Oscillations of the thermodynamic properties of a one-dimensional mesoscopic ring caused by Zeeman splitting

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It is shown that the interrelationship of spin splitting in a magnetic field and spatial quantization in a one-dimensional ballistic ring coupled with a reservoir results in mesoscopic oscillations of a new type which vanish with increasing temperature. The period of such oscillations is inversely proportional to the density of states in the spin subsystem in the ring.
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The oscillations of the thermodynamic[1] and the kinetic[2] properties of doubly-connected samples (rings) at low temperatures as a function of the magnetic flux with period equal to the magnetic flux quantum $\Phi_0 = h/e$ is a fundamental effect in mesoscopic physics, and it is a manifestation of the Aharonov–Bohm (AB) effect[3] in solids. In thermodynamics this effect leads to the existence of thermodynamically equilibrium (persistent) currents in normal (nonsuperconducting) rings. Such currents were predicted theoretically in Refs. 4 and 5 and observed experimentally in Refs. 6–8. Persistent currents are due to the AB effect in systems with a discrete spectrum. As the temperature increases, the persistent current vanishes when the temperature exceeds the characteristic splitting between the energy quantization levels of electrons in the ring.[1]

An important property of a persistent current is the parity effect,[9–12] which is that the properties of the current depend on the parity of the number of particles N_0 in the ground state for spinless electrons or on N_0 modulo 4 for electrons with spin. This effect occurs for isolated rings and for rings coupled with an electron reservoir.[11,13,14] Specifically, for an even number of electrons with spin in the ground state the period of the AB oscillations is equal to the magnetic flux quantum, while the period for odd N_0 is $\Phi_0/2$.

Although a magnetic flux formally acts only on the charge degree of freedom of a system of electrons, because of parity the spin subsystem also strongly influences the AB oscillations, an effect which is especially strong when the interelectronic interaction is taken into account.[1] Specifically, such an influence results in the existence of fractional AB oscillations with period Φ_0/N_0 in isolated systems with a small number of electrons $N_0 \geqslant 1$.[15,12,16] In systems with a large number of particles, in the general case fractional oscillations do not occur,[16] but the interaction with a spin subsystem can decrease the period of the AB oscillations by a factor of 2 or 4.[14]

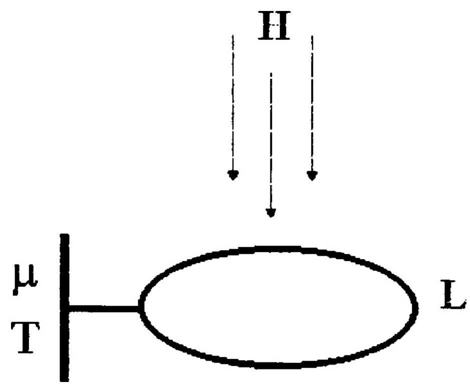

FIG. 1. One-dimensional ring of length L in a perpendicular magnetic field H. The ring is weakly coupled with an electron reservoir with chemical potential μ and temperature T.

Thus, the action on a spin subsystem will ultimately be manifested in a persistent current. The interaction with a magnetic field producing a magnetic flux in a ring lifts the spin degeneracy, and as will be shown in the present letter it results in the existence of a new type of mesoscopic oscillations in a ring coupled with a reservoir. Specifically, as the intensity of the magnetic field varies, the state of the electronic system oscillates (with a period in terms of the magnetic flux much greater than Φ_0) between a state characteristic for rings with even N_0 and a state characteristic for odd N_0 (see Ref. 14). These states differ qualitatively from one another. Specifically, as already noted, the fundamental period of the AB oscillations is Φ_0 in the first case and $\Phi_0/2$ in the second case.

