Scattering of the H_{10} mode by a cylinder with a longitudinal slot in a rectangular waveguide

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A rigorous solution is derived for the scattering of the H_{10} mode by a complicated obstacle (an open cylinder) in a rectangular waveguide. The cylinder axis is parallel to the broad waveguide wall. The problem is solved by the semi-inversion method developed for a study of the diffraction of plane waves by a grating of cylinders with a longitudinal slit [É. I. Veliev and V. P. Shestopalov, ZhVMiMF **17**, 1234 (1977)]. The properties of the scattered field are studied and new resonant effects are found for a rectangular waveguide loaded by a capacitive slotted cylinder.

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Metal inhomogeneities are widely used in rectangular waveguides in microwave technology.[1] Those inhomogeneities are used in matching elements, frequency filters, mode transducers, microwave power measurements in transmission lines, etc. Thus, there is considerable interest in a comprehensive study of the properties of the fields scattered by these inhomogeneities; such a study usually involves the solution of complicated boundary-value problems. Approximate and rigorous methods have been developed for solving the boundary-value problems for the simplest inhomogeneities.[1-12] On the other hand, for complicated obstacles we do not have effective methods for studying the scattered fields.

In this paper we report the first rigorous solution of the boundary-value problem of the scattering of an H_{10} mode by an open cylinder in a rectangular waveguide. The cylinder axis is parallel to the broad wall of the waveguide and represents a capacitive inhomogeneity (Fig. 1). We note that the solution of this problem also incorporates the solution of a simpler problem: scattering of the H_{10} mode by a solid cylinder in a rectangular waveguide. A study of the properties of the field scattered by a cylinder with a slot in a waveguide is important for understanding the ponderomotive effect of an electromagnetic field and its applications.[13]

1. FORMULATION OF THE PROBLEM. REPRESENTATIONS FOR THE SCATTERED FIELD

Figure 1a shows the structure (the notation for the structure is defined here). We assume that the walls of the cylinder with the slot and the waveguide are infinitesimally thin and are ideal conductors. We assume that the waveguide is infinite along the oz axis. The geometric dimensions of the structure are arbitrary. In addition to the Cartesian coordinate system we introduce a local cylindrical coordinate system (R, φ, z), which is erected at the inhomogeneity.

We assume that an H_{10} mode[1)] is incident from the $z < 0$ direction on the open cylinder (Fig. 1):

$$H_z^0 = i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)e^{i\beta z}, \tag{1}$$

where $\beta = \sqrt{k^2 - (\pi^2/a^2)}$, and $k = 2\pi/\lambda$.

We are to determine the diffraction field which arises

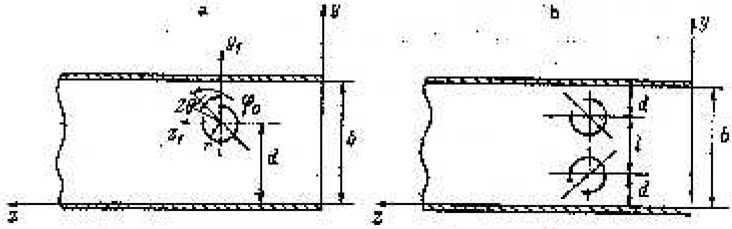

FIG. 1. The structure under consideration.

as the result of the scattering of the wave (1) by the open cylinder.

We seek the total field in the form $H_x = H_x^0 - H_x^1$, where the function H_x^1 describes the scattered wave. The function H_x must satisfy the Helmholtz equation, the standard boundary conditions at the waveguide wall, the Neumann boundary condition at the surface of the cylinder, the condition that the energy must remain finite in any bounded volume, and the radiation condition at infinity.

To determine the diffracted field we use the image method.[14] Since the total field is formed as the result of the repeated reflection of the scattered field from the waveguide walls, each reflection can be regarded as equivalent to a reflected-field source, i.e., to the multiple image of the cylinder in the waveguide walls. These sources constitute a periodic grating of open cylinders (Fig. 2). The incident wave is H-polarized with respect to the axis of the cylinder.

We write the scattered field as the sum of double-layer potentials distributed on the image sources, i.e.,

$$H_x^1 = i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)\frac{\pi}{2i}\sum_{m=-\infty}^{\infty}\int_{L_0}\mu(y',z')\frac{\partial}{\partial\rho}$$
$$\times\{H_0^{(1)}(\beta\sqrt{(z-z')^2+(y-y'-2mb)^2})$$
$$+H_0^{(1)}(\beta\sqrt{(z-z')^2+(y+y'-2mb+2d)^2})\}\,dl$$
$$= i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)\int_{L_0}\mu(y',z')\frac{\partial}{\partial\rho}G(z-z',y-y')\,dl, \qquad (2)$$

where $\mu(y', z')$ is the surface current density at the cylinder, $H_\nu^{(1)}(x)$ is the Hankel function, and $G(z-z', y-y')$ is the Green's function, which takes account of the effect of the waveguide walls and is written in the local coordinate system at the inhomogeneity.

