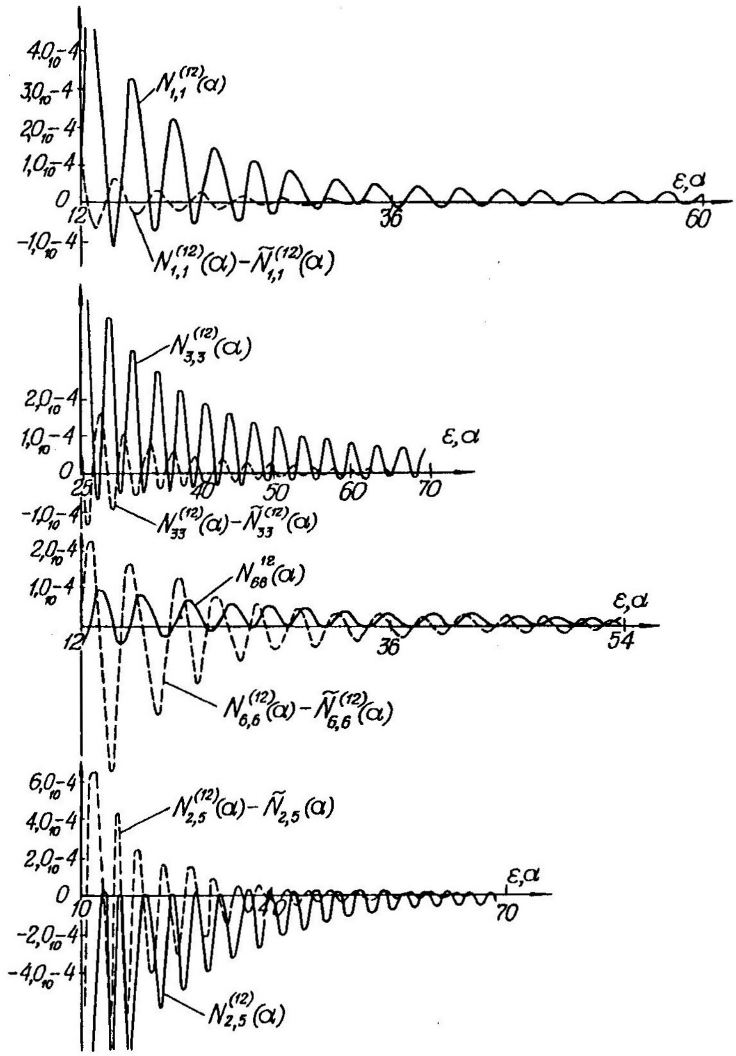

Fig. 11. Illustration of the convergence acceleration scheme for improper integrals.

For the investigation of scattering characteristics for a rectangular cylinder, a set of computer codes has been developed. The infinite SLAE (47) is solvable by the truncation method through the Crout algorithm [52], and the improper integrals are computed using the convergence acceleration procedure. The computed surface current density functions on facets and the RP for a rectangular cylinder are shown in Fig.12 for various values of the frequency parameter $\varepsilon = ka$ and the incident angle θ_0. These results show that at normal incidence ($\theta_0 = 90^o$), the current amplitude is much less in the shadow region of the facet than in the lit region, and if ε increases, the number of current oscillations evidently increases in the shadow region. At an incident angle $\theta_0 = 45^o$, two cylinder's facets are lit up uniformly and hence, the amplitudes of their currents exceed those on the shadow facets considerably. This current distribution causes the appearance of sidelobes in the RP which are comparable with the specular and shadow-forming lobes. If $\theta_0 = 90^o$ and $ka > 1$ then the RP resembles that for the flat strip (ripples of the RP near zero show a mutual interaction between the neighboring facets). For $ka < 1$, the RP is similar to that for a circular cylinder.