Let us consider a one-dimensional ballistic ring (Fig. 1) with length L, weakly coupled with an electron reservoir with chemical potential μ and temperature T low enough so that inelastic processes in the ring can be neglected. A magnetic field H, corresponding to magnetic flux $\Phi = HL^2/4\pi$ through the ring, is applied perpendicular to the plane of the ring. We shall assume that the chemical potential of the electron reservoir does not depend on the magnetic field and is the same for electrons with opposite spins. Assuming the number N_e of particles in the ring to be large, we can linearize the electron spectrum near the Fermi points and describe the interacting electrons on the basis of a Luttinger-liquid model.[17] Then the Lagrangian L_{LL} in the boson form for an electron system has the form[18]

$$L_{LL} = \frac{\hbar v_\rho}{2g_\rho}\left\{\frac{1}{v_\rho^2}\left(\frac{\partial\theta_\rho}{\partial t}\right)^2 - \left(\frac{\partial\theta_\rho}{\partial x}\right)^2\right\} + \frac{\hbar v_\sigma}{2g_\sigma}\left\{\frac{1}{v_\sigma^2}\left(\frac{\partial\theta_\sigma}{\partial t}\right)^2 - \left(\frac{\partial\theta_\sigma}{\partial x}\right)^2\right\}, \tag{1}$$

where x is the coordinate along the ring, t is the time, and v_i and g_i are the Haldane parameters $(i = \rho, \sigma)$. The indices ρ and σ mark quantities describing the charge and spin degrees of freedom. We take account of the interaction with the magnetic field and particle exchange with the reservoir by means of the following Lagrangian:

$$L_{\text{int}} = \frac{2\hbar}{L}\pi^{1/2}\left\{\frac{\partial\theta_\rho}{\partial t}\left[\frac{k_{j\rho}}{4} + \frac{\Phi}{\Phi_0}\right] + \frac{\partial\theta_\sigma}{\partial t}\frac{k_{j\sigma}}{4}\right\} + \frac{g}{2}\beta H\frac{N_\sigma}{L} + \mu\frac{N_\rho}{L}. \tag{2}$$

Here $k_{j\rho}$ and $k_{j\sigma}$ are topological numbers determined by the parity of the numbers $N_{e\uparrow}$ and $N_{e\downarrow}$ of electrons with a definite spin projection in the ring;[11,14] g is the gyromagnetic ratio for electrons in the ring; $\beta = e\hbar/(2m)$ is the Bohr magneton. The numbers of charge and spin excitations in the ring are the same, correspondingly, $N_\rho \equiv N_e = N_{e\uparrow} + N_{e\downarrow}$ and $N_\sigma = N_{e\uparrow} - N_{e\downarrow}$. In our case the interaction with a uniform magnetic field H actually reduces to the AB interaction[1] [first term in Eq. (2)] and a Zeeman interaction [second term in Eq. (2)], which does not depend on the orbital motion. We note that for a nonuniform magnetic field the Zeeman interaction results in an effective spin-orbit interaction.[19]

We shall now calculate the magnetic field dependent part $\Delta\Omega(H)$ of the thermodynamic potential of the electrons in the ring. It is known[11] that in the ballistic case this dependence is determined by the contribution of the zeroth modes of the bosonic fields. We shall assume that for $H=0$ the ground state is nonmagnetic: $N_{0\sigma}=0$. Calculations similar to those presented in Ref. 14 give

$$\Delta\Omega(H) = -T\ln(Z), \tag{3}$$

where

$$\begin{aligned} Z = \Big\{ & \theta_3(2\phi, q_\rho^4)\,\theta_3(0, q_\sigma^4)\,\theta_3(2\delta_\mu, q_{0\rho}^4)\,\theta_3(2\delta_z, q_{0\sigma}^4) \\ & + \theta_2(2\phi, q_\rho^4)\,\theta_2(0, q_\sigma^4)\,\theta_2(2\delta_\mu, q_{0\rho}^4)\,\theta_2(2\delta_z, q_{0\sigma}^4) \\ & + \theta_3\left(\left(2\phi + \frac{1}{2}\right), q_\rho^4\right)\theta_3\left(\frac{1}{2}, q_\sigma^4\right)\theta_3\left(\left(2\delta_\mu + \frac{1}{2}\right), q_{0\rho}^4\right)\theta_3\left(\left(2\delta_z + \frac{1}{2}\right), q_{0\sigma}^4\right)\Big\}. \end{aligned}$$