We will need a representation for the scattered field in the cylindrical coordinate system at the inhomogeneity and also in the form of a summation over waveguide modes. Let us find these representations.

Expanding the unknown current function $\mu(y', z')$ in a Fourier series,

$$\mu = \frac{i}{\pi^2\beta r}\sum_{n=-\infty}^{\infty}\mu_n e^{in\varphi},$$

and using the theory for combining Bessel functions, from (2) and the relation $R_0^0 = R > r$, we have

$$H_x^1 = i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)\left\{\sum_{n=-\infty}^{\infty}\mu_n J_n'(\beta r)H_n^{(1)}(\beta R_0^0)e^{in\varphi_0^0}+\sum_{p=-\infty}^{\infty}\mu_p J_p'(\beta r)\right.$$
$$\left.\times\left[\sum_{n\neq 0}H_p^{(1)}(\beta R_n^0)e^{ip\varphi_n^0}+\sum_{n=-\infty}^{\infty}H_p^{(1)}(\beta R_n^1)e^{ip(\pi-\varphi_n^1)}\right]\right\},\quad R_0^0 > r,$$

where

$$R_n^0 = \sqrt{z_1^2+(y_1-2nb)^2};\quad \varphi_n^0 = \arcsin\frac{z_1}{R_n^0},$$
$$R_n^1 = \sqrt{z_1^2+(y_1-2nb+2d)^2};\quad \varphi_n^1 = \arcsin\frac{z_1}{R_n^1}.$$

Using this theorem and (3) we find the representation for the scattered field:

$$H_x^1 = i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)\left\{\sum_{s=-\infty}^{\infty}\mu_s J_s'(\beta r)H_s^{(1)}(\beta R_0^0)e^{is\varphi_0^0}\right.$$
$$\left.+\sum_{p=-\infty}^{\infty}\mu_p J_p'(\beta r)\sum_{s=-\infty}^{\infty}J_s(\beta R_0^0)\left[Q_{s-p}^{(1)}(\beta b)+Q_{s+p}^{(2)}(\beta b,\beta d)\right]e^{is\varphi_0^0}\right\},$$
$$R_0^0 > r,$$

where

$$Q_{s+p}^{(2)}(\beta b,\beta d)=\sum_{n=-\infty}^{\infty}H_{s+p}^{(1)}(2\beta l_n)e^{i\varphi_n(s+p)};\quad l_n=\begin{cases} nb-d, & n>0,\\ -nb+d, & n\leqslant 0,\end{cases}$$
$$\varphi_n=\begin{cases}0, & n>0\\ \pi, & n\leqslant 0\end{cases};\quad Q_{s-p}^{(1)}(\beta b)=\sum_{n=1}^{\infty}H_{s-p}^{(1)}(2\beta bn)\left[1+(-1)^{s-p}\right]. \qquad (3)$$

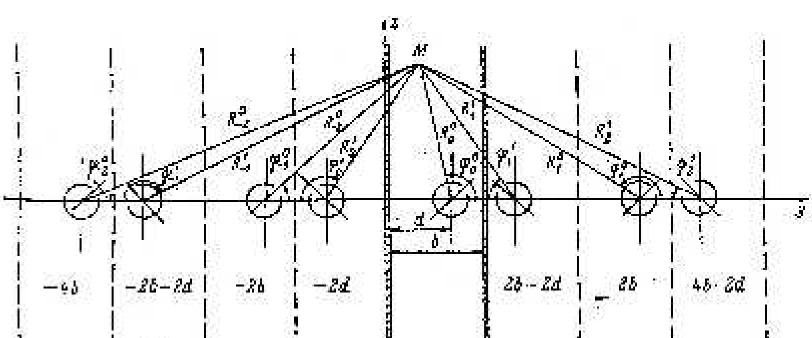

FIG. 2

A corresponding representation for H_x^1 can be found for the case $R_p^0 < r$.

We note that this expression for H_x^1 is convenient for studying the fields near the inhomogeneity.

