

# Peculiarities of nonlinear electrical conductivity of two-dimensional ballistic contacts

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Peculiarities of the conductivity of two-dimensional ballistic contacts that are sensitive to the nature of the confining potential as well as the existence of electrostatic potential inside the microconstriction are considered. It is shown that the position, amplitude, and shape of the peculiarities carry direct information about the position of quantization levels, the magnitude of potential inside the microconstriction, and the probability of passage of an electron through the contact. © 1997 American Institute of Physics. [S1063-777X(97)01108-0]

## 1. INTRODUCTION

Conducting structures of small size<sup>1</sup> are unique objects for studying the wave properties of charge carriers in solids. A quantum point contact in the form of a narrow constriction (whose size  $d$  is comparable with the electron wavelength  $\lambda_F$ ) connecting two macroscopic regions is an object of this type. Quantization of the transverse motion of electrons in the region of microconstriction changes the electron spectrum. Each transverse quantization level  $\varepsilon_n = (\pi n \hbar/d)^2/(2m)$  ( $n=1,2,\dots$ ) has a one-dimensional subband  $\varepsilon_n(p_x) = p_x^2/(2m) + \varepsilon_n$  (the  $x$ -axis is directed along the contact axis) with conductivity  $G_0 = 2e^2/h$  corresponding to it. The quantization of the conductance of two-dimensional ballistic contacts discovered in Refs. 2 and 3 is a direct experimental evidence for the existence of quasi-one-dimensional conducting subbands in a ballistic microconstriction.

It was revealed as a result of theoretical analysis<sup>4–12</sup> that conductance quantization takes place for contacts with a sharp geometry, as well as contacts with a smooth shape (adiabatic contacts). According to Landauer's multichannel formula,<sup>13</sup> the conductivity of a contact can be defined as  $G = NG_0$  ( $N$  is the number of conducting subbands). A one-dimensional subband  $n$  is conducting if the condition  $\varepsilon_n < \varepsilon_F$  is satisfied ( $\varepsilon_F$  is the Fermi energy of electrons at the banks of the structure). A variation of the gate voltage  $V_g$  changes the contact diameter, leading to a variation of the position of the quantization levels. Consequently, the number of conducting subbands varies, and this is reflected in the form of steps of equal height on the dependence  $G(V_g)$ .

A variation of the voltage  $V$  applied to the contact can also change the number of conducting subbands.<sup>14</sup> Depending on the type of conductivity, the subbands are divided into three classes: conducting subbands, for which the relation  $\varepsilon_n < \varepsilon_F - |eV|/2$  is satisfied, nonconducting subbands, for which the relation  $\varepsilon_n > \varepsilon_F + |eV|/2$  is satisfied, and subbands which conduct only in one direction:  $\varepsilon_F - |eV|/2 < \varepsilon_n < \varepsilon_F + |eV|/2$ .<sup>15</sup> The change in the nature of conductivity in a subband is manifested in the form of spikes (peaks) on the dependence  $d^2I/dV^2(V)$  arranged at  $(eV)_n = 2|\varepsilon_n - \varepsilon_F|$ .<sup>16</sup> These peculiarities were observed experimentally in Refs. 15, 17 and 18.

The position of the levels  $\varepsilon_n$  is determined by the con-

tact size and nature of the potential barrier confining the transverse motion of electrons in the contact. Normally, the “hard wall” model is used, in which the electron wave function vanishes at the contact boundary  $y=y(x)$ , and the spectrum of the transverse motion is the spectrum of a particle in a potential well (with vertical walls). However, experimental results indicate<sup>18</sup> that the “soft wall” model is preferable for  $n=1$ .<sup>19</sup> This model employs the parabolic potential  $U(x,y) = U_0(x) + \omega^2 y^2/2$  which confines the transverse motion of electrons with the harmonic oscillator spectrum. However, further investigations are needed to determine finally the nature of potential barrier in a microconstriction.

In the present work, it is shown that the peculiarities of the potential forming a microconstriction can be determined from an analysis of nonlinear singularities of conductivity of two-dimensional ballistic contacts, associated with a change in the number of subbands or a change in the contact diameter or voltage. The present communication consists of the following parts. In Sec. 2, we analyze the dependence  $dI/dV_g(V_g)$ . It will be shown in Sec. 3 that the dependence  $\partial I / \partial T(V)$  has peaks whose position corresponds to the distance between the transverse quantization levels  $\varepsilon_n$  and the Fermi level  $\varepsilon_F$ . In Sec 4, we shall show that an electrostatic potential relative to the contact edges exists in a microconstriction, and analyze its effect on nonlinear singularities of the contact conductivity.