Here

$$\theta_2(\nu, q) = 2\sum_{n=0}^{\infty} q^{(n+1/2)^2}\cos\left(2\pi\left(n + \frac{1}{2}\nu\right)\right), \quad \theta_3 = (\nu, q) = 1 + 2\sum_{n=1}^{\infty} q^{n^2}\cos(2\pi n\nu)$$

is the Jacobi theta function;[20]

$$q_{\rho(\sigma)} = \exp\left(-\frac{T}{2T^*_{\rho(\sigma)}}\right); \quad q_{0\rho(\sigma)} = \exp\left(-\frac{\pi^2 T}{8T_{0\rho(\sigma)}}\right),$$

where

$$T^*_{\rho(\sigma)} = \frac{\hbar v_{\rho(\sigma)} g_{\rho(\sigma)}}{\pi L}, \quad T_{0\rho(\sigma)} \frac{\pi\hbar v_{\rho(\sigma)}}{g_{\rho(\sigma)} L}; \quad \phi = \frac{\Phi}{\Phi_0} \bmod 1;$$

$$\delta_\mu = \frac{\mu}{4T_{0\rho}} \bmod 1; \quad \delta_z = \frac{g\beta H}{8T_{0\sigma}} \bmod 1.$$

The expression (3) was obtained for an odd number $N_{0\uparrow(\downarrow)}$ of electrons with a definite spin projection in the ground state ($T=0$, $H=0$, $\delta_\mu=0$). For an even number $N_{0\uparrow(\downarrow)}$ the formal substitution $\phi \to \phi + 0.5$ must be made in the expression (3). Comparing the expression (3) with the expressions presented in Ref. 14 it can be concluded that as a function of the parameter δ_z we obtain an expression describing a system with an even number of particles in the ground state ($\delta_z = 0$) as well as an expression describing a system with odd N_0 ($\delta_z = 1/4$).

It is obvious that the magnetic field enters in two ways in the expression for $\Delta\Omega$. First, it enters via the parameter ϕ, which gives rise to the conventional AB oscillations (with a period in terms of the magnetic flux Φ_0) in an isolated ring ($N_e = \text{const}$) and in a ring coupled with a reservoir ($\mu = \text{const}$). Second, it enters via the parameter δ_z, which also results in oscillations of the thermodynamic potential in a magnetic field. In the present letter we analyze oscillations of the second type. We note that in the present model such oscillations exist only in the $\mu = \text{const}$ regime and they are absent for an isolated ring.

Physically, the oscillations under study are due to the following. As the magnetic field increases, for electrons with a definite spin projection spin splitting results in a monotonic shift of the spatial quantization levels in the ring relative to a fixed chemical potential μ. Ultimately, this increases the number of electrons with spin projection along the field in the ground state of the ring and decreases the number of electrons with the opposite spin projection, i.e., the number of excitations in the ground state becomes different from zero, $N_{0\sigma} \neq 0$. The number of charge excitations $N_{0\rho}$ in the ground state remains unchanged. For this reason, the Coulomb blockade effect,[21] which is important for mesoscopic systems and is due to a small electrostatic capacitance of the system, has no effect on the oscillations under study.

Let us now determine the period of the oscillations which are due to the Zeeman effect. Since the function $\theta_2(v,q)$ is periodic with period 2 as a function of the first argument and the period of the function $\theta_3(v,q)$ is 1 with respect to v, we conclude from the expression (3) that the period of the oscillations under study is

$$\Delta\left(\frac{g}{2}\beta H\right) = 4T_{0\sigma}. \tag{4}$$

The quantity $4T_{0\sigma}$ determines the energy required to increase the number of spin excitations in the ring. Thus, in a model of noninteracting electrons ($g_\rho = g_\sigma = 2$; $v_\rho = v_\sigma = v_F$, where v_F is the Fermi velocity) we have $T_{0\sigma} = T_{0\rho} = \Delta_F/4$, where Δ_F is the splitting between the energy levels of electrons near the Fermi energy (for $H = 0$). Thus, taking account of the chiral and spin degeneracies, the period (4) of the oscillations corresponds to a change in the number of spin excitations by 4.