We will now write a representation for H_x^1 as a summation over waveguide modes. We use the Poisson summation formula[14] on (2); the result is

$$H_x^1 = i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)\sum_{m=-\infty}^{\infty}\begin{cases} a_m\cos\left(\frac{m\pi}{b}y\right)e^{-i\beta z\sqrt{1-\left(\frac{m\pi}{\beta b}\right)^2}}, & z > \frac{r}{2},\\ b_m\cos\left(\frac{m\pi}{b}y\right)e^{i\beta z\sqrt{1-\left(\frac{m\pi}{\beta b}\right)^2}}, & z < -\frac{r}{2},\end{cases}$$

where

$$a_m = \frac{4}{\beta b}\frac{\varepsilon_m}{\sqrt{1-\left(\frac{m\pi}{\beta b}\right)^2}}\sum_{p=-\infty}^{\infty}(-1)^p J_p'(\beta r)\mu_p\cos\left(\frac{m\pi}{b}d + p\theta_m\right),$$

$$b_m = \delta_m^0 + \frac{4}{\beta b}\frac{\varepsilon_m}{\sqrt{1-\left(\frac{m\pi}{\beta b}\right)^2}}\sum_{p=-\infty}^{\infty}J_p'(\beta r)\mu_p\cos\left(\frac{m\pi}{b}d - p\theta_m\right), \quad (7)$$

δ_m^0 is the Kronecker delta, $\theta_m = \arcsin\left(\frac{m\pi}{\beta b}\right)$, and

$$\varepsilon_m = \begin{cases} 1/2, & m = 0,\\ 1, & m \neq 0.\end{cases}$$

2. REDUCTION OF THE PROBLEM TO THE SOLUTION OF INFINITE SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS OF THE SECOND KIND

It follows from the representations for the scattered field in (4) and (6) that we must determine the Fourier amplitudes of the surface current, μ_p, in order to find this field. To find these amplitudes we impose the Neumann boundary condition on the total field at the surface of the cylinder:

$$\frac{\partial}{\partial\rho}\left[H_x^0 - H_x^1\right] = 0,\ R_p^0 = r;\ \theta < \varphi < 2\pi - \theta;\ (\varphi = \varphi_p^0). \quad (8)$$

Using the expression for the incident field in the cylindrical coordinate system as a series in Bessel functions

$$H_x^0 = i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)\sum_{p=-\infty}^{\infty}J_p(\beta R)e^{ip\varphi},$$

from (8) we have

$$\sum_{p=-\infty}^{\infty}\mu_p J_p'(\beta r)H_p^{(1)\prime}(\beta r)e^{ip\varphi} = \sum_{p=-\infty}^{\infty}J_p'(\beta r)e^{ip\varphi} - \sum_{p=-\infty}^{\infty}J_p'(\beta r)\sum_{m=-\infty}^{\infty}\mu_m J_m'(\beta r)\left[Q_{p-m}^{(1)}(\beta b) + Q_{p+m}^{(2)}(\beta b, \beta d)\right]e^{ip\varphi},$$
$$\theta < \varphi < 2\pi - \theta. \quad (9)$$

To determine the current function $\mu(\varphi)$ over the complete interval over φ, i.e., over the interval $[0, 2\pi]$, we must supplement (9) with the equation

$$\sum_{p=-\infty}^{\infty}\mu_p e^{ip\varphi} = 0,\ |\varphi| < \theta, \quad (10)$$

which follows from the obvious fact that the surface current density is zero at the slot.

We then have a system of functional equations of the first kind, (9) and (10), for the unknowns μ_p. The method of Refs. 1 and 15, which is based on the semiinversion method of Ref. 16, can be used to reduce these equations to an infinite system of linear algebraic equations of the second kind:

$$\mu_n = \sum_{m=-\infty}^{\infty}B_{nm}\mu_m + \Gamma_n,\ n = 0, \pm 1, \pm 2, \ldots, \quad (11)$$

where

$$B_{nm} = d_{nm} + N_{nm},\ n, m = 0, \pm 1, \pm 2, \ldots,$$

$$d_{nm} = \begin{cases} (-1)^{n+m}e^{i(m-n)\varphi_0}\pi\frac{|m|}{n}\delta_m V_{m-1}^{n-1}(-u), & m \neq 0,\ n \neq 0,\\ \frac{(-1)^n}{n}e^{-in\varphi_0}i\pi(\beta r)^2 J_0' H_0^{(1)\prime}V_{n-1}^{-1}(-u), & m = 0,\ n \neq 0,\\ (-1)^{m+1}e^{im\varphi_0}|m|\delta_m W_m(-u), & m \neq 0,\ n = 0,\\ -i\pi(\beta r)^2 J_0' H_0^{(1)\prime}W_0(-u), & m = 0,\ n = 0,\end{cases}$$