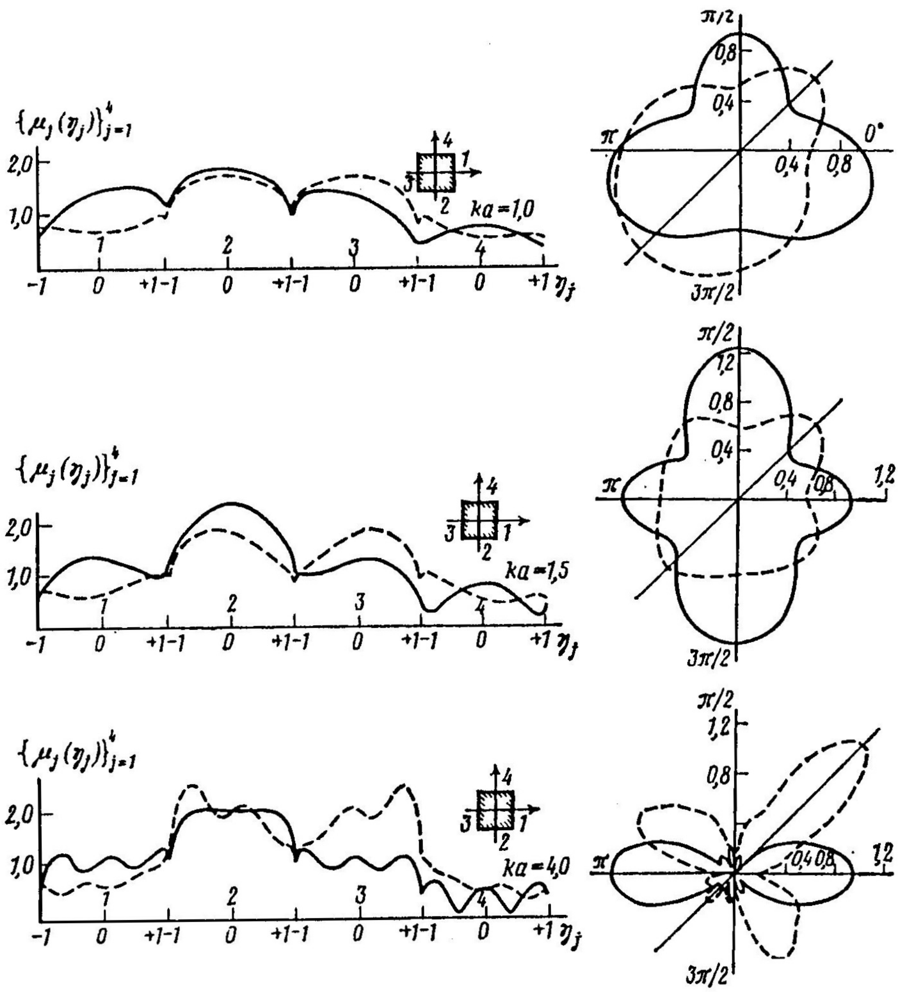

Fig. 12. Current distribution function $\mu(\eta)$ on the facets of a square rectangular cylinder and its RP at different values of ka. Solid and dashed lines denote $\theta_0 = 90°$ and $\theta_0 = 45°$, respectively.

The RP for the square cylinder of the electrical dimension $ka = 2\pi$ (see Fig.13) agrees within a graphic accuracy with the analogous result obtained on the basis of the spectral diffraction theory in [16] (for the comparison with the work [16], we give RP data in decibel scale as well).

Table 3 lists values of the total scattering cross-section for a square cylinder with different wave dimensions. Table 4 shows a connection between the accuracy of computations and the values of the truncation number for the infinite SLAE (47).

Figures 14–18 show RP and surface current functions for an infinitesimally thin strip and for a cylinder of rectangular cross-section at various values of frequency parameter, incident angle and parameter $S = a_1/a_2$ which gives the "thickness" of the cylinder. The following conclusions can be drawn from the analysis on these results.