Next, let us calculate the persistent current $I = -\partial\Omega/\partial\Phi$ in the ring. When differentiating with respect to Φ, the parameter δ_z can be assumed to be constant, since the corresponding period [see Eq. (4)] scaled to the magnetic flux (for noninteracting electrons)

$$\Delta\Phi = \Phi_0 \frac{2}{g} \frac{m}{m^*} \frac{N_0}{4}$$

(where m^* is the electron effective mass) is much greater than the period Φ_0 of the AB oscillations. The change in δ_z accompanying a change in Φ gives corrections of the order of $1/N_0$, which can be neglected. Thus, in the mesoscopic limit $N_0 \gg 1$ in a one-dimensional ring the Zeeman effect does not distort the AB oscillations, but it leads to a periodic variation of the amplitude of such oscillations as a function of the magnetic field. We present an expression for the sum of the amplitudes of all odd harmonics $I_1 = I(\phi = \frac{1}{4})$. We note that I_1 for $T \gg T_\rho^*$ is actually the amplitude of the first harmonic of the current:

$$I_1 = \frac{T}{\Phi_0} \frac{F\left(\frac{1}{4}, q_\rho\right)}{Z\left(\phi = \frac{1}{4}\right)} \theta_3\left(\frac{1}{2}, q_\rho^4\right) \theta_2(0, q_\sigma^4) \theta_2(2\delta_\mu, q_{0\rho}^4) \theta_2(2\delta_z, q_{0\sigma}^2), \tag{5}$$

where

$$F(v,q) = 2\pi \sum_{n=1}^{\infty} (-1)^n \frac{\sin(2\pi n v)}{\sinh(n \ln(1/q))}.$$

The asymptotic representation of I_1 in the limit $T \to 0$ and at high temperatures for the model of noninteracting electrons are as follows:

$$I_1/I_0 = \begin{cases} -\dfrac{1 - \exp\left(-\dfrac{\Delta_F}{T}\left(\dfrac{1}{4} - |\delta_z|\right)\right)}{1 + \exp\left(-\dfrac{\Delta_F}{T}\left(\dfrac{1}{4} - |\delta_z|\right)\right)}, & T \ll \dfrac{\Delta_F}{2\pi^2}, \quad \delta_m = 0, \quad |\delta_z| < \dfrac{1}{2}, \\ -\dfrac{16\pi T}{\Delta_F} \exp\left(-\dfrac{2\pi^2 T}{\Delta_F}\right) \cos(2\pi\delta_\mu) \cos(2\pi\delta_z), & T \gg \dfrac{\Delta_F}{2\pi^2}, \end{cases} \tag{6}$$

where $I_0 = e v_F / L$.

It follows from the expressions presented that the quantity I_1 for $\delta_z = \pm 1/4$ vanishes and the period of the AB oscillations decreases by a factor of 2. This is due to the change in the number of spin excitations in the ring by 1 (compared with $\delta_z = 0$). In this case the numbers of electrons with the opposite direction of the spin have a different parity, which, as is well known,[1,14] gives rise to a period of $\Phi_0/2$ for the AB oscillations. At the same time, I_1 changes sign for $\delta_z = 1/2$ as a result of a change in the number of spin excitations in the ring by 2. Figure 2 shows I_1 (curve *1*) as a function of the magnetic field (the parameter δ_z). The figure also shows the analogous dependence (curve *2*) for the sum of the amplitudes I_2 of the even harmonics. We note that as temperature increases, $T \gg T_{0\sigma}$, and quantization of the spectrum of the spin subsystem becomes immaterial, the oscillations under study vanish.