$$N_{nm} = \begin{cases} \frac{(-1)^{n+1}}{n}e^{-in\varphi_0}i\pi(\beta r)^2 J_m'\sum_{p=-\infty}^{\infty}J_{-p}'V_{n-1}^{p-1}(-u)\\ \quad\times\left[Q_{p-m}^{(1)}(\beta b) + Q_{p+m}^{(2)}(\beta b, \beta d)\right]e^{ip\varphi_0}, & n \neq 0,\\ -i\pi(\beta r)^2 J_m'\sum_{p=-\infty}^{\infty}J_{-p}'W_p(-u)\\ \quad\times\left[Q_{p-m}^{(1)}(\beta b) + Q_{p+m}^{(2)}(\beta b, \beta d)\right]e^{ip\varphi_0}, & n = 0,\end{cases}$$

$$\Gamma_n = \begin{cases} \frac{(-1)^{n+1}}{n}e^{-in\varphi_0}i\pi(\beta r)^2\sum_{p=-\infty}^{\infty}J_{-p}'V_{n-1}^{p-1}(-u)e^{ip\varphi_0}, & n \neq 0,\\ i\pi(\beta r)^2\sum_{p=-\infty}^{\infty}J_{-p}'W_p(-u)e^{ip\varphi_0}, & n = 0,\end{cases}$$

$\delta_m = 1 + \frac{i\pi(\beta r)^2}{|m|}J_m'H_m^{(1)\prime}$, $V_{n-1}^{m-1}(-u)$, $W_m(-u)$ are the same as in Ref. 9, $u = \cos\theta$, and the argument of the derivatives of the Bessel functions is βr.

Using the estimates for $V_{n-1}^{p-1}(-u)$ and $W_m(-u)$ (Ref. 9) and those for δ_m, $Q_{p-m}^{(1)}$, $Q_{p+m}^{(2)}$ (Refs. 1, 15, 16) and the Bessel functions, we can show that when $2r < b$ (in which the surface of the inhomogeneity does not touch the waveguide walls) the equations in (11) are Fredholm equations, since $\sum_{n,m}|B_{nm}|^2 < \infty$, $\sum_n|\Gamma_n|^2 < \infty$.

Consequently, (11) can be solved by a reduction method for arbitrary parameters.

It follows from the estimate of the norm of the matrix,

$$q = \|B_{nm}\| = \max_n\sum_{m\neq n}\frac{|B_{nm}|}{|1 - B_{nn}|},$$

that system (11) can be solved by successive approximations if $(r/b) \ll 1$ and $u = \cos\theta \to \pm 1$ (narrow slots and tapes).

Let us consider the case $\theta = 0$. In this case the inhomogeneity is a closed circular cylinder of radius r. Noting that

$$V_{n-1}^{n-1}(-1) = \frac{|n|}{m};\ V_{n-1}^{m-1}(-1) = 0,\ n \neq m;\ V_{n-1}^{-1}(-1) = (-1)^{n-1},$$
$$W_m(-1) = 0,\ m \neq 0;\ \frac{1}{W_0(-1)} = 0,$$

we find the following equation from (11):

$$x_n = -\frac{1}{H_n^{(1)'}} \sum_{m=-\infty}^{\infty} \mu_m J'_m [Q^{(1)}_{n-m}(\beta b) + Q^{(2)}_{n+m}(\beta b, \beta d)] + \frac{1}{H_n^{(1)'}},$$
$$n = 0, \pm 1, \pm 2, \ldots$$

This equation gives a rigorous solution for the scattering of an H_{10} mode by a closed circular cylinder of radius r in a rectangular waveguide if the cylinder axis is along the broad wall of the waveguide.

It is pertinent to note that Eq. (11) gives not only a rigorous solution of the problem under consideration here, but the diffraction of a plane wave by the grating in Fig. 2. A particular feature of this grating is that its period contains two mirror-image elements in the form of an open circular cylinder.

The system in (11) also gives a rigorous solution for scattering of an H_{10} mode by two open circular cylinders which are mirror images (with respect to the plane passing through their center) and which lie inside a rectangular waveguide, parallel to its broad wall (Fig. 1b). In Eq. (11) we must replace d by $(b - l)/2$, where l is the distance between the axes of the cylinders ($2b - 2r > l > 2r$).

