Quantum-size electrostatic potential in two-dimensional ballistic point contacts

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It is shown that in the strong-screening limit an electrostatic potential, which appears due to the quantization of the transverse motion of electrons, exists in a microscopic constriction. This effect increases the number of conducting one-dimensional subbands in the contact.

The experimental[1,2] observation of the conductance quantization in two-dimensional ballistic point contacts is a striking demonstration of the wave behavior of current carriers. Theoretical analysis showed that conductance quantization, in the units $G_0=2e^2/h$, as a function of the contact diameter is universal and occurs in contacts with a sharp geometry[3,4] and in contacts with a smooth geometry (Refs. 5 and 6). The reason for the conductance quantization in a microscopic constriction is that the transverse (with respect to the X axis of the contact) motion of the electrons is quantized. This accounts for the existence of one-dimensional conducting subbands with an energy

$$\epsilon_m(p_x)=\epsilon_m(0)+p_x^2/2m^*, \quad \epsilon_m(0)=\pi^2\hbar^2m^2/d^2,$$

where m^* is the electron effective mass, p_x is the electron momentum, m is the subband number, and d is the width of the microconstriction at its narrowest point. The conductance of each subband is G_0. Subbands for which the condition $\epsilon_m(0)<\epsilon_F$ is satisfied are conducting subbonds (ϵ_F is the Fermi energy of the electrons at the edges of the contact). The conductance of the contact in this case is $G=MG_0$, where M is the number of conducting subbands. Since $\epsilon_m(0)$ depends on d, it is easy to see that as the contact diameter increases with $d=m\lambda_F/2$, where λ_F is the wavelength of a Fermi electron, $m=1,2\ldots$, the next (mth) subband becomes conducting and the conductance of the contact increases abruptly by G_0.

The condition $\epsilon_m(0)<\epsilon_F$ is a consequence of applying the equality of the chemical potentials of the electrons at the edges and in the microsopic constriction. A detailed analysis shows, however, that the equality of the electrochemical potentials must be used, since at thermodynamic equilibrium a quantum electrostatic potential $\Phi(d)>0$, relative to the edges of the contact, appears in the constriction.

We shall now prove this point. We write the expression for the electron density in a two-dimensional ballistic channel of width d at zero temperature as

$$n(d)=\frac{2}{hd}\sum_m \int_{-\infty}^{+\infty} dp_x\theta(\epsilon_F-\epsilon_m(p_x)-e\Phi(d)). \tag{1}$$

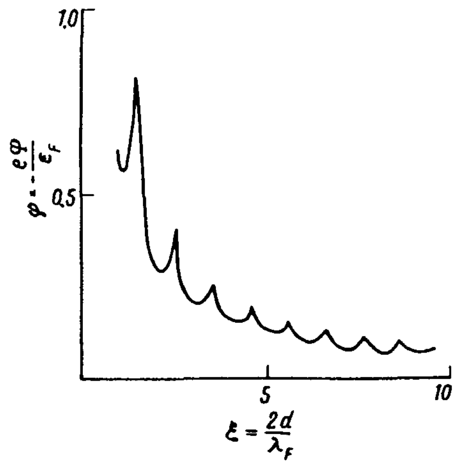

FIG. 1. Potential versus the diameter of the microconstriction.

Here $\theta(x)$ is the Heavside function. The potential $\Phi(d)$ is determined from the condition of self-consistency, which in the strong-screening limit

$$d \gg r_s, \tag{2}$$

where r_s is the screening radius, reduces to the condition of electrical neutrality. Assuming that the background formed by the positive charge is identical in the channel and at the edges (breakdown of this condition is discussed below), we have from the condition of electrical neutrality

$$n(d)=n_0, \tag{3}$$

where $n_0=2\pi/\lambda_F^2$ is the electron density at the edges. From Eqs. (1) and (3) we thus obtain the self-consistency condition which determines the magnitude of the potential:

$$\frac{4}{\pi\xi}\sum_m \left(1+\varphi-\frac{m^2}{\xi^2}\right)^{1/2} \theta\left(1+\varphi-\frac{m^2}{\xi^2}\right)=1. \tag{4}$$

Here $\varphi=-e\Phi/\epsilon_F$, and $\xi=2d/\lambda_F$.