1. For normal incidence ($\theta_0 = 0^o$) upon a wide facet of the rectangular cylinder with thickness $0.1 \leq S \leq 0.5$, the RP is close to that for a flat strip. The difference is seen in the fact that the far-field amplitude in backward direction slightly exceeds the field amplitude in the shadow region, and the RP does not vanish in the directions $\varphi = 90^o$ and $\varphi = 270^o$

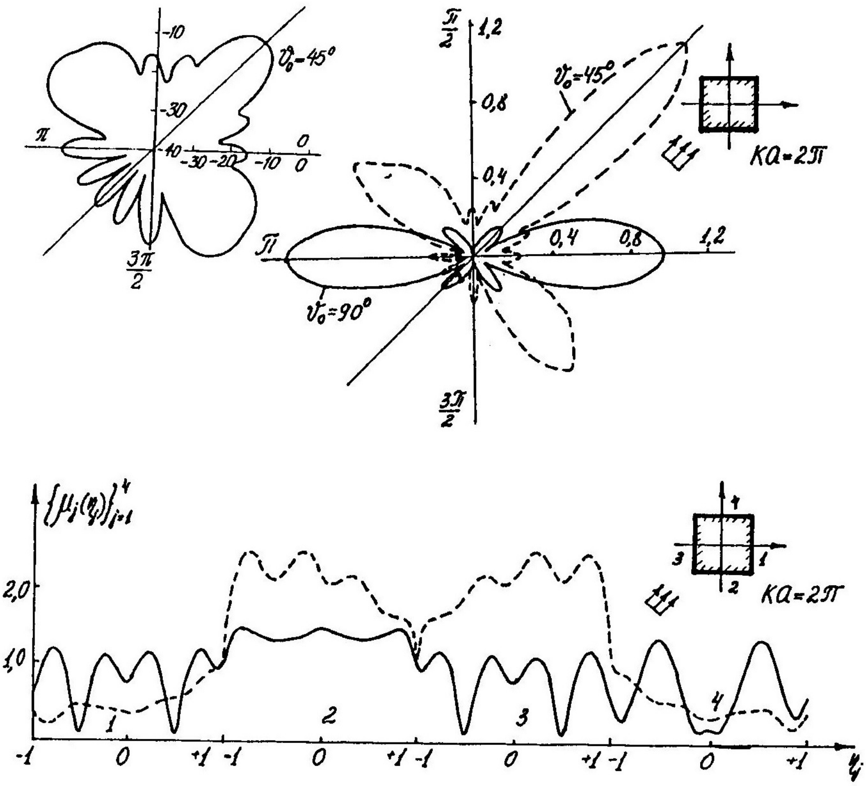

Fig. 13. Current distribution function on the facets of a square rectangular cylinder and its RP for $ka = 2\pi$. Solid and dashed lines denote $\theta_0 = 90°$ and $\theta_0 = 45°$, respectively.

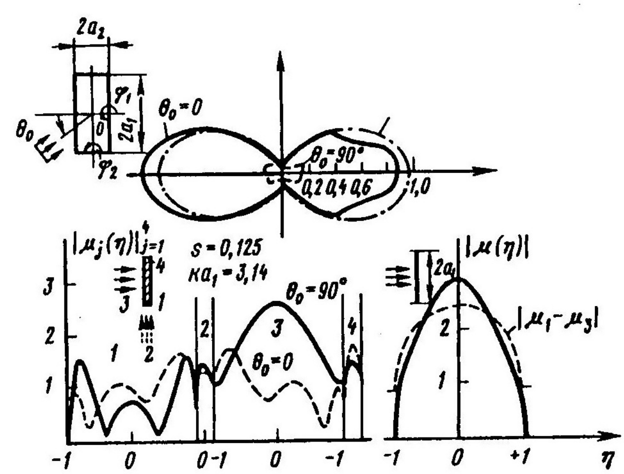

Fig. 14. RP and surface current distribution for an infinitesimally thin strip $(ka_1 = \pi)$ and a rectangular cylinder $(ka_1 = \pi,\ S \equiv a_1/a = 0.125)$. Solid and dashed lines denote $\theta_0 = 90°$ and $\theta_0 = 0°$, respectively.