The matrix elements of system (11) contain the quantities $Q^{(1)}_{p-m}(\beta b)$ and $Q^{(2)}_{p+m}(\beta b, \beta d)$, which are Schlömilch series,[17] as can be seen from (5). Since these series converge slowly in a computer solution of (11) these representations are not efficient. By using the Poisson summation formula for these quantities, however, we can find a representation in the form of rapidly converging series in terms of elementary functions. This representation is derived in Ref. 18 for the quantities $Q^{(1)}_{p-m}(\beta b)$ (see also Ref. 1) and in Ref. 15 for $Q^{(2)}_{p+m}(\beta b, \beta d)$.

3. ANALYSIS OF THE PROPERTIES OF THE SCATTERED FIELD

We begin the study of the properties of the field scattered by this inhomogeneity with the case $d = b/2$ and $\varphi_0 = 90°$. In this case the open cylinder is at the center of the waveguide and the slot points toward the incident wave.

We first introduce the notation

$$s = \frac{2r}{b}, \quad \varkappa = \frac{b}{2\pi}\beta = \frac{kb}{2\pi}\sqrt{1 - \left(\frac{\pi}{ka}\right)^2} = \frac{1}{2}\sqrt{\left(\frac{kb}{\pi}\right)^2 - \left(\frac{b}{a}\right)^2},$$

where s is the "filling factor," and $\varkappa$ is a dimensionless frequency parameter, which shows the dependence of the transformation coefficient on the wavelength and waveguide dimensions a and b.

We first treat the single-mode case. With a capacitive inhomogeneity, the parameter $\varkappa$ has a value in the range $0 \le \varkappa < 1$. The H_{10} mode is the only undamped harmonic of the diffraction spectrum. We also assume $s \ll 1$ and $\theta \to 0$, implying that the cylinder is small (since $\beta r = \pi \varkappa s < 1$) and that the longitudinal slot is narrow.

In this case the reflection and transmission coefficients for the fundamental mode ($|a_0|^2$ and $|b_0|^2$, respectively) are characteristics of the scattered field, since the total diffracted field far from the inhomogeneity can be written as the field of the fundamental mode,

$$H_z^1 = \begin{cases} i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)(e^{i\beta z} + a_0 e^{-i\beta z}), & z > \frac{r}{2}, \\ i\frac{\beta a}{\pi}\sin\left(\frac{\pi}{a}x\right)b_0 e^{i\beta z}, & z < -\frac{r}{2}. \end{cases} \quad (12)$$

Solving (11) by successive approximations, we find the following equation for the transmission coefficient:

$$b_0 = 1 - \frac{i}{\pi}\frac{(\beta r)^4 \ln \sin\frac{\theta}{2}}{1 + 2(\beta r)^2\left[1 + i\pi\left(\frac{\beta r}{2}\right)^2(1 + Q_0(\beta b))\right]\ln\sin\frac{\theta}{2}} \quad (13)$$
$$+ O\left[\theta^2 + \varkappa^4\left(\frac{r}{b}\right)^3\right],$$

where

$$Q_0(\beta b) = Q_0^{(1)}(\beta b) + Q_0^{2}\left(\beta b, \beta\frac{b}{2}\right)$$
$$= \frac{1}{\pi\varkappa} - 1 - \frac{2i}{\pi}\left(\ln\frac{\varkappa}{2} + C + \frac{\varkappa^2}{\pi}1{,}202\right),$$

and C is the Euler constant. (The reflection coefficient can be determined from the equation $|a_0^2| + |b_0|^2 = 1$, which holds in the interval $0 \le \varkappa < 1$.)

Let us examine the denominator of the transmission coefficient b_0. Equating this denominator to zero we find

$$\frac{1}{2\ln\sin\frac{\theta}{2}} + (\beta r)^2\left[1 + i\pi\left(\frac{\beta r}{2}\right)^2(1 + Q_0(\beta b))\right] = 0. \quad (14)$$

We find the complex roots of this equation by the Newton method:

$$\beta r = q_1 + iq_2; \quad q_1 = \frac{1}{\sqrt{-2\ln\sin\frac{\theta}{2}}}\left[1 + \frac{\pi \operatorname{Im} Q_0}{8\ln\sin\frac{\theta}{2}}\right] + O\left(\frac{1}{\ln^2\sin\frac{\theta}{2}}\right),$$
$$q_2 = \frac{\pi}{16\ln\sin\frac{\theta}{2}\sqrt{-2\ln\sin\frac{\theta}{2}}}[1 + \operatorname{Re} Q_0] + O\left(\frac{1}{\ln^2\sin\frac{\theta}{2}}\right).$$

For values of the parameter $\beta r = q_1$ Eq. (13) shows that transmission coefficient b_0 approaches 0 as $O[\theta^2 + \varkappa^2 \cdot (r/b)^3]$; in other words, the reflection is nearly complete.