## 2. NONLINEAR CONDUCTIVITY OF A TWO-DIMENSIONAL BALLISTIC CONTACT

We shall start from the expression for current passing through a ballistic adiabatic contact with a voltage  $V$  applied across its banks<sup>1</sup> in nonlinear response regime ( $eV \ll \varepsilon_F$ ):

$$I = \frac{2e}{h} \sum_n \int d\varepsilon T_n(\varepsilon) \left\{ f_0\left(\varepsilon - \varepsilon_F - \frac{eV}{2}\right) - f_0\left(\varepsilon - \varepsilon_F + \frac{eV}{2}\right) \right\}. \quad (1)$$

Here,  $T_n(\varepsilon)$  is the probability of passage of an electron with energy  $\varepsilon$  through the contact in channel  $n$ , and  $f_0(\varepsilon) = (\exp(\varepsilon/T) + 1)^{-1}$  is the Fermi function.

It was mentioned in the Introduction that the existence of quasi-one-dimensional conducting subbands leads to a strong

nonlinearity of the current-voltage characteristics (IVC) of a quantum ballistic contact. An increase in the voltage may bring the bottom of one of the subbands  $\varepsilon_n$  in the band of current states  $\pm eV/2$  near the Fermi level  $\varepsilon_F$ , which results in a sharp variation of the contact conductivity and is manifested in the form of a spike (peak) on the dependence of the second derivative of current  $d^2I/dV^2$  with respect to voltage  $V$ .

On the other hand, a change in the contact diameter  $d$  for a fixed voltage, accomplished by a variation of the gate voltage  $V_g$ , also leads to a change in the number of conducting subbands and hence to the emergence of conductance jumps on the  $G(V_g)$  dependence (for  $V \rightarrow 0$ ),<sup>2,3</sup> as well as current jumps on the  $I(V_g)$  dependence (for  $V \neq 0$ ).<sup>14</sup> Thus, the dependence of the first derivative of current  $dI/dV_g$  with respect to the gate voltage on  $V_g$  for  $V = \text{const}$  (just like the dependence of the second derivative of current  $d^2I/dV^2$  with respect to voltage  $V$  for  $V_g = \text{const}$ ) contains peaks corresponding to a change in the number of conducting subbands.

Let us evaluate the derivative  $dI/dV_g$  by taking into account the fact that the only quantity depending on  $V_g$  in formula (1) is  $T_n$ , which is a function of  $\varepsilon_n$ :

$$\begin{aligned} \frac{dI}{dV_g} &= \sum_n \left( \frac{dI}{dV_g} \right)_n, \\ \left( \frac{dI}{dV_g} \right)_n &= \frac{2e}{h} \int d\varepsilon \frac{dT_n}{d\varepsilon} \left( -\frac{d\varepsilon_n}{dV_g} \right) \left\{ f_0 \left( \varepsilon - \varepsilon_F - \frac{eV}{2} \right) \right. \\ &\quad \left. - f_0 \left( \varepsilon - \varepsilon_F + \frac{eV}{2} \right) \right\}. \end{aligned} \quad (2)$$

A peculiar feature of the quantity  $T_n(\varepsilon)$  is that it varies from zero (for  $\varepsilon < \varepsilon_n$ ) to unity (for  $\varepsilon > \varepsilon_n$ ) in a narrow energy interval  $\Delta\varepsilon \approx \Delta_n$  in the vicinity of  $\varepsilon_n$ . Hence we can single out two limiting cases

(1)  $4T \ll \Delta_n$  ( $T$  is the temperature at the contact banks). In this case, we can put  $f_0(\varepsilon) = \theta(-\varepsilon)$ :

$$\left( \frac{dI}{dV_g} \right)_n = \frac{2e}{h} \left( -\frac{d\varepsilon_n}{dV_g} \right) \left\{ T_n \left( \varepsilon_F + \frac{eV}{2} \right) - T_n \left( \varepsilon_F - \frac{eV}{2} \right) \right\}, \quad (3a)$$

or, for  $V \ll \Delta_n$

$$\left( \frac{dI}{dV_g} \right)_n = \frac{2e^2}{h} V \left( -\frac{d\varepsilon_n}{dV_g} \right) \frac{dT_n}{d\varepsilon}. \quad (3b)$$