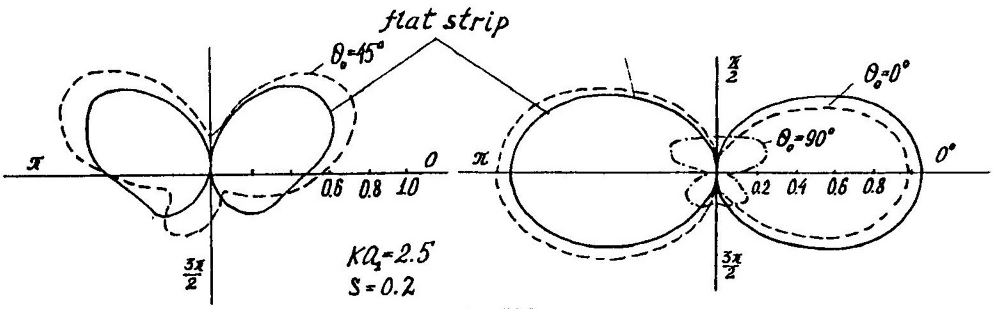

Fig. 15. Comparison of RP for a flat strip (solid line) and a rectangular cylinder with $S=0.2$ (dashed line) at $ka_1=2.5$.

Table 3. Frequency dependence of the total SCS for a rectangular cylinder.

ka	$\sigma_s^h/4a(\theta_0=90^o)$	$\sigma_s^h/4a(\theta_0=45^o)$
0.4	0.36577	0.36989
0.8	0.62534	0.65542
1.2	0.90282	0.70981
1.6	1.1668	0.73359
2.0	1.15348	0.83477
2.2	1.1105	0.90396
2.6	1.0394	0.9791
3.0	0.9791	1.1476
3.4	0.98767	1.18519
3.8	0.94205	1.31141
4.2	0.7041	1.3839

Table 4. Computational accuracy of the total SCS depending on the truncation order N_1.

	$ka=1.5,\ \theta_0=90^o$
N_1	$\sigma_s^h/4a$
3	1.1529
5	1.1515
7	1.1507
9	1.1505
11	1.1505

due to the contribution of the side facets. However, if an integer number of half-wavelengths fits across the wide facet of rectangular cylinder, the difference between its RP and RP of a flat strip decreases.

2. At oblique incidence (see Figs.15,16) on a thin rectangular cylinder (on a flat strip of finite thickness), the RP becomes asymmetric as compared to the RP of an infinitesimally thin strip. The thickness of the strip has a noticeable effect on the RP. The current amplitude on the side facet is large because the function of the current density on the cylinder facets is piecewise continuous in the case of H-polarized wave scattering. In other words, in this case, the surface current (by passing the edge) effectively penetrates the shadow zone through the side facets.

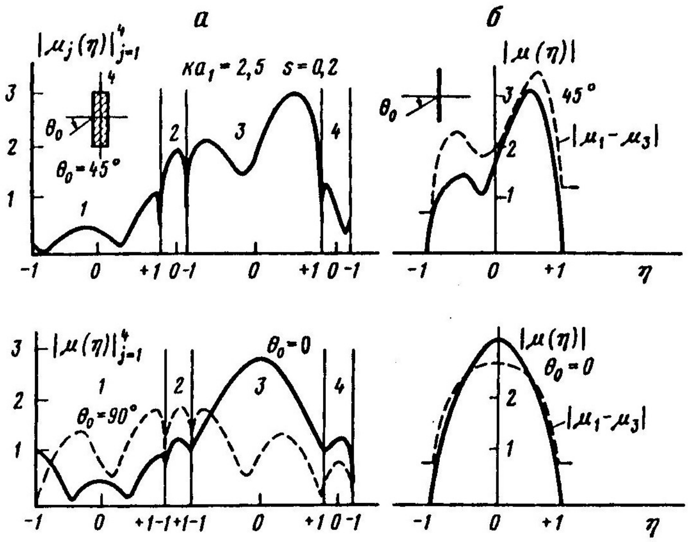

Fig. 16. Surface current distribution on facets of a rectangular cylinder with $S=0.2$ (a) and on a flat strip (b) at $ka_1=2.5$. Top and bottom results denote oblique and normal incidence, respectively.