If we take the limit $\theta = 0$ in (13) (corresponding to the case in which the inhomogeneity is a closed circular cylinder), we can find an expression for $b_0 = 1 + O[\varkappa^2 \cdot (r/b)^3]$. We then obtain almost nearly total transmission. This result, which is familiar from waveguide theory,[2] has a simple physical meaning: The incident wave induces negligible currents at a capacitive inhomogeneity such as a small closed circular cylinder ($\beta r < 1$). An inhomogeneity of this kind only causes a slight perturbation of the incident field. This is why such inhomogeneities are not used in practice.[2]

The presence of a narrow longitudinal slot in the wall of a small cylinder ($\beta r < 1$) leads to a qualitatively new resonant effect: total reflection of the incident wave. This happens because the slot in the cylinder gives rise to a resonant increase in the surface current when $\beta r = q_1$; the ultimate result is nearly total reflection of the incident wave. Figures 3 and 4 show the transmission and reflection coefficients as functions of the frequency $\varkappa$ and the filling factor s. These curves are obtained by solving (11) by the reduction method on an M-222 computer with the help of (7). It follows from Figs. 3a, 3b, and 4a that

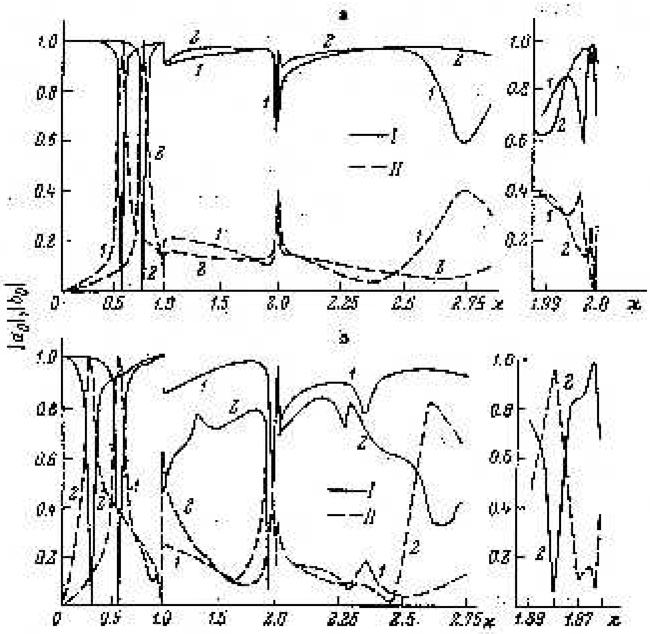

FIG. 3. Variation of the transmission coefficient (I) and the reflection coefficient (II) with the frequency parameter $\varkappa$. a) $\theta = 5°$, $\varphi_1 = 90°$; 1) s = 0.2; 2) 0.15. b: $\theta = 15°$, $\varphi_1 = 90°$, 1) s = 0.25; 2) 0.5.

total reflection of the incident wave occurs if $\varkappa < 1$ and if the slot is narrow, for various values of s. As the slot becomes wider, the quality factors for the resonant values of $|a_0|$ and $|b_0|$ decrease and shift toward larger values of s (Fig. 4a). For wide slots, the resonant values of $|a_0|$ and $|b_0|$ vanish (Fig. 4b), and the properties of the scattered field of this inhomogeneity approach those of the fields scattered by a capacitive iris.[8]

When $\varkappa \geq 1$ the scattered H_{10} mode loses energy by conversion to higher-order H_{tn} modes. This result is demonstrated clearly in Figs. 3a and 3b: At $\varkappa = 1$ and 2 we observe resonant changes in $|a_0|$ and $|b_0|$ due to the appearance of new undamped harmonics in the diffraction spectrum.

Furthermore, in the region $\varkappa > 1$, for various values of $\varkappa$, there are minima (maxima) for the transmission (reflection) coefficient. These minima are observed at values of the parameter βr approximately equal to the roots of the derivatives of the Bessel functions, ν'_{mn} (m, n = 0, 1, 2, ...). For example, for curve 1 in Fig. 3b this minimum is observed at the values $\beta r = 2.0065 = \nu'_{11} + 0.1653$; for curves 2 it is observed at $\beta r = 2.1037 = \nu'_{11} + 0.2625$ and $\beta r = 3.6264 = \nu'_{21} - 0.5723$. These resonant values of $|a_0|$ and $|b_0|$ and the similar behavior of $|a_0|$ and $|b_0|$ in the single-mode case, $\varkappa < 1$, are consequences of the fact that the incident wave excites quasimodes in an open cylinder. These quasimodes are studied in part in Ref. 19. In particular, the resonant reflection in the region $\varkappa < 1$ is associated with the excitation of a slot wave in the open cylinder; when $\varkappa > 1$ it is associated with the excitation of waveguide modes. The deviation of βr from ν'_{mn} is due to the presence of the slot and the effect of the waveguide walls.