(2)  $4T \gg \Delta_n$ . In this case, we can put  $T_n(\varepsilon) = \theta(\varepsilon - \varepsilon_n)$ :

$$\left( \frac{dI}{dV_g} \right)_n = \begin{cases} \frac{2e}{h} \left( -\frac{d\varepsilon_n}{dV_g} \right) \Omega(\varepsilon_n - \varepsilon_F + eV/2), & V \gg T, \\ \frac{e^2}{2h} \frac{V}{T} \left( -\frac{d\varepsilon_n}{dV_g} \right) \cosh^{-2} \left( \frac{\varepsilon_n - \varepsilon_F}{T} \right), & V \ll T. \end{cases} \quad (4)$$

Here,  $\Omega(x) = [1 + \exp(-x/T) + \exp((x-V)/T)]^{-1}$ . Let us also consider the expression for the peak amplitude [for  $\varepsilon_n(V_g) = \varepsilon_F$ ] for an arbitrary relation between  $V$  and  $T$ :

$$\left( \frac{dI}{dV_g} \right)_{\varepsilon_n = \varepsilon_F} = \frac{2e}{h} \left( -\frac{d\varepsilon_n}{dV_g} \right) A_n(V, T),$$

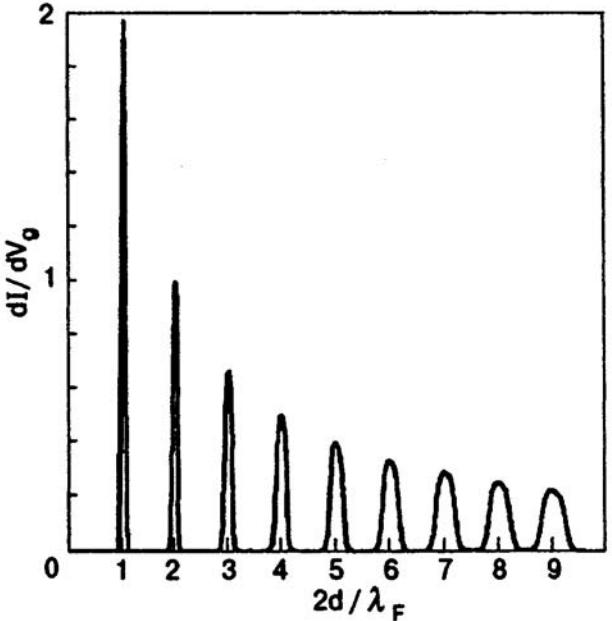


FIG. 1. Dependence of  $dI/dV_g$  on contact diameter for  $T=0.01\varepsilon_F$  and  $V=0.1\varepsilon_F$ .

$$A_n(V, T) = \tanh \left( \frac{V}{4T} \right). \quad (5)$$

Let us analyze the above expressions. The dependence  $dI/dV_g(V_g)$  consists of a sequence of peaks arranged at  $\varepsilon_n(V_g) = \varepsilon_F$  and corresponding to the “passage” of the quantization level  $n$  through the current interval  $\pm eV/2$  in the vicinity of the Fermi level  $\varepsilon_F$ :  $\varepsilon_F - eV/2 < \varepsilon_n(V_g) < \varepsilon_F + eV/2$ . The amplitudes of the peaks are defined by the quantities  $d\varepsilon_n/dV_g$  and  $A_n(V_g, V, T)$ . For  $4T \gg \Delta_n$ , the dependences  $A_n(V_g)$  are identical for all  $n$ . Hence, the ratio of the peak amplitudes in this case is determined exclusively by the quantity  $d\varepsilon_n/dV_g$ . In the following, we shall consider two traditionally used models of the potential corresponding to the motion of electrons in the contact region, viz., the “hard wall” and the “soft wall” models.<sup>19</sup>

In the “hard wall” model, the contact boundary is assumed to be impermeable to electrons with any energy. The quantization levels  $\varepsilon_n = (\pi n \hbar/d)^2/(2m)$  depends on the contact diameter  $d$  which is assumed to be proportional to the gate voltage  $V_g$ . In this model, the variable  $V_g$  can be replaced by the variable  $d$  (it is more convenient to use the dimensionless quantity  $\xi = 2d/\lambda_F$ ). Computing  $d\varepsilon_n/d\xi = -2\varepsilon_n/\xi$ , we can easily show that the ratio of peak amplitudes in the “hard wall” model (for  $4T \gg \Delta_n$ ) is defined as (Fig. 1)

$$\left( \frac{dI}{dV_g} \right)_1 : \left( \frac{dI}{dV_g} \right)_2 : \left( \frac{dI}{dV_g} \right)_3 : \dots = \frac{1}{1} : \frac{1}{2} : \frac{1}{3} : \dots \quad (6)$$