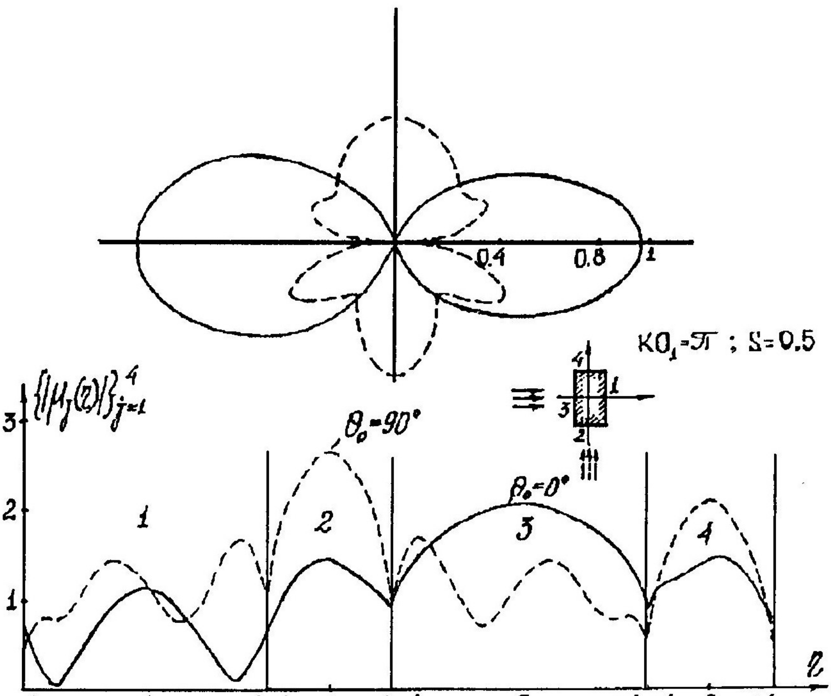

Fig. 17. RP and surface current distribution on facets of a rectangular cylinder for $ka_1=\pi$, $S=$ 0.5. Solid and dashed lines denote $\theta_0=0°$ and $\theta_0=90°$, respectively.

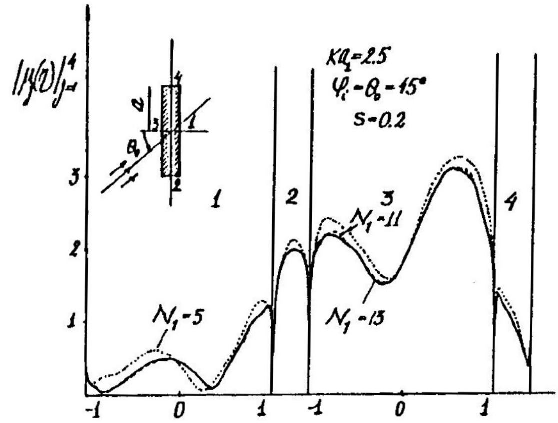

Fig. 18. Comparative plots of the surface current distribution on facets of a rectangular cylinder for different values of the truncation order. $N_1 = 5$ (short-dashed line), $N_1 = 11$ (long-dashed line), $N_1 = 13$ (solid line).

Note that at oblique plane wave incidence on a rectangular cylinder of small thickness, it is necessary to increase the truncation order N_1 of the infinite SLAE for obtaining all the physical quantities with the preassigned accuracy. It is obvious from the plots of the current density distributions on the cylinder's facets for different N_1 (Fig.18). With a decrease of the strip "thickness" (value of the parameter S), a greater N_1 is required.

6. E-POLARIZED PLANE WAVE SCATTERING BY A REGULAR POLYGONAL CYLINDER

Let us formulate the problem for an E-polarized plane wave

$$e_z^0 = e^{ik(\alpha_0 x + \sqrt{1-\alpha_0^2} y)}, \quad (\alpha_0 = \cos\theta_0)$$

incident on the PC. It is necessary to find the total electric field component e_z in the form

$$e_z = e_z^0 + e_z^p = e_z^0 + \sum_{j=1}^{N} e_z^{(j)}$$

where the functions $\{e_z^{(j)}\}_{j=1}^N$ denote the fields generated by the corresponding currents on the PC facets. The field is to satisfy the Helmholtz equation outside the cylinder surface and the Dirichlet boundary condition

$$\left[e_z^0 + \sum_{j=1}^{N} e_z^{(j)} \right]_L = 0, \qquad L = \bigcup_{j=1}^{N} L_j \tag{76}$$

on the cylinder's surface, the Sommerfeld radiation condition in far zone and the Meixner condition (10) at the edges of the PC.