It has been shown[20] that resonances in a slotted cylinder caused by an H-polarized electromagnetic wave causes the mechanical force acting on the cylinder to increase sharply. It is then important from the practical standpoint to determine the mechanical force acting on such a cylinder.

A unit length of the cylinder is subjected to a force[20]

$$\begin{Bmatrix}\langle F_z\rangle\\ \langle F_y\rangle\end{Bmatrix} = \frac{a}{4\pi^2}\sin\left(\frac{\pi}{a}x\right)\begin{Bmatrix}\mathrm{Im}\\ -\mathrm{Re}\end{Bmatrix}\sum_{m=-\infty}^{\infty}\left[\mu^*_{m+1}\left(\frac{m(m+1)}{(\beta r)^2}\right)\right.$$
$$\left.\pm\mu^*_{m-1}\left(\frac{m(m-1)}{(\beta r)^2}+1\right)\right]\left[\frac{2i}{\pi\beta r}\mu_m + 2J_m(\beta r)\left(i^m - \mu_m H^{(1)\prime}_m(\beta r)\right)\right]$$
$$= \frac{a}{4\pi^2}\sin\left(\frac{\pi}{a}x\right)\begin{Bmatrix}\langle F^0_z\rangle\\ \langle F^0_y\rangle\end{Bmatrix}, \tag{16}$$

where $\langle F_z\rangle$ and $\langle F_y\rangle$ are the average values of the force over the oscillation period.

By integrating (16) over the height of the cylinder we find the force exerted on the cylinder as a whole:

$$\begin{Bmatrix}\langle F_z\rangle\\ \langle F_y\rangle\end{Bmatrix} = 2\frac{a^2}{4\pi^3}\begin{Bmatrix}\langle F^0_z\rangle\\ \langle F^0_y\rangle\end{Bmatrix}. \tag{17}$$

Equation (17) is used to calculate the dependence of the components $\langle F_{z,y}\rangle$ on the parameter $\varkappa$; the results are shown in Fig. 5. Evidently the longitudinal component $\langle F_z\rangle$ is nearly zero everywhere except in the vicinity of the resonant values of $\varkappa$, near which it increases greatly. The transverse force component is zero everywhere, as in the case of a closed cylinder.

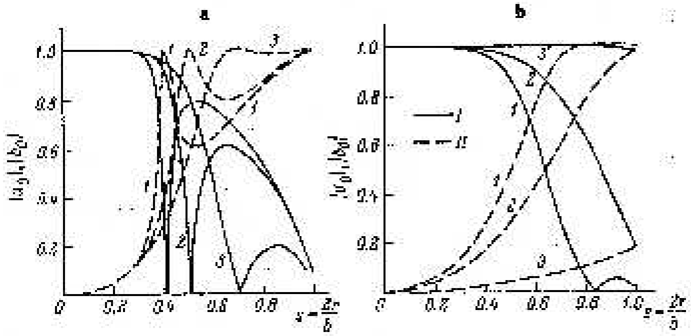

FIG. 4. Variation of the transmission coefficient (I) and the reflection coefficient (II) with the parameter $s = 2r/b$ ($\varphi_1 = 90°$). a) $\varkappa = 0.2525$, 1) $\theta = 1$; 2) 5; 3) 20°. b) $\varkappa = 0.2525$, 1) $\theta = 30$; 2) 90, 3) 150.

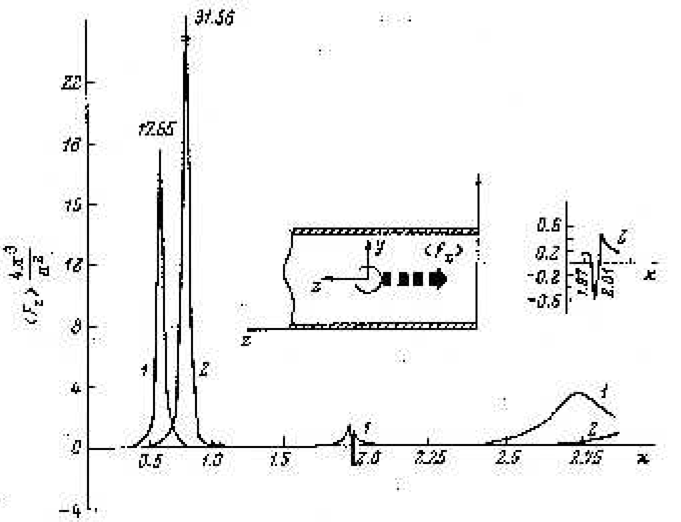

FIG. 5. Variation of the force components with the frequency parameter $\varkappa$. $\theta = 5°$, $\varphi_1 = 90°$. 1) s = 0.2; 2) 0.15.