In the “soft wall” model, the transverse motion of electrons is confined by a parabolic potential. The quantization levels are equidistant:  $\varepsilon_n = U_0(V_g) + \hbar\omega(n+1/2)$ . It is assumed that the gate voltage does not change the relative separation between levels, i.e., the frequency  $\omega$ , but varies only the potential  $U_0$  inside the contact. The dependence

$U_0(V_g)$  is assumed to be linear. In this case, the quantity  $d\varepsilon_n/dV_g = \text{const}$  and does not depend on  $n$ . Hence all peaks in the “soft wall” model (for  $4T \gg \Delta_n$ ) have the same height:

$$\left( \frac{dI}{dV_g} \right)_1 : \left( \frac{dI}{dV_g} \right)_2 : \left( \frac{dI}{dV_g} \right)_3 : \dots = 1:1:1:\dots . \quad (7)$$

A comparison of formulas (6) and (7) leads to the conclusion that an analysis of the relative amplitudes of the dependence  $dI/dV_g(V_g)$  makes it possible to choose between various models of the potential confining the transverse motion of electrons in the constriction region.

As the temperature decreases ( $4T \ll \Delta_n$ ), formulas (6) and (7) remain valid for  $V \gg \Delta_n$  [see formula (3a)]. However, an additional factor  $dT_n/d\varepsilon \approx 1/\Delta_n$  appears for  $V \ll \Delta_n$  (see formula (3b)). In the “soft wall” model,<sup>19</sup>  $\Delta_n \approx \text{const}$ , and formula (7) remains valid. In the “hard wall” model,  $\Delta_n \approx 1/n$  (see, for example, Refs. 4 and 6) and amplitudes of all peaks become equal. It should be mentioned, however, that Zagorskin and Kulik<sup>11</sup> obtained  $\Delta_n = \text{const}$  for a contact with special geometry in the “hard wall” model. Hence formula (6) remains valid in this model for the entire range of variation of temperature  $T$  and bias voltage  $V$ .

An analysis of the dependence of the width  $(\Delta V_g)_n$  of peaks on their number  $n$  also makes it possible to choose between models of confining potential in the contact:

$$(\Delta V_g)_n = \begin{cases} n \cdot \max(T, V, \Delta_n) & \text{in the hard wall model,} \\ \max(T, V, \Delta_n) & \text{in the soft wall model.} \end{cases} \quad (8)$$

Moreover, an analysis of the dependence of the peak width on temperature  $T$  and bias voltage  $V$  allows us to determine experimentally the quantity  $\Delta_n$ .

An increase in the bias voltage  $V$  or temperature  $T$  increases the width of peaks and the peaks merge. In this case, the dependence  $dI/dV_g(V_g)$  becomes smooth and attains a classical asymptotic form. In the classical (not quantum) limit ( $d \gg \lambda_F$ ), the current  $I$  passing through a ballistic contact is equal to  $4e^2 dV/(h\lambda_F)$ , and hence  $dI/d\xi = G_0 V$  ( $G_0 = 2e^2/h$  is the conductance quantum). The current attains the classical value when the bias voltage  $V$  or the width of temperature blurring of Fermi steps becomes equal to the separation between quantization levels (see Fig. 2):

$$\Delta\varepsilon_n \approx \max(V, 4T). \quad (9)$$

### 3. TEMPERATURE AND FIELD SPECTROSCOPY OF TRANSVERSE QUANTIZATION LEVELS IN A CONTACT

In the preceding section, we considered the peculiarities of the  $I-V_g$  characteristics of a contact associated with the “passage” of the quantization level  $\varepsilon_n$  through the current interval  $\pm eV/2$  in the vicinity of the Fermi level. In this section, we shall consider the IVC nonlinearity associated with the “passage” of the quantization levels through the region of temperature blurring of the edge of the Fermi step. This nonlinearity is manifested on the dependence of  $\partial I/\partial T$  on the bias voltage  $V$ . Differentiating formula (1) with respect to temperature, we obtain