It can be shown that the fields $\{e_z^{(j)}\}_{j=1}^N$ may be represented as [44]:

$$e_z^{(j)} = \frac{i}{2\pi} \int_{-\infty}^{\infty} e_j(\alpha) e^{i\varepsilon_j [\alpha\eta_j' + \sqrt{1-\alpha^2}|\zeta_j'|]} \frac{d\alpha}{\sqrt{1-\alpha^2}} \tag{77}$$

where $\varepsilon_j(\equiv ka_j)$, $\eta_j'\left(\equiv \frac{x_j'}{a_j}\right)$, and $\zeta_j'\left(\equiv \frac{y_j'}{a_j}\right)$ are the normalized coordinates, and the functions $\{e_j(\alpha)\}_{j=1}^N$ are the Fourier transforms of the current density functions $\{\rho_j(\eta)\}_{j=1}^N$ on the PC facets. The definition of these functions is extended such that they vanish outside the interval under consideration, i.e., $\rho(\eta) = 0$ for $|\eta| > 1$.

The behavior of the functions $\{\rho_j(\eta)\}_{j=1}^N$ is to be associated with the Meixner condition (10). In order to take into account the Meixner condition, it is convenient to expand these functions in terms of a complete orthogonal system of the Gegenbauer polynomials with the corresponding weighting factor:

$$\rho_j(\eta) = (1-\eta^2)^{\nu-1} \sum_{m=0}^{\infty} \rho_m^{(j)} C_m^{\nu-\frac{1}{2}}(\eta), \qquad \frac{1}{2} \le \nu < 1 \tag{78}$$

The problem is then to find the unknown coefficients $\{\rho_m^{(j)}\}_{m=0}^{\infty}$. The Fourier transform of (78) can be represented as

$$e_j(\alpha) = \frac{2\pi}{\Gamma(\nu-\frac{1}{2})} \sum_{m=0}^{\infty} (-i)^m \rho_m^{(j)} \beta_m^{(\nu-\frac{1}{2})} \frac{J_{m+\nu-1/2}(\varepsilon_j\alpha)}{(2\varepsilon_j\alpha)^{\nu-1/2}}, \qquad \nu > \frac{1}{2} \tag{79}$$

where

$$\beta_m^{(\nu-1/2)} = \frac{\Gamma(m+2\nu-1)}{\Gamma(m+1)}$$

Now it is clear from (79) that $e_j(\alpha) \sim O(\alpha^{-\nu})$ for $|\alpha| \to \infty$; in other words, there is a relation between the decreasing order of the Fourier transform of the function $\rho_j(\eta)$ and their behavior near the endpoints of the interval $\eta \in [-1,1]$. The fulfilment of the boundary condition (76) yields the following coupled system of dual integral equations with the kernel in trigonometric functions, which should be solved for the functions $\{e_j(\alpha)\}_{j=1}^N$:

$$\begin{cases} \int\limits_{-\infty}^{\infty} e_j(\alpha) e^{i\varepsilon_j\alpha\eta} \dfrac{d\alpha}{\sqrt{1-\alpha^2}} = -2\pi i \varepsilon^{i\varepsilon_j\eta B_j(\alpha_0) + ikr_{0j}D_j(\alpha_0)} \\ \qquad - \sum\limits_{g\neq j=1}^{N} \int\limits_{-\infty}^{\infty} e_q(\alpha) e^{i\varepsilon_j\eta B_{jq}(\alpha) + ikl_{jq}D_{jq}(\alpha)} d\alpha, \quad |\eta| < 1 \\ \int\limits_{-\infty}^{\infty} e_j(\alpha) e^{i\varepsilon_j\alpha\eta} d\alpha = 0, \quad |\eta| > 1 \end{cases} \tag{80}$$

Here the quantities $\{B_j(\alpha_0)\}_{j=1}^N, \{D_j(\alpha_0)\}_{j=1}^N, \{B_{jq}\}_{j,q=1}^N$, and $\{D_{j,q}\}_{j,q=1}^N$, have been defined in connection with (8). The functions $\{e_j(\alpha)\}_{j,q=1}^N$ are subjected to the relationship

$$\int\limits_{-\infty}^{\infty} \frac{|e_j(\alpha)|^2}{|\alpha|+1} d\alpha < \infty, \qquad j = 1,\ldots,N \tag{81}$$

which follows from the condition of finite energy in any bounded domain [38,44]. The class of functions satisfying these conditions is denoted by $\tilde{L}_2(-\infty,\infty)$.