We note that an open cylinder in a rectangular waveguide in the longwave region experiences a force which is several times stronger (depending on s) than that experienced by the same cylinder in free space. For example, with $s = 0.15$ we have $\langle F_Z \rangle \sim 12 \langle F_Z^1 \rangle$, while for $s = 0.2$ we have $\langle F_Z \rangle \sim 7 \langle F_Z^1 \rangle$, where $\langle F_Z^1 \rangle$ is the force component acting on a slotted cylinder in free space.[20]

As noted in Ref. 20, the resonant properties of this type of inhomogeneity make it a unique device for sensing the ponderomotive effect of an electromagnetic field. Particular interest attaches to the resonance in the longwave region since the inhomogeneity is small ($\beta r < 1$). (For example, on curve 1 in Fig. 5 at resonance we have $\beta r = 0.3758$; on curve 2 we have $\beta r = 0.3808$).

CONCLUSION

This analysis of the properties of the field scattered by a capacitive inhomogeneity – a cylinder with a longitudinal slot – shows that a small obstacle of this kind with a narrow longitudinal slot can cause total reflection of an incident H_{10} wave. These new properties of this type of inhomogeneity may find applications in microwave technology for measuring microwave power levels in transmission lines, etc.

A formal rigorous solution of the scattering of an H_{10} mode by an inductive inhomogeneity is given in Ref. 15.

[1]For brevity, we omit the time factor $e^{-i\omega t}$.

[1]É. I. Veliev and V. P. Shestopalov, ZhVMiMF 17, 1234 (1977).

[2]H. Meinke and F. Gundlach, Radioengineering Handbook [Russian translation], Vol. 1, Gosenergoizdat, Moscow–Leningrad (1960).

[3]J. Schwinger, Zarubezhnaya Radioelektronika No. 3 (1970).

[4]L. Levin, Advanced Theory of Waveguides, Iliffe, London (1951).

[5]N. A. Khizhnyak, in: Radioengineering [in Russian], No. 4, Izd. KhGU, Khar'kov (1967).

[6]V. V. Nikol'skii, Variational Methods for Interior Problems of Electrodynamics [in Russian], Nauka, Moscow (1967).

[7]A. S. Il'inskii and T. N. Galishnikova, in: Computational Methods and Programming [in Russian], No. 20, Izd. MGU, Moscow (1973).

[8]Ya. N. Fel'd, Radiotekh. Elektron. 23, 1 (1978).

[9]V. P. Shestopalov, Method of the Riemann–Hilbert Problem in the Theory of the Diffraction and Propagation of Electromagnetic Waves [in Russian], Izd. KhGU, Khar'kov (1971).

[10]V. N. Vol'man and A. G. Sarkis'yants, Radiotekhnika 30, 43 (1975).

[11]G. Sh. Kevanishvili, Radiotekh. Elektron. 20, 1810 (1975).

[12]A. V. Moshinskii and V. K. Berezovskii, Radiotekh. Elektron. 22, 1350 (1977).

[13]R. A. Valitov (editor), Ponderomotive Effects of Electromagnetic Fields [in Russian], Sov. Radio, Moscow (1975).

[14]P. M. Morse and H. Feshbach, Methods of Theoretical Physics, Vol. 1, McGraw-Hill, New York (1953).

[15]É. I. Veliev, Author's Abstract, Candidate's Dissertation [in Russian], Khar'kov (1977).

[16]V. P. Shestopalov, L. N. Litvinenko, S. A. Masalov, and V. G. Sologub, Wave Diffraction by Gratings [in Russian], Izd. KhGU, Khar'kov (1971).

[17]I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products [in Russian], Nauka, Moscow (1971).

[18]V. Twersky, Archive for Rational Mechanics and Analysis 8, 323 (1961).

[19]É. I. Veliev, A. I. Nosich, and V. P. Shestopalov, Radiotekh. Elektron. 22, 466 (1977).

[20]A. I. Nosich and V. P. Shestopalov, Pis'ma Zh. Tekh. Fiz. 4, 114 (1978) [Sov. Tech. Phys. Lett. 4, 49 (1978)].

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