ESTIMATION OF ROUND-OFF ERRORS IN SOLVING BOUNDARY VALUE PROBLEMS OF MATHEMATICAL PHYSICS

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When solving various practically important problems, such as boundary value problems of mathematical physics for partial differential equations, it is necessary to calculate derivatives of complex functions. To solve the problem of taking into account rounding errors, it is proposed to use constructive tools of interval analysis, which allow estimating rounding errors using automatic differentiation methods.

The computational process can be represented as r computational steps, each of which performs basic operations and stores their values in intermediate variables a_j [1]. The computational process is considered as a sequence of operations that are performed when calculating the value of a function. For a function with n variables, the computational process is represented as follows:

$$a_1 \leftarrow c_1(b_{11}, b_{12})$$
$$...$$
$$a_j \leftarrow c_j(b_{j1}, b_{j2})$$
$$...$$
$$a_r \leftarrow c_r(b_{r1}, b_{r2}).$$

The number of intermediate variables is equal to the number of computational steps, *r*. When calculating the value of a function, the result, as a rule, is approximate due to the occurrence of rounding errors. The execution of calculations taking into account rounding errors is considered [2].

The use of formulas for automatic differentiation of the algebra of differential tuples in interval analysis allows one to obtain constructive formulas for calculating the upper bound of rounding errors, absolute bounds of rounding errors, and probability estimates. The constructiveness of the method is that the obtained formulas are based on the use of automatic differentiation formulas from the algebra of differential tuples, which is used to solve the boundary value problem. This allows one to implement the process of solving the boundary value problem on the same calculations of derivatives and simultaneously take into account rounding errors, which optimizes the computational process.

References (translated):
1.   Masao Iri, Koichi Kubota, Kazuto Murota. Geometrical optimization and fast automatic differentiation // Yugoslav Journal of Operations research – 1991. Vol. 2 – 121-134 p.
2.   Huiskes M. Automatic Differentiation and Uncertainty Analysis // IIASA Interim Report. – 1998. – 43 p.