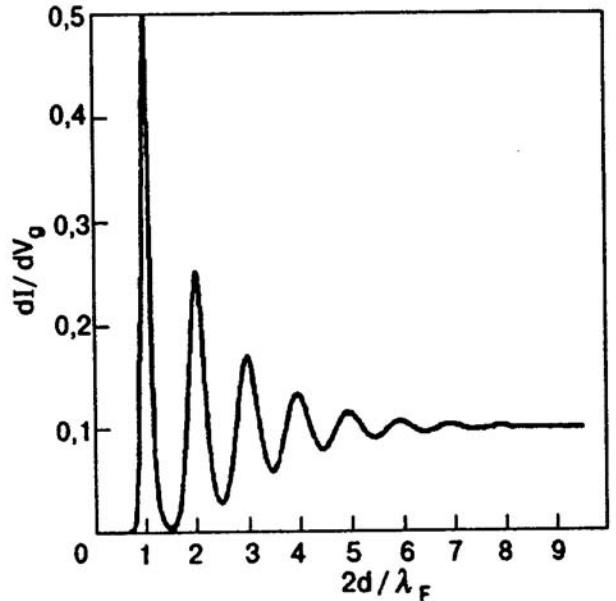


FIG. 2. Dependence of  $dI/dV_g$  on contact diameter for  $T, V > \Delta_n$ . The values of parameters are  $T = 0.1\varepsilon_F$  and  $V = 0.1\varepsilon_F$ .

$$\frac{\partial I}{\partial T} = \frac{2e}{h} \sum_n \int d\varepsilon \frac{dT_n}{d\varepsilon} \left\{ \Psi\left(\frac{\varepsilon - \varepsilon_F - eV/2}{T}\right) - \Psi\left(\frac{\varepsilon - \varepsilon_F + eV/2}{T}\right) \right\}. \quad (10)$$

Here,  $\Psi(x) = \ln[1+\exp(x)] - x[1+\exp(-x)]^{-1}$ . Formula (10) is simplified in two limiting cases:

(a) for  $4T \gg \Delta_n$

$$\frac{\partial I}{\partial T} = \frac{2e}{h} \sum_n \left\{ \Psi\left(\frac{\varepsilon_n - \varepsilon_F - eV/2}{T}\right) - \Psi\left(\frac{\varepsilon_n - \varepsilon_F + eV/2}{T}\right) \right\}; \quad (11)$$

(b) for  $4T \ll \Delta_n$

$$\frac{\partial I}{\partial T} = 6.6T \frac{e}{h} \sum_n \left\{ \frac{dT_n}{d\varepsilon} (\varepsilon_F + eV/2) - \frac{dT_n}{d\varepsilon} (\varepsilon_F - eV/2) \right\}. \quad (12)$$

Thus, the dependence  $\partial I/\partial T(V)$  consists of a set of positive and negative peaks arranged at  $(eV)_n = 2|\varepsilon_n - \varepsilon_F|$  (see Fig. 3, curve 1). In the case (a) corresponding to the situation when the region of temperature blurring of the Fermi step exceeds  $\Delta_n$ , the shape of the peak does not depend on  $n$  and is determined by the function  $\Psi(x)$ . Note that the amplitude of the peak in this case does not depend on its number and temperature:  $(\partial I/\partial T)_{\varepsilon_n = \varepsilon_F} = (2e/h)\ln 2$ , while the peak width is proportional to temperature. As the temperature decreases ( $4T \ll \Delta_n$ ), the shape of the peak is determined by the energy derivative of the transmission coefficient  $dT_n/d\varepsilon(\varepsilon)$ . In this case, the width of the peak is independent of temperature (and is equal to  $\Delta_n$ ) and the amplitude is proportional to the ratio  $T/\Delta_n$ . The effect vanishes at  $T=0$ .

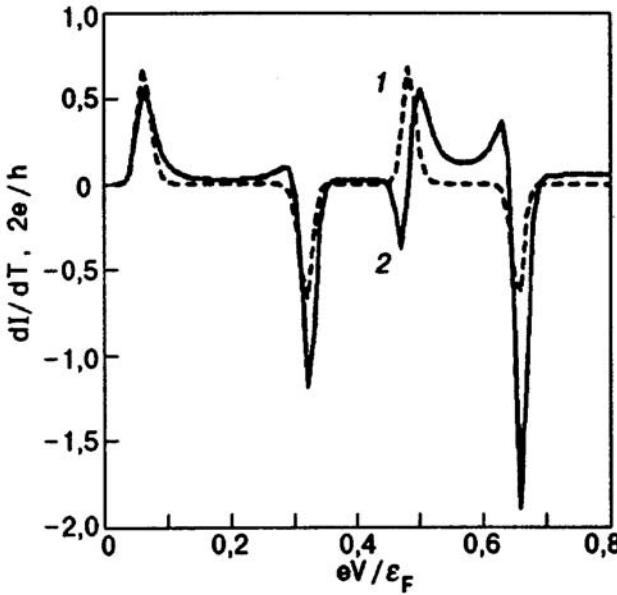


FIG. 3. Dependence of  $\partial I / \partial T$  on voltage  $V$ . Curve 1 is obtained without consideration of the potential at the center of microconstriction. Curve 2 takes the potential  $\Phi$  into account. The values of parameters are  $T = 0.002\epsilon_F$  and  $d = 5.25\lambda_F$ .