The function $\varphi(\xi)$ is shown in Fig 1. In order of magnitude, the value of the potential Φ is equal to the splitting between the transverse-quantization levels $(\epsilon_m-\epsilon_{m+1})/e$ near the Fermi energy, and as the width of the channel increases $(\xi\to\infty)$, it approaches zero $(\varphi\to 0)$.

Taking into account the quantum electrostatic potential, we can write the condition determining the number of conducting one-dimensional subbands in the form

$$\epsilon_m(0)+e\Phi(d)<\epsilon_F. \tag{5}$$

The channel width d_m, for which the next (mth) conducting subband appears, is

$$d_m=\frac{\lambda_F}{2}\frac{m}{\sqrt{1+\varphi_m}}. \tag{6}$$

The potential (in units of ϵ_F), in this case is

$$\varphi_m=1-\frac{\pi}{4}m\left(\sum_{k=1}^{m-1}\sqrt{1-\frac{k^2}{m^2}}\right)^{-1}. \tag{7}$$

The local maxima in the function $\varphi(\xi)$ (see Fig. 1), which lie approximately at half-integer values of the parameter ξ, correspond to an increase in the number of conducting subbands. The jumps in the function $G(d)$ therefore occur at $\xi_m \approx m-0.5$.

The self-consistency equation (3) was obtained under the assumption of strong screening (2). For the systems employed in Refs. 1 and 2 , the screening radius is of the order of the Fermi wavelength. The results presented here are therefore valid in the region of the steps [in the function $G(d)$] with large numbers, $m \gg 1$. For $d \sim \lambda_F$ a charged layer, which prevents the potential in the channel from increasing, is formed near the channel edges at distances of the order of r_s. As the channel width decreases further ($d<\lambda_F$), the potential Φ no longer compensates for the increase in energy $\epsilon_1(0) \sim 1/d^2$ and the channel becomes nonconducting ($G=0$). (In the limit of strong screening the channel would remain conducting even when $d \rightarrow 0$.)

We note that this effect, i.e., the appearance of a potential difference between the constriction and the contact edges, should occur in three-dimensional ballistic contacts under conditions where quantization of the transverse motion of electrons near the microscopic constriction is important.

We now return to the condition (3). In Ref. 7 it is predicted that the electron density is a nonmonotonic function of the width of a two-dimensional channel. However, the measurements performed in Ref. 8 showed that the function $n_0(d)$ is monotonic. Such a monotonic dependence could be due to the characteristic features of the formation of a two-dimensional electron layer[9] and should be taken into account as follows: First, the decrease in the Fermi energy of the electrons in the constriction $\epsilon_F(d)=\epsilon_F n_0(d)/n_0$ leads to the appearance of a potential difference $e\Phi=\epsilon_F(1-n_0(d)/n_0)$ between the constriction and the contact edges (analog of the contact potential difference). This will result in a replacement in Eq. (1) and in the condition (5) of the Fermi energy ϵ_F at the edges by the Fermi energy $\epsilon_F(d)$ in the constriction. Second, on the right-hand side of the electrical neutrality condition (3) the electron density n_0 at the edges should be replaced by the quantity $n_0(d)$.

In conclusion, we note that we have predicted here a new effect, which can be described as the appearance of a nonmonotonic (depending on the width of the microscopic constriction) potential difference between the constriction and the contact edges. This potential difference is not associated with the electrostatic potential that forms the microscopic constriction in a two-dimensional electron layer. This is an equilibrium potential difference, in terms of thermodynamics, which arises exclusively as a result of the quantum nature of the electron motion in the microscopic constriction.

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