In summary, for sufficiently strong magnetic fields the spin splitting can result in an additional (by π) change in the phase of the AB oscillations or in a decrease in the period of the oscillations by a factor of 2. Let us estimate the characteristic change ΔH of the magnetic field [see expression (4)] for which this effect can be observed. A persistent current has been observed experimentally in ballistic rings produced in a two-dimensional electron gas in a GaAsAs/GaAs heterostructure.[8] In this case $L \approx 10^{-5}$ m

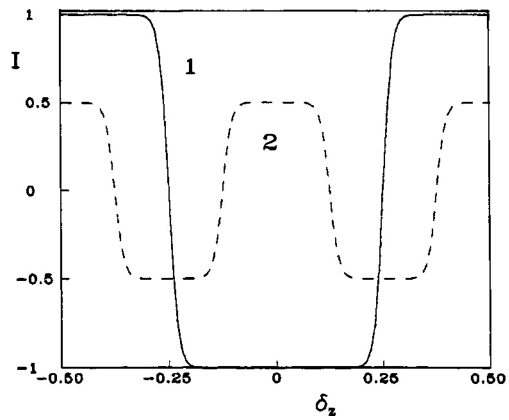

FIG. 2. Sum of the amplitudes of odd I_1 (*1*) and even I_2 (*2*) harmonics of the persistent current in units of $I_0 = ev_F/L$ versus the magnetic field (the parameter δ_z) for noninteracting electrons. $T/\Delta_F = 0.01$, $\delta_\mu = 0$.

and $v_F = 2.6\times 10^5$ ms^{-1}. Assuming the gyromagnetic ratio $g=2$, in the model of noninteracting electrons we obtain $\Delta H = 1.8$ T, which corresponds to $\delta_z = 1$. We note that the period of the AB oscillations is $\Delta H_{AB} \approx 5\times 10^{-4}$ T.

In the present letter we studied the influence of the Zeeman effect on the thermodynamic properties of a uniform ballistic ring coupled with a reservoir and containing interacting electrons in a perpendicular magnetic field. It was shown that spin splitting causes the properties of the ring to oscillate with a nonuniversal period in terms of the magnetic flux period. This period is proportional to the product of the magnetic flux quantum $\Phi_0 = h/e$ and the number N_0 of particles in the ring. This effect introduces an additional phase change in the AB oscillations as a function of the magnetic field as a result of a change in the number of spin excitations in the ring.

[1] A. A. Zvyagin and I. V. Krive, Fiz. Nizk. Temp. **21**, 687 (1995) [Low Temp. Phys. **21**, 533 (1995)].
[2] S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).
[3] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 484 (1959).
[4] I.O. Kulik, JETP Lett. **11**, 275 (1970).
[5] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. A **96**, 365 (1983).
[6] L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[7] V. Chandrasekhar, R. A. Webb, M. J. Brady *et al.*, Phys. Rev. Lett. **67**, 3578 (1991).
[8] D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).
[9] A. J. Leggett, in *Granular Nanoelectronics*, edited by D. K. Ferry, J. R. Barker, C. Jacoboni, NATO ASI, Ser. B, vol .**251** (Plenum, New York, 1991) p. 297.
[10] F. V. Kusmartev, JETP Lett. **53**, 28 (1991).
[11] D. Loss, Phys. Rev. Lett. **69**, 343 (1992).
[12] N. Yu and M. Fowler, Phys. Rev. B **45**, 11795 (1992).
[13] M. V. Moskalets, Eur. Phys. J. B **7**, 645 (1999).
[14] M. V. Moskalets, Physica E **4**, 17 (1999).
[15] F. V. Kusmartsev, J. Phys.: Condens. Matter **3**, 3199 (1991).
[16] F. V. Kusmartsev, J. E. Weisz, R. Kishore, and M. Takahashi, Phys. Rev. B **49**, 16234 (1994).
[17] F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).
[18] C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).
[19] D. Loss and P. M. Goldbart, Phys. Rev. B **45**, 13544 (1992).

[20] H. Bateman, *Higher Transcendental Functions*, edited by A. Erdélyi (McGraw-Hill Book Company, Inc., New York, 1953).
[21] D. V. Averin and K. K. Likharev, in *Mesoscopic Phenomena in Solids*, edited by B. Altshuler, P. A. Lee, and R. A. Webb (Elsevier, Amsterdam, 1991).

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