Since the unknown functions $\{e_j(\alpha)\}_{j=1}^N$ are expressed via the unknown coefficients $\{\rho_m^{(j)}\}_{m=0}^{\infty}$, the DIE system (80) is to be solved for $\{\rho_m^{(j)}\}_{m=0}^{\infty}$. To find them, first of all, principal and completely continuous parts (in the space $\tilde{L}_2(-\infty,\infty)$) of the integral operator of the left-hand side of the inhomogeneous equation in the DIE (80) will be separated. This procedure is realized by introducing the function $\delta(\alpha)$ by the formula

$$\frac{1}{\sqrt{1-\alpha_2}} = \frac{i}{|\alpha|}[1-\delta(\alpha)]; \qquad \delta(\alpha) \underset{|\alpha|\to\infty}{\sim} O\left(\frac{1}{\alpha^2}\right) \tag{82}$$

Let us substitute (82) and (79) into the inhomogeneous equation of the system (80) and recall that

$$e^{i\varepsilon_j\alpha\eta} = \left(\frac{2}{\varepsilon_j\alpha}\right)^{\nu-1/2}\Gamma\left(1-\frac{1}{2}\right)\sum_{k=0}^{\infty} i^k\left(k+\nu+\frac{1}{2}\right)J_{k+\nu-\frac{1}{2}}(\varepsilon_j\alpha)C_k^{\nu-\frac{1}{2}}(\eta); \qquad \frac{1}{2}<\nu<1$$

Then in view of the completeness of the set of Gegenbauer polynomials, the infinite coupled SLAE for the unknowns $\{\rho_m^{(j)}\}_{m=0}^{\infty}$ is obtained as follows:

$$\sum_{m=0}^{\infty} Z_m^{(j)}\left[1+(-1)^{k+m}\right]N_{km}^{(\nu-\frac{1}{2})} = \Gamma_k^{(j)} - \sum_{q=1,q\neq j}^{N}\sum_{m=0}^{\infty} Z_m^{(q)}P_{km}^{jq}, \qquad j=1,2,\ldots,N \tag{83}$$

In (83), the following notation has been used:

$$Z_m^{(j)} = (-i)^m\rho_m^{(j)}\beta_m^{(\nu-\frac{1}{2})}\frac{2\pi}{\Gamma\left(\nu-\frac{1}{2}\right)}; \qquad K_{\nu-\frac{1}{2}}(\varepsilon_j) = \left(\frac{2}{\varepsilon_j}\right)^{2\nu-1}\frac{2\Gamma^2\left(\nu+\frac{1}{2}\right)}{\Gamma(2\nu)}$$

$$N_{km}^{(\nu-\frac{1}{2})} = \begin{cases} N_{00}^{(\nu-\frac{1}{2})} = 2\displaystyle\int_0^{\infty} J_{\nu-\frac{1}{2}}^2(\varepsilon_j\alpha)\frac{d\alpha}{\alpha^{2\nu-1}\sqrt{1-a^2}}, & k=m=0 \\ i\left(C_{km}^{(\nu-\frac{1}{2})} - d_{km}^{(\nu-\frac{1}{2})}\right), & k+m\neq 0 \end{cases}$$

$$C_{km}^{(\nu-\frac{1}{2})} = K_{\nu-\frac{1}{2}}(\varepsilon_j)\int_0^{\infty} J_{k+\nu-\frac{1}{2}}(\varepsilon_j\alpha)J_{m+\nu-\frac{1}{2}}(\varepsilon_j\alpha)\frac{d\alpha}{\alpha^{2\nu}}$$