Thus, an analysis of the dependence  $\partial I / \partial T(V)$  makes it possible to determine the position of quantization levels  $(eV)_n = 2|\epsilon_n - \epsilon_F|$  in the contact (for a fixed contact diameter  $d$ ). Moreover, the temperature dependence of the peak width leads to the value of  $\Delta_n$ . Note that the position of quantization levels can also be determined from the position of the peaks on the dependence  $d^2 I / dV^2(V)$ ,<sup>16</sup> although only broad singularities are observed instead of peaks in actual experiments.<sup>18</sup>

#### 4. EFFECT OF QUANTUM ELECTROSTATIC POTENTIAL ON THE CONDUCTIVITY OF A BALLISTIC CONTACT

We shall show that one-dimensionalization of electron spectrum in the region of microconstriction leads to the existence of a potential difference in thermodynamic equilibrium between the constriction and contact banks (quantum electrostatic potential).<sup>20</sup> We shall study the effect of this potential difference on nonlinear singularities of the conductivity of the contact considered in the previous section.

For the sake of simplicity, we shall consider a ballistic contact in the form of a channel of width  $d$  and length  $L > d$ . The potential difference between the contact and the banks is denoted by  $\Phi(d)$ . In this case, the electron number density  $n(d)$  in the contact can be written in the form

$$n(d) = \frac{2}{hd} \sum_n \int dp_x f_0[\epsilon_n + e\Phi(d) + p_x^2/(2m) - \epsilon_F]. \quad (13)$$

Here,  $\epsilon_F$  is the chemical potential of electrons at the banks (the potential of the banks is assumed to be zero). The potential  $\Phi(d)$  is determined from the self-consistency condition which, in the limit of strong screening

$$L, d \gg r_s \quad (14)$$

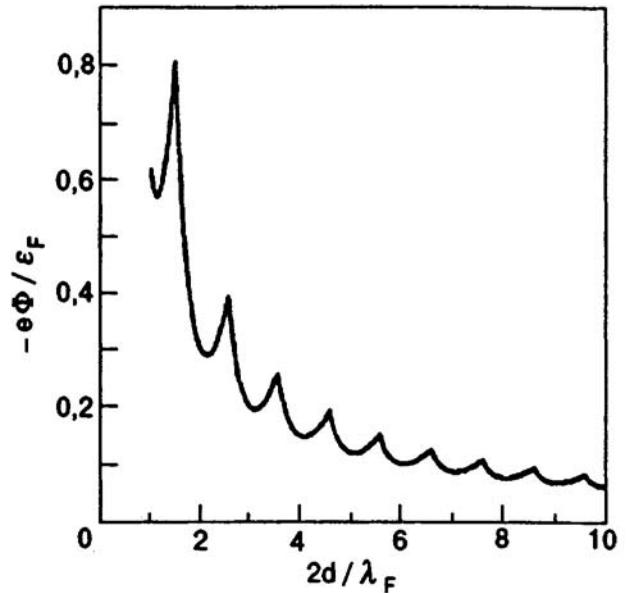


FIG. 4. Dependence of potential  $\Phi$  on the microconstriction diameter.

( $r_s$  is the screening radius) is reduced to the condition of electrical neutrality. Assuming that the background of positive charge is the same at the banks and in the channel, we obtain from the electroneutrality condition

$$n(d) = n_0, \quad (15)$$

where  $n_0 = 2\pi/\lambda_F^2$  is the electron density at the banks. Thus, formulas (13) and (15) lead to the self-consistency condition which determines the quantity  $\Phi(d)$ :

$$\frac{\lambda_F^2}{\pi h d} \sum_n \int dp_x f_0[\epsilon_n + e\Phi(d) + p_x^2/(2m) - \epsilon_F] = 1. \quad (16)$$

In the “hard wall” model at zero temperature ( $T=0$ ), this relation gives

$$\frac{4}{\pi \xi} \sum_n (1 + \varphi(d) - n^2/\xi^2)^{1/2} \theta(1 + \varphi(d) - n^2/\xi^2) = 1. \quad (17)$$

Here,  $\varphi(d) = -e\Phi(d)/\epsilon_F$ . The dependence  $\varphi(d)$  is plotted in Fig. 4. The peaks on this dependence correspond to the “inclusion” of the next conductivity channel, which occurs when the following condition is satisfied:

$$\epsilon_n + e\Phi(d) = \epsilon_F. \quad (18)$$

The corresponding channel width  $d_n$  is given by

$$d_n = \frac{\lambda_F}{2} \frac{n}{(1 + \varphi_n)^{1/2}}, \quad (19)$$

The potential in the channel is defined as

$$\varphi_n = \frac{\pi}{4} n \left( \sum_{k=1}^{n-1} \left( 1 - \frac{k^2}{n^2} \right)^{1/2} \right)^{-1} - 1. \quad (20)$$

It can easily be shown that, if we take into account the quantum electrostatic potential (QEP)  $\Phi(d)$ , the “inclusion” of a conducting subband with number  $n$  occurs for  $\xi_n \approx n - 0.5(n \gg 1)$ .