$$= \frac{\Gamma^2\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{k+m}{2}\right)}{\Gamma\left(\nu+\frac{m-k+1}{2}\right)\Gamma\left(\nu+\frac{k-m+1}{2}\right)\Gamma\left(2\nu+\frac{k+m}{2}\right)}, \qquad k+m\neq 0$$

$$d_{km}^{(\nu-\frac{1}{2})} = K_{\nu-\frac{1}{2}}(\varepsilon_j)\int_0^{\infty}\delta(\alpha)J_{k+\nu-\frac{1}{2}}(\varepsilon_j\alpha)J_{m+\nu-\frac{1}{2}}(\varepsilon_j\alpha)\frac{d\alpha}{\alpha^{2\nu}}, \qquad k+m\neq 0$$

$$P_{km}^{jq} = K_{\nu-\frac{1}{2}}(\varepsilon_j)\left(\frac{\varepsilon_j}{\varepsilon_q}\right)\int_{-\infty}^{\infty} J_{k+\nu-\frac{1}{2}}(\varepsilon_j B_{jq}(\alpha))J_{m+\nu-\frac{1}{2}}(\varepsilon_q\alpha)\frac{e^{i\hat{k}l_{jq}D_{jq}(\alpha)}}{\sqrt{1-\alpha^2}}\frac{d\alpha}{[\alpha Bjq(\alpha)]^{\nu-\frac{1}{2}}}$$

$$\Gamma_k^{(j)} = K_{\nu-\frac{1}{2}}(\varepsilon_j)2\pi e^{i\hat{k}r_{0j}D_j(\alpha_0)}\frac{J_{k+\nu-\frac{1}{2}}(\varepsilon_j B_j(\alpha_0))}{[\alpha B_j(\alpha_0)]^{\nu-\frac{1}{2}}}; \qquad \hat{k} = \frac{2\pi}{\lambda} \tag{84}$$

Here, the unknowns $\{Z_n^{(j)}\}_{n=0}^{\infty}$ belong to the class of numerical sequences $l_2\left(\nu-\frac{1}{2}\right)$ defined as

$$l_2\left(\nu-\frac{1}{2}\right) = \left\{Z_n^{(j)} : |Z_0^{(j)}|c + \sum_{n=1}^{\infty}|Z_n^{(j)}|C_{nn}^{(\nu-\frac{1}{2})} < \infty\right\}, \qquad c: const$$

because the functions $\{e_j(\alpha)\}_{j=1}^{N}$ are taken from $\tilde{L}_2(-\infty,\infty)$ (see (81)). Aftrer transforming the original unknowns to the new ones belonging to the space l_2, the Fredholm SLAE of the second kind may be generated. Hence, the unknowns can be found with any preassigned accuracy by means of the truncation method. One can make sure that for the elements of matrices $\left\{d_{km}^{(\nu-1/2)}\right\}_{k,m=0}^{\infty}$ and $\left\{P_{km}^{(\nu-1/2)}\right\}_{k,m=0}^{\infty}$, their estimates, asymptotic expressions and recurrence relations analogous to those obtained in Sections 2.2, 2.3 and 4 are derived.

7. CONCLUSION

The diffraction of an H-polarized wave scattering by a rectangular (square) cylinder has been studied as one of the simplest problems associated with polygonal cylinders. First of all, this selection has been motivated by the possibility of principal investigations such as the comparison of various methods and the scattered field feature analysis.

This problem analysis is far from covering all the advantages of this method. The basic ideas combining the partial inversion of the operator and the moment method (the hybrid approach) are fruitful and can be effectively used in many problems of electromagnetics.

In view of this, the following directions of possible applications of the proposed method can be pointed out:

I. Scattering problems for polygonal cylinders formed by

a) an intersection of circular cylinders (the contour of PC cross-section is a joint of circular arcs (see [53]));

b) an intersection of a circular cylinder with planes (the contour of PC cross-section is a joint of straight intervals and circular arcs (see [54]));

II. Scattering problems for an arbitrary PC with the facets such as flat strips and similar problems for the piecewise continuous cylindrical surfaces (see Fig.1f,g).

III. The developed method for solving the dual integral equations with the kernel in trigonometric functions can be generalized to those with the kernel in Bessel functions appearing in problems of wave scattering by disks (including a disk of finite thickness).

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