Upon a decrease in the contact diameter ( $d \approx \lambda_F$ ), the screening radius ( $r_s \approx \lambda_F$ ) becomes comparable with  $d$ , and the strong screening approximation becomes invalid. In this case, a charge layer formed near the channel edges hampers an increase in the potential inside the channel upon a decrease in its diameter. Hence the variation of the potential  $\Phi$  will no longer compensate the increase in the energy  $\varepsilon_1 \approx 1/d^2$  and the channel will become nonconducting. Note that the channel would have remained conducting in the strong screening limit even for  $d \rightarrow 0$ .

In the current state, the potential  $\Phi$  also depends on the voltage applied to the contact. In this case, the self-consistency condition assumes the form

$$\frac{\lambda_F^2}{\pi h d} \sum_n \int dp_x \theta(p_x) \{ f_0 [\varepsilon_n + e\Phi(d, V, T) + p_x^2/(2m) - \varepsilon_F - eV/2] + f_0 [\varepsilon_n + e\Phi(d, V, T) + p_x^2/(2m) - \varepsilon_F + eV/2] \} = 1. \quad (21)$$

Formulas (16) and (21) are also applicable for a contact of arbitrary shape. The quantity  $d$  stands for the contact size at the narrowest position. Moreover, the change in the contact diameter over a distance  $\sim \lambda_F$  must be small:  $d(\ln d)/dx \ll \lambda_F^{-1}$ . Note that the expressions presented above were obtained under the assumption  $T_n = \theta(\varepsilon - \varepsilon_n)$ , which is valid at least for  $4T \gg \Delta_n$ .

Let us now study how the existence of a potential  $\Phi(d, V, T)$  changes the results obtained in previous sections. Note that the results obtained in this section are applicable for a broad contact ( $d > \lambda_F$ ) for which the “hard wall” model of confining potential is more suitable.

The existence of the potential  $\Phi$  can be taken into account easily by replacing  $\varepsilon_n$  with  $\varepsilon_n + e\Phi$ . As a result, the expression for  $dI/dV_g$  assumes the form

$$\frac{dI}{dV_g} = \frac{2e}{h} \sum_n \int d\varepsilon \frac{dT_n}{d\varepsilon} \left( -\frac{d\varepsilon_n}{dV_g} - e \frac{d\Phi}{dV_g} \right) \left\{ f_0 \left( \varepsilon - \varepsilon_F - \frac{eV}{2} \right) - f_0 \left( \varepsilon - \varepsilon_F + \frac{eV}{2} \right) \right\}. \quad (22)$$

It can easily be shown that the peak amplitude for  $4T \gg \Delta_n$  is proportional to the quantity

$$\left( \frac{dI}{dV_g} \right)_n \approx \frac{(1 + \varphi_n)^{3/2}}{n}, \quad (23)$$

where  $\varphi_n$  is defined by formula (20) for  $T=0$  and  $V=0$ . If  $T$  and  $V$  are not equal to zero, we must solve Eq. (21) numerically to obtain the quantity  $\varphi_n$ . It follows from formula (23) that, if we take QEP into consideration, the ratios of peak amplitudes on the dependence  $dI/dV_g(V_g)$  no longer satisfy the simple relation (6), which can be used for experimentally determining the potential inside the microconstriction.

The expression for  $\partial I/\partial T$  also changes and assumes the following form for  $4T \gg \Delta_n$ :

$$\begin{aligned} \frac{\partial I}{\partial T} = & \frac{2e}{h} \sum_n \left\{ \Psi \left( \frac{\varepsilon_n + e\Phi - \varepsilon_F - eV/2}{T} \right) \right. \\ & - \Psi \left( \frac{\varepsilon_n + e\Phi - \varepsilon_F + eV/2}{T} \right) - e \frac{\partial \Phi}{\partial T} \left[ f_0 \left( \varepsilon_n + e\Phi - \varepsilon_F - \frac{eV}{2} \right) - f_0 \left( \varepsilon_n + e\Phi - \varepsilon_F + \frac{eV}{2} \right) \right] \left. \right\}. \end{aligned} \quad (24)$$

It follows from the above expression that the presence of QEP leads to the asymmetry of positive and negative peak amplitudes in the dependence  $\partial I/\partial T(V)$  (see Fig. 3, curve 2).

## 5. CONCLUSION

In this work, we have shown that the investigation of nonlinear singularities of two-dimensional ballistic contacts makes it possible to obtain direct information about the nature of electrostatic potential forming the microconstriction. For example, the measurement of relative amplitude of peaks on the dependence  $dI/dV_g(V_g)$  determines whether the confining potential is impenetrable for electrons or the transverse electron movement occurs in the parabolic (“soft wall”) potential.

It is also shown that the presence of electrostatic potential in the microconstriction region is a characteristic feature of quantum ballistic contacts. This potential emerges due to a difference in the nature (dimensions) of the electron spectrum in the region of microconstriction and at the contact edges. It should be observed that in the “soft wall” model<sup>19</sup> which is applicable for  $d \approx \lambda_F$ , the electrostatic potential in the microconstriction region is induced by the gate voltage  $V_g$ . According to our investigations, the electrostatic potential in the contact does not vanish upon an increase in the contact diameter ( $d > \lambda_F$ ), although its physical nature changes. In this case, the potential in the microconstriction is not connected with the gate voltage  $V_g$  determining the shape of the contact, but is due entirely to the manifestation of the quantum nature of electron motion in the contact.

<sup>1</sup> Y. Imry, in *Directions in Condensed Matter Physics* [ed. by G. Grinstein and G. Mazenko], World Scientific, Singapore (1986).

<sup>2</sup> B. J. van Wees, H. van Houten, C. W. J. Beenakker *et al.*, Phys. Rev. Lett. **60**, 848 (1988).

<sup>3</sup> D. A. Wharam, T. J. Thornton, R. Newbury *et al.*, J. Phys. C **21**, L209 (1988).

<sup>4</sup> L. I. Glazman, G. B. Lesovik, D. E. Khmel'nitskii, and R. I. Shekhter, Pis'ma Zh. Éksp. Teor. Fiz. **48**, 218 (1988) [JETP Lett. **48**, 238 (1988)].

<sup>5</sup> I. B. Levinson, Pis'ma Zh. Éksp. Teor. Fiz. **48**, 273 (1988) [JETP Lett. **48**, 301 (1988)].

<sup>6</sup> A. Szafer and A. D. Stone, Phys. Rev. Lett. **62**, 300 (1989).

<sup>7</sup> G. Kirczenow, Phys. Rev. **B39**, 10452 (1989).

<sup>8</sup> L. Escapa and N. Garsia, J. Phys. Cond. Mat. **1**, 2125 (1989).

<sup>9</sup> D. van der Marel and E. G. Haanappel, Phys. Rev. **B39**, 7811 (1989).

<sup>10</sup> A. Kawabata, J. Phys. Soc. Jpn. **58**, 372 (1989).

<sup>11</sup> A. M. Zagoskin and I. O. Kulik, Fiz. Nizk. Temp. **16**, 911 (1990) [Sov. J. Low Temp. Phys. **16**, 533 (1990)].

<sup>12</sup> E. Castano and G. Kirczenow, Phys. Rev. **B45**, 1514 (1992).

<sup>13</sup> M. Büttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. **B31**, 6207 (1985).

- <sup>14</sup>L. I. Glazman and A. V. Khaetskii, Pis'ma Zh. Éksp. Teor. Fiz. **48**, 546 (1988) [JETP Lett. **48**, 591 (1988)].
- <sup>15</sup>L. P. Kouwenhoven, B. J. van Wees, C. J. P. M. Harmans *et al.*, Phys. Rev. **B39**, 8040 (1989).
- <sup>16</sup>A. M. Zagoskin, Pis'ma Zh. Éksp. Teor. Fiz. **52**, 1043 (1990) [JETP Lett. **52**, 435 (1990)].
- <sup>17</sup>N. K. Patel, L. Martin-Moreno, M. Pepper *et al.*, J. Phys. Cond. Mat. **2**, 7247 (1990).

- <sup>18</sup>N. K. Patel, J. T. Nichols, L. Martin-Moreno *et al.*, Phys. Rev. **B44**, 13549 (1991).
- <sup>19</sup>M. Büttiker, Phys. Rev. **B41**, 7906 (1995).
- <sup>20</sup>M. V. Moskalets, Pis'ma Zh. Éksp. Teor. Fiz. **62**, 702 (1995) [JETP Lett. **62**, 719 (1995)].

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